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DECOMPOSITIONS OF TOPOLOGICAL FUNCTIONS

Otchana Thevar Ravi[1,*] <siingam@yahoo.com>
Subramanian Jeyashri[2] <jeyabala1@yahoo.com>
Stanis Laus Pious Missier[3] <spmissier@yahoo.com>

[1]Department of Mathematics, P. M. Thevar College, Usilampatti, Madurai Dt, Tamil Nadu, India.
[2]Department of Mathematics, Mother Teresa Women's University, Kodaikanal, Tamil Nadu, India
[3]Department of Mathematics, V. O. Chidambaram College, Thoothukudi, Tamil Nadu, India.

Abstaract − We obtain new classes of sets by using λ-closed sets in topological spaces and study their basic properties; and their connections with other kind of topological sets. Moreover new decompositions of topological functions are obtained.

Keywords − *λ-α-closed set, λ-s-closed set, λ-p-closed set, λ-β-closed set, λ-b-closed set.*

1 Introduction

In 1986, Maki [24] introduced the notion of Λ-sets in topological spaces. A Λ-set is a set A which is equal to its kernel (= saturated set) i.e to the intersection of all open supersets of A. Arenas et al. [4] introduced and investigated the notion of λ-closed sets by involving Λ-sets and closed sets. In 1965, Njastad [29] introduced α-open sets which have been considered as an important research tool in the field of topology.

In this paper, we introduce generalized λ-closed sets in topological spaces. In Section 3, we obtain characterizations of generalized λ-closed sets. In Section 4, we obtain some decompositions of topological functions.

2 Preliminaries

Throughout this paper (X, τ) and (Y, σ) (or X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure of A and the interior of A respectively.

We recall the following definitions and remark which are useful in the sequel.

Definition 2.1. *A subset A of a topological space (X, τ) is called*

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* *Corresponding Author.*

1. *α-open [29] if* $A \subseteq int(cl(int(A)))$;
2. *preopen [26] if* $A \subseteq int(cl(A))$;
3. *semi-open [22] if* $A \subseteq cl(int(A))$;
4. *β-open [1] if* $A \subseteq cl(int(cl(A)))$;
5. *b-open [3] if* $A \subseteq int(cl(A)) \cup cl(int(A))$;

The complements of the above mentioned open sets are called their respective closed sets.

The collection of all α-open (resp. semi-open, preopen, β-open, b-open) sets is denoted by $\alpha O(X)$ *(resp.* $SO(X)$, $PO(X)$, $\beta O(X)$, $bO(X)$*).*

The preclosure [31] (resp. semi-closure [14], α-closure [27], β-closure [1], b-closure [3]) of a subset A of X, denoted by pcl(A) (resp. scl(A), $\alpha cl(A)$, $\beta cl(A)$, *bcl(A)), is defined to be the intersection of all preclosed (resp. semi-closed, α-closed, β-closed, b-closed) sets of* (X, τ) *containing A. It is known that pcl(A) (resp. scl(A),* $\alpha cl(A)$, $\beta cl(A)$, *bcl(A)) is a preclosed (resp. semi-closed, α-closed, β-closed, b-closed) set.*

Definition 2.2. *A subset A of a topological space* (X, τ) *is called*

1. *generalized closed (briefly g-closed) [23] if* $cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open.*
2. *α-generalized closed (briefly αg-closed) [25] if* $\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open.*
3. *a generalized semiclosed (briefly gs-closed) [7] if* $scl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open.*
4. *a generalized preclosed (briefly gp-closed) [8] if* $pcl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open.*
5. *a generalized semi-preclosed (briefly gsp-closed) [15] if* $\beta cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open.*
6. *a generalized b-closed (briefly gb-closed) [17] if* $bcl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open.*

The complements of the above mentioned closed sets are called their respective open sets.

Definition 2.3. *A subset A of a topological space* (X, τ) *is called*

1. *Λ-set if* $A = A^{\wedge}$ *where* $A^{\wedge} = \cap \{G : A \subseteq G,\ G \in \tau\}$ *[24].*
2. Λ_α*-set if* $A = \Lambda_\alpha(A)$ *where* $\Lambda_\alpha(A) = \cap \{G : A \subseteq G,\ G \in \alpha O(X)\}$*[13].*
3. Λ_s*-set if* $A = \Lambda_s(A)$ *where* $\Lambda_s(A) = \cap \{G : A \subseteq G,\ G \in SO(X)\}$*[12].*
4. Λ_p*-set if* $A = \Lambda_p(A)$ *where* $\Lambda_p(A) = \cap \{G : A \subseteq G,\ G \in PO(X)\}$*[19].*
5. Λ_β*-set (=* Λ_{sp}*-set [30]) if* $A = \Lambda_{sp}(A)$ *where* $\Lambda_{sp}(A) = \cap \{G : A \subseteq G,\ G \in \beta O(X)\}$.

