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SOFT β-OPEN SETS AND THEIR APPLICATIONS

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Abstract – First of all, we focused on soft β-open sets, soft β-closed sets, soft β-interior and soft β-closure over the soft topological space and investigated some properties of them. Secondly, we defined the concepts soft β-continuity, soft β-irresolute and soft β-homeomorphism on soft topological spaces. We also obtained some characterizations of these mappings. Finally, we observed that the collection $S\beta r\text{-}h(X,\tau,E)$ was a soft group.

Keywords – *Soft sets, Soft topology, Soft β-open sets, Soft β-interior, Soft β-closure, Soft β-continuity.*

1 Introduction

Molodtsov [14], in 1999, presented the soft theory as a new mathematical tool for tackling with ambiguities that known mathematical tools cannot hold. He has indicated a few aplications of soft theory for finding solutions to many practical problems such as economics, social science, engineering, medical science, etc.

Recently, papers about soft sets and their applications in various fields have increased largely. With a fixed number of parameters Shabir and Naz [15] came up with some notions of soft topological spaces defined on the initial universe . The authors defined soft open sets, soft interior, soft closed sets, soft closure, and soft seperation axioms. Chen [7] presented soft semi open sets and of the some related properties. With a fixed number of parameters Gunduz Aras et al. [4] came up with some soft continuous mappings defined on the initial universe. Mahanta and Das [12] presented and classified many forms of soft functions, such as irresolute, semicontinuous and semiopen soft functions. Arockiarani and Lancy [5] presented soft $g\beta$-closed and soft $gs\beta$-closed sets in soft topological spaces and with the aid of these presented sets they found out some properties.

In the present study, firstly, we focused soft β-open sets, soft β-closed sets, soft β-interior and soft β-closure over the soft topological space and investigated some properties of them. Secondly, we defined the concepts soft β-continuity, soft β-irresolute and soft β-homeomorphism on soft topological spaces. We also obtained some characterizations of these mappings. Finally, we observed that the collection $S\beta r\text{-}h(X,\tau,E)$ was a soft group.

This study is a part of correspoinding author's MSc thesis.

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2 Preliminary

Let U be an initial universe set and E be a collection of all probable parameters with respect to U. Here the parameters are characteristics or properties of objects in U. Let $P(U)$ denote the power set of U, and let $A \subseteq E$.

Definition 2.1. [14] A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \longrightarrow P(U)$. In other words, a soft set over U is a parametrized family of subsets of the universe U. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A).

Definition 2.2. [13] For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if (i) $A \subseteq B$, and (ii)$\forall e \in A$, $F(e) \subseteq G(e)$ are identical approximations. We write (F, A) $\widetilde{\subseteq}$ (G, B). (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A) \widetilde{\supseteq} (G, B)$.

Definition 2.3. [13] A soft set (F, A) over U is said to be

(*i*) null soft set denoted by Φ, if $\forall e \in A$, $F(e) = \phi$.
(*ii*) absolute soft set denoted by $\widetilde{A}$, if $\forall e \in A$, $F(e) = U$.

Definition 2.4. For two soft sets (F, A) and (G, B) over a common universe U,

(*i*) [13] union of two soft sets of (F, A) and (G, B) is the soft set (H, C), where $C = A \cup B$, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e) & , \text{ if } e \in A - B \\ G(e) & , \text{ if } e \in B - A \\ F(e) \cup G(e) & , \text{ if } e \in A \cap B \end{cases}$$

We write $(F, A) \,\widetilde{\cup}\, (G, B) = (H, C)$.

(*ii*) [9] intersection of (F, A) and (G, B) is the soft set (H, C), where $C = A \cap B$, and $\forall e \in C$, $H(e) = F(e) \cap G(e)$. We write $(F, A) \,\widetilde{\cap}\, (G, B) = (H, C)$.

Let X be an initial universe set and E be the non-empty set of parameters.

Definition 2.5. [15] Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ is read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$. Note that for any $x \in X$. $x \notin (F, E)$, if $x \notin F(e)$ for some $e \in E$.

Definition 2.6. [15] Let Y be a non-empty subset of X, then $\widetilde{Y}$ denotes the soft set (Y, E) over X for which $Y(e) = Y$, for all $e \in E$. In particular, (X, E), will be denoted by $\widetilde{X}$.

Definition 2.7. [3] The relative complement of a soft set (F, E) is denoted by $(F, E)'$and is defined by $(F, E)' = (F', E)$ where $F' : E \longrightarrow P(U)$ is a mapping given by $F'(e) = U - F(e)$ for all $e \in E$.

Definition 2.8. [15] Let τ be the collection of soft sets over X, then τ is said to be soft topology on X if

(1) Φ, $\widetilde{X}$ belong to τ
(2) the union of any number of soft sets in τ belongs to τ
(3) the intersection of any two soft sets in τ belongs to τ

The triplet (X, τ, E) is called a soft topological space over X. The members of τ are said to be soft open sets in X.

We will denote all soft open sets(resp. soft closed sets) in X as $S.O(X)$ (resp. $S.C(X)$).

Definition 2.9. [15] Let (X, τ, E) be a soft topological space over X. A soft set (F, E) over X is said to be a soft closed set in X, if its relative complement $(F, E)'$ belongs to τ.

Definition 2.10. Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then

a) soft interior[10] of the soft set (F, E) is denoted by $(F, E)^{\circ}$ and is defined as the union of all soft open sets contained in (F, E). Thus $(F, E)^{\circ}$ is the largest soft open set contained in (F, E).

b) soft closure[15] of (F, E), denoted by $\overline{(F, E)}$ is the intersection of all soft closed super sets of (F, E). Clearly $\overline{(F, E)}$ is the smallest soft closed set over X which contains (F, E).

We will denote interior(resp. closure) of the soft set (F, E) as $int\,(F, E)$ (resp.$cl\,(F, E)$).