6. *Λ_b-set if $A = \Lambda_b(A)$ where $\Lambda_b(A) = \cap \{G : A \subseteq G, G \in bO(X)\}$[11].*

Remark 2.4. *In a topological space, every α-closed set is αg-closed but not conversely [25].*

Definition 2.5. *A subset A of a topological space (X, τ) is called*

1. *locally closed set (briefly lc-set)[18] if $A = L \cap F$, where L is open and F is closed.*
2. *αlc^*-set [21] if $A = L \cap F$, where L is open and F is α-closed.*
3. *slc*-set [5] if $A = L \cap F$, where L is open and F is semi-closed.*
4. *λ-closed set [4] if $A = L \cap F$, where L is Λ-set and F is closed.*

Definition 2.6. *A function $f : X \to Y$ is called*

1. *continuous [9] if $f^{-1}(V)$ is closed in X for every closed subset V of Y.*
2. *α-continuous [27] if $f^{-1}(V)$ is an α-closed in X for every closed subset V of Y.*
3. *αg-continuous [20] if $f^{-1}(V)$ is an αg-closed in X for every closed subset V of Y.*
4. *αlc^*-continuous [21] if $f^{-1}(V)$ is αlc^*-set in X for every closed subset V of Y.*
5. *semi-continuous [22] if $f^{-1}(V)$ is semi-closed in X for every closed subset V of Y.*
6. *gs-continuous [32] if $f^{-1}(V)$ is gs-closed in X for every closed subset V of Y.*
7. *slc*-continuous [5] if $f^{-1}(V)$ is slc*-set in X for every closed subset V of Y.*
8. *precontinuous [26] if $f^{-1}(V)$ is preclosed in X for every closed subset V of Y.*
9. *gp-continuous [6] if $f^{-1}(V)$ is gp-closed in X for every closed subset V of Y.*
10. *gsp-continuous [15] if $f^{-1}(V)$ is gsp-closed in X for every closed subset V of Y.*
11. *gb-continuous [17] if $f^{-1}(V)$ is gb-closed in X for every closed subset V of Y.*
12. *β-continuous [1] if $f^{-1}(V)$ is β-closed in X for every closed subset V of Y.*
13. *b-continuous [16] if $f^{-1}(V)$ is b-closed in X for every closed subset V of Y.*

3 Characterizations of generalized λ-closed sets

Definition 3.1. *A subset A of a topological space (X, τ) is called*

1. *αg^*-closed [28] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open.*
2. *sg*-closed [28] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.*
3. *pg*-closed [28] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is preopen.*

4. *βg^*-closed [28] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is β-open.*

5. *bg*-closed if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is b-open.*

Definition 3.2. *A subset A of a topological space (X, τ) is called*

1. *αlc-set [2] if A = L $\cap$ F where L is α-open and F is closed.*

2. *slc-set [10] if A = L $\cap$ F where L is semi-open and F is closed.*

3. *plc-set [10] if A = L $\cap$ F where L is preopen and F is closed.*

4. *βlc-set [10] if A = L $\cap$ F where L is β-open and F is closed.*

5. *blc-set if A = L $\cap$ F where L is b-open and F is closed.*

Definition 3.3. *A subset A of a topological space (X, τ) is called λ-α-closed if A = L $\cap$ F, where L is Λ-set and F is an α-closed set.*

Proposition 3.4. *Every λ-closed set is λ-α-closed but not conversely.*

Example 3.5. *Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{a\}, X\}$. Then $\{b\}$ is λ-α-closed but not λ-closed.*

Lemma 3.6. *For a subset A of a topological space (X, τ), the following conditions are equivalent.*