Proposition 2.11. [10] Let (X, τ, E) be a soft topological space over X and (F, E) and (G, E) be a soft set over X. Then

a) $int\,(int\,(F, E)) = int\,(F, E)$
b) $(F, E) \tilde{\subseteq} (G, E)$ imples $int\,(F, E) \tilde{\subseteq} int(G, E)$
c) $cl\,(cl\,(F, E)) = cl\,(F, E)$
d) $(F, E) \tilde{\subseteq} (G, E)$ imples $cl\,(F, E) \tilde{\subseteq} cl(G, E)$

Definition 2.12. [6] Let (F, E) be a soft set X. The soft set (F, E) is called a soft point , denoted by (x_e, E) *or* x_e , if for the element $e \in E$, $F(e) = \{x\}$ and $F(e') = \phi$ for all $e' \in E - \{e\}$.

Definition 2.13. [18] The *soft point* x_e is said to belong to the soft set (G, E), denoted by $x_e \in (G, E)$, if for the element $e \in E$, $F(e) \tilde{\subseteq} G(e)$.

Definition 2.14. [18] A soft set (G, E) in a soft topological space (X, τ, E) is called a soft neighborhood of the soft point x_e if there exists an open soft set (H, E) such that $x_e \in (H, E) \tilde{\subseteq} (G, E)$. A soft set (G, E) in a soft topological space (X, τ, E) is called a soft neighborhoood of the soft set (F, E) if there exists an open soft set (H, E) such that $(F, E) \in (H, E) \tilde{\subseteq} (G, E)$.The neighborhood system of a soft point x_e, denoted by $N_{\tau}(x_e)$, is the family of all its neigborhoods.

Definition 2.15. [11] Let (X, τ, E) be a soft topological space. A soft point $x_e \in cl(F, E)$ if and only if each soft neighborhood of x_e intersects (F, E).

3 Soft β-open Sets and Soft β-closed Sets

Definition 3.1. A soft set (F, E) in a soft topological space (X, τ, E) is said to be

a) soft *semi*-open[7] if $(F, E) \tilde{\subseteq} cl(int(F, E))$.
b) soft *pre*-open[5] if $(F, E) \tilde{\subseteq} int(cl(F, E))$.
c) soft α-open if[5] if $(F, E) \tilde{\subseteq} int(cl(int(F, E)))$.
d) soft β-open (soft β-closed)[5] if $(F, E) \tilde{\subseteq} cl(int(cl(F, E)))$ $(int(cl(int(F, E))) \tilde{\subseteq} (F, E))$.
e) soft regular-open (soft regular-closed)[16] if $(F, E) = int(cl(F, E))$ $((F, E) = cl(int(F, E)))$

We will denote all the soft β-open (resp. soft *semi*-open, soft *pre*-open, soft α-open, soft β-closed, soft regular-open, soft regular-closed) sets in X as $S.\beta.O(X)$ (resp. $S.S.O(X)$, $S.P.O(X)$, $S.\alpha.O(X)$, $S.\beta.C(X)$, $S.R.O(X)$, $S.R.C(X)$).

Remark 3.2. It is clear that $S.\beta.O(X)$ contains each of $S.S.O(X)$, $S.P.O(X)$ and $S.\alpha.O(X)$, and the following diagram shows this fact.

$$\begin{array}{ccccc} soft\ open\ set & \longrightarrow & soft\ \alpha\text{-}open\ set & \longrightarrow & soft\ semi\text{-}open\ set \\ & & \downarrow & & \downarrow \\ & & soft\ pre\text{-}open\ set & \longrightarrow & soft\ \beta\text{-}open\ set \end{array}$$

The converses need not be true, in general, as show in the following examples.

Example 3.3. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), ..., (F_7, E)\}$ where $(F_1, E), (F_2, E), ..., (F_7, E)$ are soft sets over X, which is defined as follows: $F_1(e_1) = \{x_1, x_2\}$, $F_1(e_2) = \{x_1, x_2\}$, $F_2(e_1) = \{x_2\}$, $F_2(e_2) = \{x_1, x_3\}$, $F_3(e_1) = \{x_2, x_3\}$, $F_3(e_2) = \{x_1\}$, $F_4(e_1) = \{x_2\}$, $F_4(e_2) = \{x_1\}$, $F_5(e_1) = \{x_1, x_2\}$, $F_5(e_2) = X$, $F_6(e_1) = X$, $F_6(e_2) = \{x_1, x_2\}$, $F_7(e_1) = \{x_2, x_3\}$, $F_7(e_2) = \{x_1, x_3\}$ [7]. Then τ defines a soft topology on X and hence (X, τ, E) is a soft topological space over X. Now we give a soft set (H, E) in (X, τ, E) is defined as follows: $H(e_1) = \phi$, $H(e_2) = \{x_1\}$. Then, (H, E) is a soft *pre*-open set but not a soft α-open set, also it is a soft β-open set but not a soft *semi*-open set .

Example 3.4. Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, where (F_1, E), (F_2, E), (F_3, E), are soft sets over X, defined as follows. $F_1(e_1) = \{x_1, x_3\}$, $F_1(e_2) = \phi$, $F_2(e_1) = \{x_4\}$, $F_2(e_2) = \{x_4\}$, $F_3(e_1) = \{x_1, x_3, x_4\}$, $F_3(e_2) = \{x_4\}$. Then τ defines a soft topology on X. Hence (X, τ, E) is a soft topological space over X. Now we give two soft sets (H, E) and (K, E) in (X, τ, E) are defined as follows: $H(e_1) = \{x_2, x_3\}$, $H(e_2) = \{x_3\}$, $K(e_1) = \{x_2, x_4\}$, $K(e_2) = \{x_1, x_4\}$. Then (H, E) is a soft β-open set which is not soft *pre*-open and (K, E) is a soft *semi*-open set which is not soft α-open.

Theorem 3.5. (a) For every soft open set (F, E) in a soft topological space X and every $(G, E) \widetilde{\subseteq} X$ we have $(F, E) \widetilde{\cap}\, cl(G, E) \widetilde{\subseteq} cl((F, E) \widetilde{\cap} (G, E))$; (b) For every soft closed set (F, E) in a soft topological space X and every $(G, E) \widetilde{\subseteq} X$ we have $int((F, E) \widetilde{\cup} (G, E)) \widetilde{\subseteq} (F, E) \widetilde{\cup}\, int(G, E)$.

Proof. (a) Let x_e be a soft point on (X, τ, E). $x_e \in (F, E) \widetilde{\cap}\, cl(G, E) \Longrightarrow x_e \in (F, E)$ and $x_e \in cl(G, E)$. $x_e \in cl(G, E) \Longleftrightarrow \forall\ (K, E) \in N_\tau(x_e)$, $(K, E) \widetilde{\cap} (G, E) \neq \Phi$. Since $(K, E) \widetilde{\cap} (F, E) \in N_\tau(x_e)$, $(K, E) \widetilde{\cap} (F, E) \widetilde{\cap} (G, E) \neq \Phi$. Then, $x_e \in cl((F, E) \widetilde{\cap} (G, E))$.

(b) It can be proved by taking the complement of $(F, E) \widetilde{\cap}\, cl(G, E) \widetilde{\subseteq} cl((F, E) \widetilde{\cap} (G, E))$ in (a). □

Theorem 3.6. If (F, E) is soft open and (G, E) is soft β-open, then $(F, E) \widetilde{\cap} (G, E)$ is soft β-open.

Proof. Using Theorem 3.5(a) we obtain $(F, E) \widetilde{\cap} (G, E) \widetilde{\subseteq} (F, E) \widetilde{\cap}\, cl(int(cl\ (G, E))) \widetilde{\subseteq} cl[(F, E) \widetilde{\cap}\, int\ (cl(G, E))] = cl[int((F, E) \widetilde{\cap}\, cl(G, E))] \widetilde{\subseteq} cl[int[cl\ ((F, E) \widetilde{\cap} (G, E))]]$ which completes the proof. □

Theorem 3.7. If (F, E) is soft closed and (G, E) is soft β-closed, then $(F, E) \widetilde{\cup} (G, E)$ is soft β-closed.

Proof. Using Theorem 3.5(b) we obtain $int[cl[int((F, E) \widetilde{\cup} (G, E))]] \widetilde{\subseteq} int[cl\ ((F, E) \widetilde{\cup}\, int(G, E))] = int((F, E) \widetilde{\cup}\, cl(int(G, E))) \widetilde{\subseteq} (F, E) \widetilde{\cup}\, int(cl(int\ (G, E))) \widetilde{\subseteq} (F, E) \widetilde{\cup} (G, E)$ which completes the proof. □

Theorem 3.8. $S.S.O(X) \widetilde{\cup}\, S.P.O(X) \widetilde{\subseteq}\, S.\beta.O(X)$

Proof. Let $(F, E) \in S.S.O(X)$ and $(G, E) \in S.P.O(X)$. Then, $(F, E) \widetilde{\subseteq} cl(int(F, E)) \widetilde{\subseteq} cl(int(cl(F, E)))$ and $(G, E) \widetilde{\subseteq} int(cl(G, E)) \widetilde{\subseteq} cl(int(cl\ (G, E)))$. Therefore, $(F, E) \widetilde{\cup} (G, E) \widetilde{\subseteq} cl(int(cl(F, E))) \widetilde{\cup}\, cl(int(cl(G, E))) = cl[int(cl(F, E)) \widetilde{\cup}\, int(cl(G, E))] \widetilde{\subseteq} cl[int(cl(F, E) \widetilde{\cup}\, cl(G, E))] = cl[int[cl\ ((F, E) \widetilde{\cup} (G, E))]]$. □

Theorem 3.9. $S.S.C(X) \widetilde{\cup}\, S.P.C(X) \widetilde{\subseteq}\, S.\beta.C(X)$

Proof. Easy. □

Now we define the notion of soft supratopology is weaker than soft topology.

Definition 3.10. [17, 8] Let τ be the collection of soft sets over X, then τ is said to be soft supratopology on X if

(1) Φ, $\widetilde{X}$ belong to τ
(2) the union of any number of soft sets in τ belongs to τ

We give the following property for soft β-open sets.

Proposition 3.11. The collection $S.\beta.O(X)$ of all soft β-open sets of a space (X, τ, E) forms a soft supratopology.

Proof. (1) is obvious

(2) Let $(F_i, E) \in S.\beta.O(X)$ for $\forall i \in I = \{1, 2, 3.....\}$. Then, for $\forall i \in I$, $(F_i, E) \tilde{\subseteq} cl(int(cl(F_i, E))) \Longrightarrow \tilde{\bigcup}_{i\in I}(F_i, E) \tilde{\subseteq} \tilde{\bigcup}_{i\in I}(cl(int(cl(F_i, E)))) = cl(\tilde{\bigcup}_{i\in I} (int\ (cl(F_i, E)))) \tilde{\subseteq} cl(int(\tilde{\bigcup}_{i\in I}(cl(F_i, E)))) = cl(int(cl(\tilde{\bigcup}_{i\in I}(F_i, E))))$ □

The intersection of two soft β-open sets need not be a soft β-open set as is illustrated by the following example.

Example 3.12. Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where (F_1, E), $(F_2, E), (F_3, E)$ are soft sets over X, defined as follows. $F_1(e_1) = \{x_1\}$, $F_1(e_2) = \{x_2\}$, $F_2(e_1) = \{x_1, x_2\}$, $F_2(e_2) = \{x_2\}$, $F_3(e_1) = \{x_1\}$, $F_3(e_2) = \{x_1, x_2\}$. Then τ defines a soft topology on X and hence (X, τ, E) is a soft topological space over X. Now we give two soft sets (G, E), (H, E) in (X, τ, E) which are defined as follows: $G(e_1) = \{x_2\}$, $G(e_2) = \{x_2\}$, $H(e_1) = \{x_1, x_2\}$, $H(e_2) = \{x_1\}$. Then, (G, E) and (H, E) are soft β-open sets over X, therefore, $(G, E) \tilde{\cap} (H, E) = \{\{x_2\}, \phi\}$ and $cl(int(cl((G, E) \tilde{\cap} (H, E)))) = \Phi$. Hence, $(G, E) \tilde{\cap} (H, E)$ is not a soft β-open set.

We have the following proposition by using relative complements.

Proposition 3.13. Arbitrary intersection of soft β-closed sets is soft β-closed.