1. *A is λ-α-closed.*

2. *$A = L \cap \alpha cl(A)$ where L is a Λ-set.*

3. *$A = A^{\wedge} \cap \alpha cl(A)$.*

Lemma 3.7. *In a space X, the following statements hold.*

1. *Every α-closed set is λ-α-closed but not conversely.*

2. *Every Λ-set is λ-α-closed but not conversely.*

3. *Every α-closed set is αlc^*-set but not conversely.*

4. *Every αlc^*-set is λ-α-closed.*

Example 3.8. *Let X and τ be as in Example 3.5. Then*

1. *$\{a\}$ is λ-α-closed but not α-closed.*

2. *$\{b\}$ is λ-α-closed but not Λ-set.*

3. *$\{a\}$ is αlc^*-set but not α-closed.*

Lemma 3.9. *A subset $A \subset (X, \tau)$ is αg-closed if and only if $\alpha cl(A) \subset A^{\wedge}$.*

Theorem 3.10. *For a subset A of a topological space (X, τ), the following conditions are equivalent.*

1. *A is α-closed.*

2. *A is αg-closed and αlc^*-set.*

3. *A is αg-closed and λ-α-closed.*

Proof. (1) $\Rightarrow$ (2) and (2) $\Rightarrow$ (3) : Obvious.
(3) $\Rightarrow$ (1) : Since A is αg-closed, by Lemma 3.9, αcl(A) $\subset$ A^. Since A is λ-α-closed, by Lemma 3.6, A = A^ $\cap$ αcl(A) = αcl(A). Hence A is α-closed.

Remark 3.11. *The following Example shows that the concepts of αg-closed set and αlc^*-set are independent of each other.*

Example 3.12. *Let X and τ be as in Example 3.5. Then $\{a, b\}$ is αg-closed but not αlc^*-set in (X, τ). Moreover, $\{a\}$ is αlc^*-set but not αg-closed in (X, τ).*

Remark 3.13. *The following Example shows that the concepts of αg-closed set and λ-α-closed set are independent of each other.*

Example 3.14. *Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{a, b\}, X\}$. Then $\{a, c\}$ is αg-closed but not λ-α-closed in (X, τ). Moreover, $\{a, b\}$ is λ-α-closed but not αg-closed in (X, τ).*

Definition 3.15. *A subset A of a topological space (X, τ) is called*

1. *λ-s-closed if $A = L \cap F$, where L is Λ-set and F is semi-closed.*

2. *λ-p-closed if $A = L \cap F$, where L is Λ-set and F is preclosed.*

3. *λ-β-closed if $A = L \cap F$, where L is Λ-set and F is β-closed.*

4. *λ-b-closed if $A = L \cap F$, where L is Λ-set and F is b-closed.*

Definition 3.16. *A subset A of a topological space (X, τ) is called*

1. *plc^*-set if $A = L \cap F$, where L is open and F is preclosed.*

2. *βlc^*-set if $A = L \cap F$, where L is open and F is β-closed.*

3. *blc^*-set if $A = L \cap F$, where L is open and F is b-closed.*

Lemma 3.17. *A subset $A \subset (X, \tau)$ is*

1. *gs-closed if and only if $scl(A) \subset A^\wedge$.*

2. *gp-closed if and only if $pcl(A) \subset A^\wedge$.*

3. *gsp-closed if and only if $\beta cl(A) \subset A^\wedge$.*

4. *gb-closed if and only if $bcl(A) \subset A^\wedge$.*

Corollary 3.18. *For a subset A of a topological space (X, τ), the following conditions are equivalent.*

1. *(a) A is semi-closed.*
   (b) A is gs-closed and slc^*-set.
   (c) A is gs-closed and λ-s-closed.

2. (a) *A is preclosed.*
   (b) *A is gp-closed and plc*-set.*
   (c) *A is gp-closed and λ-p-closed.*
3. (a) *A is β-closed.*
   (b) *A is gsp-closed and βlc^*-set.*
   (c) *A is gsp-closed and λ-β-closed.*
4. (a) *A is b-closed.*
   (b) *A is gb-closed and blc*-set.*
   (c) *A is gb-closed and λ-b-closed.*

Proof. The proof is similar to that of Lemma 3.6, Lemma 3.17 and Theorem 3.10.