Proof. Let $(F_i, E) \in S.\beta.C(X)$ for $\forall i \in I = \{1, 2, 3.....\}$. Then, for $\forall i \in I$, $(F_i, E) \tilde{\supseteq} int(cl(int(F_i, E))) \Longrightarrow \tilde{\bigcap}_{i\in I}(F_i, E) \tilde{\supseteq} \tilde{\bigcap}_{i\in I}(int(cl(int(F_i, E)))) = int(\tilde{\bigcap}_{i\in I}(cl(int(F_i, E)))) \tilde{\supseteq} int(cl(\tilde{\bigcap}_{i\in I}(int(F_i, E)))) = int(cl(int (\tilde{\bigcap}_{i\in I}(F_i, E))))$. The union of two soft β-closed sets need not be soft β-closed set as is illustrated by the following example. □

Example 3.14. Let (X, τ, E) be as in Example 3.12. Now we give two soft sets (G, E), (H, E) in (X, τ, E) which are defined as follows: $G(e_1) = \{x_1\}$, $G(e_2) = \{x_1\}$, $H(e_1) = \phi$, $H(e_2) = \{x_2\}$. Then, (G, E) and (H, E) are soft β-closed sets over X , therefore, $(G, E) \tilde{\cup} (H, E) = \{\{x_1\}, \{x_1, x_2\}\}$ and $int(cl(int((G, E) \tilde{\cup} (H, E)))) = \widetilde{X}$. Hence, $(G, E) \tilde{\cup} (H, E)$ is not a soft β-closed set.

Theorem 3.15. For any soft set (F, E) of a soft topological space X the following conditions are equivalent: (a) $(F, E) \in S.\beta.O(X)$ (b) $cl(F, E) \in S.R.C(X)$.

Proof. $(a) \to (b)$ Let (F, E) be a soft β-open set. Then $(F, E) \tilde{\subseteq} cl(int(cl\ (F, E)))$. This implies $cl(F, E) = cl(int(cl(F, E)))$ that is $cl(F, E) \in S.R.C\ (X)$. $(b) \to (a)$ is obvious. □

Theorem 3.16. For any soft set (F, E) of a soft topological space X the following conditions are equivalent: (a) $(F, E) \in S.\beta.C(X)$ (b) $int(F, E) \in S.R.O(X)$.

Theorem 3.17. Each soft β-open set which is soft semi-closed is soft semi-open .

Proof. $(F, E) \in S.\beta.O(X) \Longrightarrow (F, E) \tilde{\subseteq} cl(int(cl(F, E)))$ and $(F, E) \in S.S.C(X) \Longrightarrow int(cl(F, E)) \tilde{\subseteq} (F, E)$. Then $int(cl(F, E)) \tilde{\subseteq} (F, E) \tilde{\subseteq} cl(int\ (cl(F, E)))$. Since $int(cl(F, E)) = (U, E)$ is a soft open set, we can write $(U, E) \tilde{\subseteq} (F, E) \tilde{\subseteq} cl(U, E)$. Hence (F, E) is a soft semi-open set. □

Corollary 3.18. If a soft set (F, E) in a soft topological space (X, τ, E) is soft β-closed and soft *semi*-open, then (F, E) is soft *semi*-closed.

Theorem 3.19. If (F, E) is soft α-open and (G, E) is soft β-open then $(F, E) \tilde{\cap} (G, E)$ is soft β-open.

Proof. $(F, E) \tilde{\cap} (G, E) \tilde{\subseteq} int(cl(int(F, E))) \tilde{\cap} cl(int(cl(G, E))) \tilde{\subseteq} cl[int(cl\ (int(F, E))) \tilde{\cap} int(cl(G, E))] = cl[int[cl(int(F, E)) \tilde{\cap} int(cl(G, E))]] \tilde{\subseteq} cl[int[\ cl[int(F, E) \tilde{\cap} int(cl(G, E))]]] = cl[int[int(F, E) \tilde{\cap} cl(G, E)]] \tilde{\subseteq} cl[int[cl[int\ (F, E) \tilde{\cap} (G, E)]]] \tilde{\subseteq} cl[int[cl[(F, E) \tilde{\cap} (G, E)]]]$. □

Corollary 3.20. If (F, E) is soft α-closed and (G, E) is soft β-closed then $(F, E) \tilde{\cup} (G, E)$ is soft β-closed.

Proposition 3.21. In an indiscrete soft topological space (X, τ, E), each soft β-open is soft *pre*-open.

Proof. If $(F,E) = \Phi$, then (F,E) is soft β-open and soft *pre*-open. Let $(F,E) \neq \Phi$, then, $(F,E) \in S.\beta.O(X) \Longrightarrow (F,E) \tilde{\subseteq} cl(int(cl(F,E))) = \tilde{X} = (int(cl(F,E))$. Hence (F,E) is soft *pre*-open. □

Theorem 3.22. A soft set (F,E) in a soft topological space (X,τ,E) is soft β-closed if and only if $cl(\tilde{X} - cl(int(F,E))) - (\tilde{X} - cl(F,E)) \tilde{\supseteq} cl(F,E) - (F,E)$.

Proof. $cl(\tilde{X}-cl(int(F,E)))-(\tilde{X}-cl(F,E)) \tilde{\supseteq} cl(F,E)-(F,E) \Longleftrightarrow (\tilde{X}-int(cl(int(F,E))))-(\tilde{X}-cl(F,E)) \tilde{\supseteq} cl(F,E)-(F,E) \Longleftrightarrow (\tilde{X}-int(cl(int\ (F,E)))) \tilde{\cap} cl(F,E) \tilde{\supseteq} cl(F,E)-(F,E) \Longleftrightarrow (\tilde{X} \tilde{\cap} cl(F,E))-[int(cl(int\ (F,E))) \tilde{\cap} cl(F,E)] \tilde{\supseteq} cl(F,E) - (F,E) \Longleftrightarrow cl(F,E) - int(cl(int(F,E))) \tilde{\supseteq} cl(F,E) - (F,E) \Longleftrightarrow (F,E) \tilde{\supseteq} int(cl(int(F,E))) \Longleftrightarrow (F,E)$ is soft β-closed. □

Theorem 3.23. Each soft β-open and soft α-closed set is soft closed.

Proof. Let $(F,E) \in S.\beta.O(X), (F,E) \tilde{\subseteq} cl(int(cl(F,E)))$, since (F,E) is soft α-closed $cl(int(cl(F,E))) \tilde{\subseteq} (F,E)$, then $cl(int(cl(F,E))) \tilde{\subseteq} (F,E) \tilde{\subseteq} cl(int(cl(F,E))), (F,E) = cl(int(cl(F,E)))$ which is soft closed. □

Corollary 3.24. Each soft β-closed and soft α-open set is soft open.

Definition 3.25. [2]Let (F,E) be a soft subset of (X,τ,E) then the soft beta-closure of (F,E), denoted by $S\beta cl(F,E)$, is the soft intersection of all soft β-closed subsets of X containing (F,E).

Theorem 3.26. Let (F,E) be a soft subset of X. Then $S\beta cl(F,E) = (F,E) \tilde{\cup} int(cl(int(F,E)))$.

Proof. We observe that $int[cl[int[(F,E) \tilde{\cup} int(cl(int(F,E)))]]] \tilde{\subseteq} int[cl[int\ [(F,E) \tilde{\cup} cl(int(F,E))]]] \tilde{\subseteq} int[cl\ [int(F,E) \tilde{\cup} cl(int(F,E))]] = int[cl(int\ (F,E)) \tilde{\cup} cl(int(F,E)] = int(cl(int(F,E))) \tilde{\subseteq} (F,E) \tilde{\cup} int(cl(int\ (F,E)))$. Hence $(F,E) \tilde{\cup} int(cl(int(F,E)))$ is soft β-closed and thus $S\beta cl(F,E) \tilde{\subseteq} (F,E) \tilde{\cup} int(cl(int(F,E)))$. On the other hand, since $S\beta cl(F,E)$ is soft β-closed, we have $int(cl(int(F,E))) \tilde{\subseteq} int(cl(int(S\beta cl(F,E))))$ $\tilde{\subseteq} S\beta cl(F,E)$ and hence $(F,E) \tilde{\cup} int(cl(int(F,E))) \tilde{\subseteq} S\beta cl(F,E)$. □

Definition 3.27. [2]Let (F,E) be a soft subset of (X,τ,E) then the soft beta-interior of (F,E), denoted by $S\beta int(F,E)$, is the soft union of all soft β-open subsets of X contained in (F,E).

Theorem 3.28. Let (F,E) be a soft subset of X. Then $S\beta int(F,E) = (F,E) \tilde{\cap} cl(int(cl(F,E)))$.

Proof. We observe that $(F,E) \tilde{\cap} cl(int(cl(F,E))) \tilde{\subseteq} cl(int(cl(F,E))) = cl[int[cl(F,E) \tilde{\cap} int(cl(F,E))]] \tilde{\subseteq} cl[int[cl[(F,E) \tilde{\cap} int(cl(F,E))]]] \tilde{\subseteq} cl[int[cl\ [(F,E) \tilde{\cap} cl(int(cl(F,E)))]]]$. Hence $(F,E) \tilde{\cap} cl(int(cl(F,E)))$ is soft β-open and thus $(F,E) \tilde{\cap} cl(int(cl(F,E))) \tilde{\subseteq} S\beta int(F,E)$. On the other hand, since $S\beta int(F,E)$ is soft β-open, we have $S\beta int(F,E) \tilde{\subseteq} cl(int(cl(S\beta int\ (F,E)))) \tilde{\subseteq} cl(int(cl(F,E)))$ and hence $S\beta int(F,E) \tilde{\subseteq} (F,E)$ $\tilde{\cap}\ cl(int(cl(F,E)))$. □

Corollary 3.29. (a) $S\beta int((F,E)') = (S\beta cl(F,E))'$ (b) $S\beta cl((F,E)') = (S\beta int(F,E))'$

The following theorem is an easy consequence of the definitions of soft α-open and soft β-open sets.

Theorem 3.30. *a*) $(F,E) \in S.\alpha.O(X)$ if and only if $S\beta cl(F,E) = int(cl(int\ (F,E)))$, *b*) $(F,E) \in S.\alpha.C(X)$ if and only if $S\beta int(F,E) = cl(int(cl(F,E)))$.

Proof. $(a) \Longrightarrow$ Let $(F,E) \in S.\alpha.O(X)$, then $(F,E) \tilde{\subseteq} int(cl(int(F,E)))$. $S\beta cl(F,E) = (F,E) \tilde{\cup} int(cl(int\ (F,E))) = int(cl(int(F,E)))$.

$\Longleftarrow S\beta cl(F,E) = int(cl(int(F,E))) = (F,E) \tilde{\cup} int(cl(int(F,E)))$, then $(F,E) \tilde{\subseteq} int(cl(int(F,E)))$.

(b) Easy □

Theorem 3.31. Let (F,E) be a soft subset of X. Then $S\beta int(S\beta cl(F,E)) = S\beta cl(S\beta int(F,E))$.

Proof. We have $S\beta int(S\beta cl(F,E)) = S\beta cl(F,E) \tilde{\cap} cl(int(cl(S\beta cl(F,E)))) = [(F,E) \tilde{\cup} int(cl(int(F,E)))] \tilde{\cap} cl[int[cl[(F,E) \tilde{\cup} int(cl(int(F,E)))]]] = [(F,E) \tilde{\cup} int(cl(int(F,E)))] \tilde{\cap} cl(int(cl(F,E))) = [(F,E) \tilde{\cap} cl(int(cl\ (F,E)))] \tilde{\cup} [int(cl(int(F,E))) \tilde{\cap} cl(int(cl(F,E)))] = [(F,E) \tilde{\cap} cl(int(cl(F,E)))] \tilde{\cup} int(cl(int(F,E))) = [(F,E) \tilde{\cap} cl(int(cl(F,E)))] \tilde{\cup} int[cl[int[(F,E) \tilde{\cap} cl(int(cl\ (F,E)))]]] = S\beta int(F,E) \tilde{\cup} int(cl(int(S\beta int(F,E)))) = S\beta cl(S\beta int(F,E))$ □