Remark 3.19. *The following Examples show that the concepts of*

1. *gs-closed set and slc*-set are independent of each other.*
2. *gs-closed set and λ-s-closed set are independent of each other.*
3. *gp-closed set and plc*-set are independent of each other.*
4. *gp-closed set and λ-p-closed set are independent of each other.*
5. *gsp-closed set and βlc^*-set are independent of each other.*
6. *gsp-closed set and λ-β-closed set are independent of each other.*
7. *gb-closed set and blc*-set are independent of each other.*
8. *gb-closed set and λ-b-closed set are independent of each other.*

Example 3.20. *Let X and τ be as in Example 3.14. Then*

1. *$\{a, c\}$ is gs-closed but not slc*-set in (X, τ). Moreover, $\{a, b\}$ is slc*- set but not gs-closed in (X, τ).*
2. *$\{b, c\}$ is gs-closed but not λ-s-closed in (X, τ). Moreover, $\{a, b\}$ is λ-s-closed but not gs-closed in (X, τ).*

Example 3.21. *Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$. Then*

1. *$\{a, b\}$ is gp-closed but not plc*-set in (X, τ). Moreover, $\{a, c\}$ is plc*-set but not gp-closed in (X, τ).*
2. *$\{a, b\}$ is gp-closed but not λ-p-closed in (X, τ). Moreover, $\{a\}$ is λ-p-closed but not gp-closed in (X, τ).*

Example 3.22. *Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{b\}, \{a, b\}, X\}$. Then*

1. *$\{b, c\}$ is gsp-closed but not βlc^*-set in (X, τ). Moreover, $\{b\}$ is βlc^*- set but not gsp-closed in (X, τ).*

2. $\{b, c\}$ *is gsp-closed but not* λ-β-*closed in* (X, τ). *Moreover,* $\{a, b\}$ *is* λ-β-*closed but not gsp-closed in* (X, τ).

3. $\{b, c\}$ *is gb-closed but not blc*-set in* (X, τ). *Moreover,* $\{a, b\}$ *is blc*-set but not gb-closed in* (X, τ).

4. $\{b, c\}$ *is gb-closed but not* λ-*b-closed in* (X, τ). *Moreover,* $\{b\}$ *is* λ-*b-closed but not gb-closed in* (X, τ).

Remark 3.23. *We have the following diagrams for the subsets we stated above:*

Diagram 1.

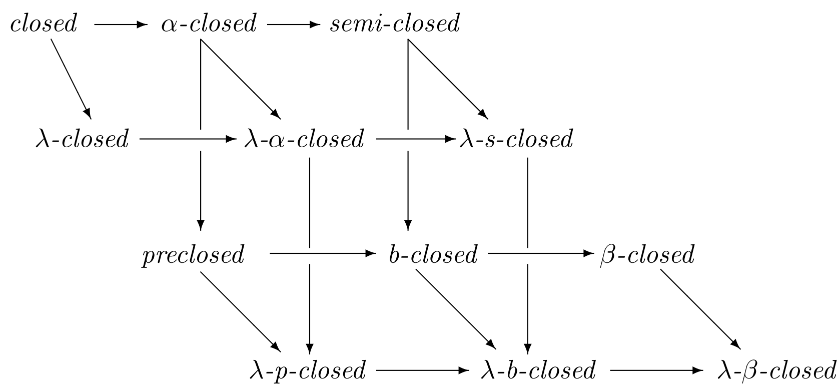
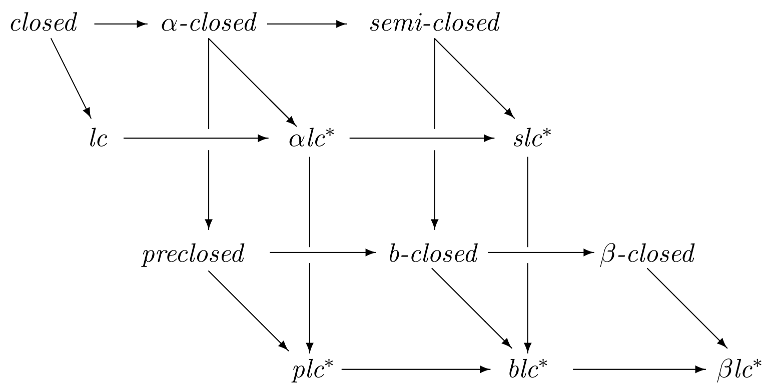


Diagram 2.