Corollary 3.32. (a) $(F,E)\,\tilde{\cup}\, S\beta int(S\beta cl(F,E)) = S\beta cl(F,E)$ (b) $(F,E)\,\tilde{\cap}\, S\beta int(S\beta cl(F,E)) = S\beta int(F,E)$

Proof. (a) $(F,E)\,\tilde{\cup}\, S\beta int(S\beta cl(F,E)) = (F,E)\,\tilde{\cup}\, [S\beta cl(F,E)\,\tilde{\cap}\, cl(int(cl\ (S\beta cl(F,E))))] = (F,E)\,\tilde{\cup}\, [[(F,E)\,\tilde{\cup}\, int(cl(int(F,E)))]\,\tilde{\cap}\, cl[int[cl[(F,E)\,\tilde{\cup}\, int(cl(int(F,E)))]]]] = (F,E)\,\tilde{\cup}\, [[(F,E)\,\tilde{\cup}\, int(cl(int(F,E)))]\,\tilde{\cap}\, cl(int(cl\ (F,E)))] = [(F,E)\,\tilde{\cup}\, int(cl(int(F,E)))]\,\tilde{\cap}\, [(F,E)\,\tilde{\cup}\, cl(int(cl(F,E)))] = [(F,E)\,\tilde{\cup}\, int(cl(int(F,E)))] = S\beta cl(F,E)$

(b) Easy ☐

Theorem 3.33. For any soft subset (F,E) of a soft topological space X the following conditions are equivalent: (a) $(F,E) \in S.\beta.O(X)$ (b) $(F,E)\,\tilde{\subseteq}\, S\beta int\,[\, S\beta cl(F,E)]$.

Proof. (a) →(b) Let $(F,E) \in S.\beta.O(X)$. Then $(F,E)\,\tilde{\subseteq}\, cl(int(cl(F,E)))$. $S\beta int(S\beta cl(F,E)) = S\beta cl(F,E)\,\tilde{\cap}\, cl(int(cl(S\beta cl(F,E)))) = [(F,E)\,\tilde{\cup}\, int(cl\ (int(F,E)))]\,\tilde{\cap}\, cl[int[cl[(F,E)\,\tilde{\cup}\, int(cl(int(F,E)))]]] = [(F,E)\,\tilde{\cup}\, int(cl(int\ (F,E)))\,\tilde{\cap}\, cl(int(cl(F,E)))] = [(F,E)\,\tilde{\cap}\, cl(int(cl(F,E)))]\,\tilde{\cup}\, [int(cl(int(F,E)\ ))\,\tilde{\cap}\, cl(int(cl(F,E)))] = (F,E)\,\tilde{\cup}\, int(cl(int(F,E)))\,\tilde{\supseteq}\, (F,E)$.

(b) →(a) $(F,E)\,\tilde{\subseteq}\, S\beta int\,[\, S\beta cl(F,E)] = S\beta cl(F,E)\,\tilde{\cap}\, cl(int(cl(S\beta cl\ (F,E)))) = [(F,E)\,\tilde{\cup}\, int(cl(int\ (F,E)))]\,\tilde{\cap}\, cl[int[cl[(F,E)\,\tilde{\cup}\, int(cl(int(F,E)))\ ]]] = [(F,E)\,\tilde{\cup}\, int(cl(int(F,E)))]\,\tilde{\cap}\, cl(int(cl(F,E)))$. Hence $(F,E)\,\tilde{\subseteq}\, cl(int(cl\ (F,E)))$. ☐

3.1 Soft β-continuous Mappings

We define the notion of soft β-continuity by using soft β-open sets.

Definition 3.34. Let (X,τ,E) and (Y,τ',E) be two soft topological spaces. A function $f : (X,\tau,E) \longrightarrow (Y,\tau',E)$ is said to be

a) soft *semi*-continuons[12] if $f^{-1}((G,E))$ is soft *semi*-open in (X,τ,E), for every soft open set (G,E) of (Y,τ',E).

b) soft *pre*-continuons[1] if $f^{-1}((G,E))$ is soft *pre*-open in (X,τ,E), for every soft open set (G,E) of (Y,τ',E).

c) soft α-continuons if[1] $f^{-1}((G,E))$ is soft α-open in (X,τ,E), for every soft open set (G,E) of (Y,τ',E).

d) soft β-continuons if $f^{-1}((G,E))$ is soft β-open in (X,τ,E), for every soft open set (G,E) of (Y,τ',E).

e) soft β-irresolute if $f^{-1}((G,E))$ is soft β-open in (X,τ,E), for every soft β-open set (G,E) of (Y,τ',E).

It is clear that the class of soft β-continuity contains each of classes soft *semi*-continuous and soft *pre*-continuous, the implications between them and other types of soft continuities are given by the following diagram.

$$\begin{array}{ccccc} soft\ continuity & \longrightarrow & soft\ \alpha\text{-}continuity & \longrightarrow & soft\ semi\text{-}continuity \\ & & \downarrow & & \downarrow \\ & & soft\ pre\text{-}continuity & \longrightarrow & soft\ \beta\text{-}continuity \end{array}$$

The converses of these implications do not hold, in general, as show in the following examples.

Example 3.35. Let $X = Y = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and let the soft topology on X be soft indiscrete and on Y be soft discrete. If we get the mapping $f : (X,\tau,E) \longrightarrow (Y,\tau',E)$ defined as $f(x_1) = x_2$, $f(x_2) = x_1$, $f(x_3) = x_3$ then f is soft β-continuous but not soft *semi*-continuous.

Example 3.36. Let $X = Y = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$. Then $\tau = \{\Phi, \widetilde{X}, (F_1,E), (F_2,E), (F_3,E)\}$ is a soft topological space over X and $\tau' = \{\Phi, \widetilde{Y}, (G_1,E), (G_2,E)\}$ is a soft topological space over Y. Here (F_1,E), $(F_2,E), (F_3,E)$ are soft sets over X and $(G_1,E), (G_2,E)$ are soft sets over Y,defined as follows: $F_1(e_1) = \{x_1\}$, $F_1(e_2) = \{x_1\}$, $F_2(e_1) = \{x_2\}$, $F_2(e_2) = \{x_2\}$, $F_3(e_1) = \{x_1, x_2\}$, $F_3(e_2) = \{x_1, x_2\}$ and $G_1(e_1) = \{x_1\}$, $G_1(e_2) = \{x_1\}$, $G_2(e_1) = \{x_1, x_2\}$, $G_2(e_2) = \{x_1, x_2\}$.