Definition 3.24. *A subset A of a topological space* (X, τ) *is called*

1. λ-αg^*-*closed if* $A = L \cap F$, *where L is a* Λ_α-*set and F is closed.*

2. *λ-sg*-closed if $A = L \cap F$, where L is a Λ_s-set and F is closed.*

3. *λ-pg*-closed if $A = L \cap F$, where L is a Λ_p-set and F is closed.*

4. *λ-βg*-closed if $A = L \cap F$, where L is a Λ_{sp}-set and F is closed.*

5. *λ-bg*-closed if $A = L \cap F$, where L is a Λ_b-set and F is closed.*

Lemma 3.25. 1. *Every αlc-set (resp. slc-set, plc-set, βlc-set, blc-set) is λ-αg*-closed (resp. λ-sg*-closed, λ-pg*-closed, λ-βg*-closed, λ-bg*-closed).*

2. *Every Λ_α-set (resp. Λ_s-set, Λ_p-set, Λ_{sp}-set, Λ_b-set) is λ-αg*-closed (resp. λ-sg*-closed, λ-pg*-closed, λ-βg*-closed, λ-bg*-closed).*

Lemma 3.26. 1. *A subset $A \subset (X, \tau)$ is αg*-closed if and only if $cl(A) \subset \Lambda_\alpha(A)$.*

2. *A subset $A \subset (X, \tau)$ is sg*-closed if and only if $cl(A) \subset \Lambda_s(A)$.*

3. *A subset $A \subset (X, \tau)$ is pg*-closed if and only if $cl(A) \subset \Lambda_p(A)$.*

4. *A subset $A \subset (X, \tau)$ is βg*-closed if and only if $cl(A) \subset \Lambda_\beta(A)$.*

5. *A subset $A \subset (X, \tau)$ is bg*-closed if and only if $cl(A) \subset \Lambda_b(A)$.*

Lemma 3.27. *For a subset A of a topological space (X, τ), the following conditions are equivalent.*

1. *A is λ-αg*-closed.*

2. *$A = L \cap cl(A)$ where L is a Λ_α-set.*

3. *$A = \Lambda_\alpha(A) \cap cl(A)$.*

Theorem 3.28. *For a subset A of a topological space (X, τ), the following conditions are equivalent.*

1. (a) *A is closed.*
   (b) *A is αg*-closed and αlc-set.*
   (c) *A is αg*-closed and λ-αg*-closed.*

2. (a) *A is closed.*
   (b) *A is sg*-closed and slc-set.*
   (c) *A is sg*-closed and λ-sg*-closed.*

Remark 3.29. *The following Examples show that the concepts of*

1. *αg*-closed set and αlc-set are independent of each other.*

2. *αg*-closed set and λ-αg*-closed set are independent of each other.*

3. *sg*-closed set and slc-set are independent of each other.*

4. *sg*-closed set and λ-sg*-closed set are independent of each other.*

Example 3.30. *Let X and τ be as in Example 3.14. Then*

1. $\{a,\ c\}$ *is* αg^**-closed but it is neither* αlc*-set nor* λ-αg^**-closed in* X.

2. $\{a,\ b\}$ *is both* αlc*-set and* λ-αg^**-closed but not* αg^**-closed in* X.

Example 3.31. *Let* $X = \{a,\ b,\ c\}$ *with* $\tau = \{\emptyset,\ \{b,\ c\},\ X\}$. *Then*

1. $\{a,\ b\}$ *is* sg^**-closed but it is neither slc-set nor* λ-sg^**-closed in* X.

2. $\{b,\ c\}$ *is both slc-set and* λ-sg^**-closed but not* sg^**-closed in* X.

Remark 3.32. *We have the following diagrams for the subsets we stated above:*

Diagram 3.

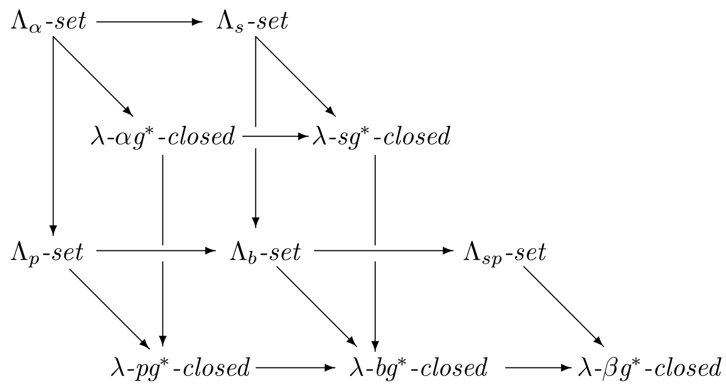
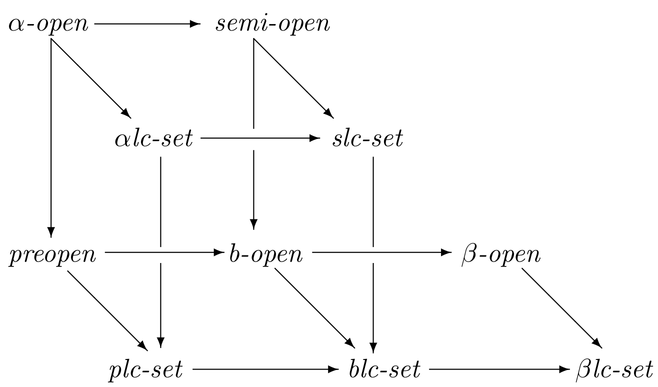


Diagram 4.