If we get the mapping $f : (X, \tau, E) \longrightarrow (Y, \tau', E)$ defined as $f(x_1) = x_1, \quad f(x_2) = x_3, \quad f(x_3) = x_2$ then f is soft β-continuous but not soft *pre*-continuous, since $f^{-1}(G_2) = \{\{x_1, x_3\}, \{x_1, x_3\}\}$ is not a soft *pre*-open set over X.

We give some characterizations of soft β-continuity.

Theorem 3.37. Let $f : (X, \tau, E) \longrightarrow (Y, \tau', E)$ be a soft mapping, then the following statements are equivalent.

a) f is soft β-continuous.

b) For each soft point (x_e, E) over X and each soft open (G, E) containing $f(x_e, E) = (f(x)_e, E)$ over Y, there exists a soft β-open set (F, E) over X containing (x_e, E) such that $f(F, E) \tilde{\subseteq} (G, E)$.

c) The inverse image of each soft closed set in Y is soft β-closed in X.

d) $int(cl(int(f^{-1}(G, E)))) \tilde{\subseteq} f^{-1}(cl(G, E))$ for each soft set (G, E) over Y.

e) $f(int(cl(int(F, E)))) \tilde{\subseteq} cl(f(F, E))$ for each soft set (F, E) over X.

Proof. $(a) \Longrightarrow (b)$ Since $(G, E) \tilde{\subseteq} Y$ containing $f(x_e, E) = (f(x)_e, E)$ is soft open, then $f^{-1}(G, E) \in S.\beta.O(X)$. Soft set $(F, E) = f^{-1}(G, E)$ which contains (x_e, E), therefore $f(F, E) \tilde{\subseteq} (G, E)$.

$(a) \Longrightarrow (c)$ Let $(G, E) \in S.C(Y)$,then $(\tilde{Y} - (G, E)) \in S.O(Y)$.Since f is soft β-continuous, $f^{-1}(\tilde{Y} - (G, E)) \in S.\beta.O(X)$. Hence $[\tilde{X} - f^{-1}(G, E)] \in S.\beta.O(X)$. Then $f^{-1}(G, E) \in S.\beta.C(X)$

$(c) \Longrightarrow (d)$ Let (G, E) be a soft set over Y, then $f^{-1}(cl(G, E)) \in S.\beta.C(X)$. $f^{-1}(cl(G, E)) \tilde{\supseteq} int(cl(int (f^{-1}(cl(G, E))))) \tilde{\supseteq} int(cl(int(f^{-1}(G, E))))$

$(d) \Longrightarrow (e)$ Let (F, E) be a soft set over X and $f(F, E) = (G, E)$. Then, according to (d) $int(cl(int (f^{-1}(f(F, E))))) \tilde{\subseteq} f^{-1}(cl(f(F, E)) \Longrightarrow int(cl(int(F, E))) \tilde{\subseteq} f^{-1}(cl(f(F, E)) \Longrightarrow f(int(cl(int(F, E)))) \tilde{\subseteq} cl(f(F, E))$

$(e) \Longrightarrow (a)$ Let $(G, E) \in S.O(Y)$, $(H, E) = \tilde{Y} - (G, E)$ and $(F, E) = f^{-1}(H, E)$, by (e) $f(int(cl(int (f^{-1}(H, E))))) \tilde{\subseteq} cl(f(f^{-1}(H, E))) \tilde{\subseteq} cl(H, E) = (H, E)$, so $int(cl(int(f^{-1}(H, E)))) \tilde{\subseteq} f^{-1}(H, E)$. Then $f^{-1}(H, E) \in S.\beta.C(X)$, thus (by (c)) f is soft β-continuous. □

Remark 3.38. The composition of two soft β-continuous mappings need not be soft β-continuous, in general, as shown by the following example.

Example 3.39. Let $X = Z = \{x_1, x_2, x_3\}$, $Y = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2\}$. Then $\tau = \{\Phi, \tilde{X}, (F, E)\}$ is a soft topological space over X, $\tau' = \{\Phi, \tilde{Y}, (G, E)\}$ is a soft topological space over Y and $\tau'' = \{\Phi, \tilde{Z}, (H_1, E), (H_2, E)\}$ is a soft topological space over Z. Here (F, E) is a soft set over X, (G, E) is a soft set over Y and $(H_1, E), (H_2, E)$ are soft sets over Z defined as follows: $F(e_1) = \{x_1\}$, $F(e_2) = \{x_1\}$, $G(e_1) = \{x_1, x_3\}$, $G(e_2) = \{x_1, x_3\}$, $H_1(e_1) = \{x_3\}$, $H_1(e_2) = \{x_3\}$, $H_2(e_1) = \{x_1, x_2\}$, $H_2(e_2) = \{x_1, x_2\}$.

If we get the identity mapping $I : (X, \tau, E) \longrightarrow (Y, \tau', E)$ and $f : (Y, \tau', E) \longrightarrow (Z, \tau'', E)$ defined as $f(x_1) = x_1$, $f(x_2) = f(x_4) = x_2$, $f(x_3) = x_3$. It is clear that each of I and f is soft β-continuous but $f \ o \ I$ is not soft β-continuous, since $(f \ o \ I)^{-1}(H_1, E) = \{\{x_3\}, \{x_3\}\}$ is not a soft β-open set over X.

Definition 3.40. A function $f : (X, \tau, E) \longrightarrow (Y, \tau', E)$ is called a soft β-homeomorphism (resp.soft βr-homeomorphism) if f is a soft β-continuous bijection (resp. sorf β-irresolute bijection) and $f^{-1} : (Y, \tau', E) \longrightarrow (X, \tau, E)$ is a soft β-continuous (soft β-irresolute).

Now we can give the following definition by taking the soft space (X, τ, E) instead of the soft space (Y, τ', E).

Definition 3.41. For a soft topological space (X, τ, E), we define the following two collections of functions:

$S\beta$-$h(X, \tau, E) = \{f \mid f : (X, \tau, E) \longrightarrow (X, \tau, E)$ is a softβ-continuous bijection, $f^{-1} : (X, \tau, E) \longrightarrow (X, \tau, E)$ is softβ-continuous$\}$

$S\beta r$-$h(X, \tau, E) = \{f \mid f : (X, \tau, E) \longrightarrow (X, \tau, E)$ is a softβ-irresolute bijection, $f^{-1} : (X, \tau, E) \longrightarrow (X, \tau, E)$ is softβ-irresolute$\}$

Theorem 3.42. For a soft topological space (X, τ, E), $S\text{-}h(X, \tau, E) \tilde{\subseteq} S\beta r\text{-}h(X, \tau, E) \tilde{\subseteq} S\beta\text{-}h(X, \tau, E)$, where $S\text{-}h(X, \tau, E) = \{f \mid f : (X, \tau, E) \longrightarrow (X, \tau, E)$ is a soft-homeomorphism$\}$.

Proof. First we show that every soft-homeomorphism $f : (X, \tau, E) \longrightarrow (Y, \tau', E)$ is a soft βr-homeomorphism. Let $(G, E) \in S.\beta.O(Y)$, then $(G, E) \tilde{\subseteq} cl(int(cl(G, E)))$. Hence, $f^{-1}((G, E)) \tilde{\subseteq} f^{-1}(cl(int(cl(G, E)))) = cl(int(cl\ (f^{-1}(G, E))))$ and so $f^{-1}((G, E)) \in S.\beta.O(X)$. Thus, f is soft β-irresolute. In a similar way, it is shown that f^{-1} is soft β-irresolute. Hence, we have that $S\text{-}h(X, \tau, E) \tilde{\subseteq} S\beta r\text{-}h(X, \tau, E)$.

Finally, it is obvious that $S\beta r\text{-}h(X, \tau, E) \tilde{\subseteq} S\beta\text{-}h(X, \tau, E)$, because every soft β-irresolute function is soft β-continuous. □

Theorem 3.43. For a soft topological space (X, τ, E), the collection $S\beta r\text{-}h(X, \tau, E)$ forms a group under the composition of functions.

Proof. If $f : (X, \tau, E) \longrightarrow (Y, \tau', E)$ and $g : (Y, \tau', E) \longrightarrow (Z, \tau'', E)$ are soft βr-homeomorphism, then their composition $gof : (X, \tau, E) \longrightarrow (Z, \tau'', E)$ is a soft βr-homeomorphism. It is obvious that for a bijective soft βr-homeomorphism $f : (X, \tau, E) \longrightarrow (Y, \tau', E)$, $f^{-1} : (Y, \tau', E) \longrightarrow (X, \tau, E)$ is also a soft βr-homeomorphism and the identity $1 : (X, \tau, E) \longrightarrow (X, \tau, E)$ is a soft βr-homeomorphism. A binary operation $\alpha : S\beta r\text{-}h(X, \tau, E) \times S\beta r\text{-}h(X, \tau, E) \longrightarrow S\beta r\text{-}h(X, \tau, E)$ is well defined by $\alpha(a, b) = boa$, where $a, b \in S\beta r\text{-}h(X, \tau, E)$ and boa is the composition of a and b. By using the above properties, the set $S\beta r\text{-}h(X, \tau, E)$ forms a group under composition of functions. □

Theorem 3.44. The group $S\text{-}h(X, \tau, E)$ of all soft homeomorphisms on (X, τ, E) is a subgroup of $S\beta r\text{-}h(X, \tau, E)$.

Proof. For any $a, b \in S\text{-}h(X, \tau, E)$, we have $\alpha(a, b^{-1}) = b^{-1}o\ a \in S\text{-}h(X, \tau, E)$ and $1_X \in S\text{-}h(X, \tau, E) \neq \varnothing$. Thus, using (Theorem 4.10) and (Theorem 4.11), it is obvious that the group $S\text{-}h(X, \tau, E)$ is a subgroup of $S\beta r\text{-}h(X, \tau, E)$. □

For a soft topological space (X, τ, E), we can construct a new group $S\beta r\text{-}h(X, \tau, E)$ satisfying the property: if there exists a homeomorphism $(X, \tau, E) \cong (Y, \tau', E)$, then there exists a group isomorphism $S\beta r\text{-}h(X, \tau, E) \cong S\beta r\text{-}h(X, \tau, E)$.

Corollary 3.45. Let $f : (X, \tau, E) \longrightarrow (Y, \tau', E)$ and $g : (Y, \tau', E) \longrightarrow (Z, \tau'', E)$ be two functions between soft topological spaces.

a) For a soft βr-homeomorphism $f : (X, \tau, E) \longrightarrow (Y, \tau', E)$, there exists an isomorphism, say $f_* : S\beta r\text{-}h(X, \tau, E) \longrightarrow S\beta r\text{-}h(X, \tau, E)$, defined $f_*(a) = f\ o\ a\ o\ f^{-1}$, for any element $a \in S\beta r\text{-}h(X, \tau, E)$.
b) For two soft βr-homeomorphisms $f : (X, \tau, E) \longrightarrow (Y, \tau', E)$ and $g : (Y, \tau', E) \longrightarrow (Z, \tau'', E)$, $(gof)_* = g_* o\ f_* : S\beta r\text{-}h(X, \tau, E) \longrightarrow S\beta r\text{-}h(Z, \tau'', E)$ holds.
c) For the identity function $1_X : (X, \tau, E) \longrightarrow (X, \tau, E)$, $(1_X)_* = 1 : S\beta r\text{-}h(X, \tau, E) \longrightarrow S\beta r\text{-}h(X, \tau, E)$ holds where 1 denotes the identity isomorphism.

Proof. Straightforward . □

4 Conclusion

We obtain some properties of two operators called soft β-interior and soft β-closure. Besides, in soft topological spaces, two new varieties of continuity via soft soft β-open and soft β-homeomorphism with soft β-irresolute homeomorphism are defined and given some characterizations of these notions. Of course, the most important the family of soft β-irresolute homeomorphism was a soft group. Therefore, one can say that this paper is applying to algebra.

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