4 Decompositions of Topological Functions

Definition 4.1. *A function* $f : (X,\ \tau) \to (Y,\ \sigma)$ *is called*

1. λ-α*-continuous if* $f^{-1}(V)$ *is a* λ-α*-closed set in* X *for every closed subset* V *of* Y.

2. *λ-s-continuous if $f^{-1}(V)$ is a λ-s-closed set in X for every closed subset V of Y.*

3. *λ-p-continuous if $f^{-1}(V)$ is a λ-p-closed set in X for every closed subset V of Y.*

4. *λ-β-continuous if $f^{-1}(V)$ is a λ-β-closed set in X for every closed subset V of Y.*

5. *λ-b-continuous if $f^{-1}(V)$ is a λ-b-closed set in X for every closed subset V of Y.*

Definition 4.2. *A function $f : (X, \tau) \to (Y, \sigma)$ is called*

1. *αg^*-continuous if $f^{-1}(V)$ is an αg^*-closed set in X for every closed subset V of Y.*

2. *sg^*-continuous if $f^{-1}(V)$ is a sg^*-closed set in X for every closed subset V of Y.*

3. *αlc-continuous if $f^{-1}(V)$ is an αlc-set in X for every closed subset V of Y.*

4. *slc-continuous if $f^{-1}(V)$ is a slc-set in X for every closed subset V of Y.*

5. *λ-αg^*-continuous if $f^{-1}(V)$ is an λ-αg^*-closed set in X for every closed subset V of Y.*

6. *λ-sg^*-continuous if $f^{-1}(V)$ is a λ-sg^*-closed set in X for every closed subset V of Y.*

7. *plc^*-continuous if $f^{-1}(V)$ is a plc^*-set in X for every closed subset V of Y.*

8. *βlc^*-continuous if $f^{-1}(V)$ is a βlc^*-set in X for every closed subset V of Y.*

9. *blc^*-continuous if $f^{-1}(V)$ is a blc^*-set in X for every closed subset V of Y.*

We have the following decompositions of topological functions.

Theorem 4.3. *Let $f : X \to Y$ be a function. Then the following are equivalent.*

1. *f is α-continuous.*

2. *f is αg-continuous and αlc^*-continuous.*

3. *f is αg-continuous and λ-α-continuous.*

Proof. It follows from Theorem 3.10.

Theorem 4.4. *Let $f : X \to Y$ be a function. Then the following are equivalent.*

1. *f is semi-continuous.*

2. *f is gs-continuous and slc^*-continuous.*

3. *f is gs-continuous and λ-s-continuous.*

Proof. It follows from Corollary 3.18 (1).

Theorem 4.5. *Let $f : X \to Y$ be a function. Then the following are equivalent.*

1. *f is precontinuous.*
2. *f is gp-continuous and plc*-continuous.*
3. *f is gp-continuous and λ-p-continuous.*

Proof. It follows from Corollary 3.18(2).

Theorem 4.6. *Let $f : X \to Y$ be a function. Then the following are equivalent.*

1. *f is β-continuous.*
2. *f is gsp-continuous and βlc*-continuous.*
3. *f is gsp-continuous and λ-β-continuous.*

Proof. It follows from Corollary 3.18(3).

Theorem 4.7. *Let $f : X \to Y$ be a function. Then the following are equivalent.*

1. *f is b-continuous.*
2. *f is gb-continuous and blc*-continuous.*
3. *f is gb-continuous and λ-b-continuous.*

Proof. It follows from Corollary 3.18(4).

Theorem 4.8. *Let $f : X \to Y$ be a function. Then the following are equivalent.*

1. *f is continuous.*
2. *f is αg^*-continuous and αlc-continuous.*
3. *f is αg^*-continuous and λ-αg^*-continuous.*

Proof. It follows from Theorem 3.28(1).

Theorem 4.9. *Let $f : X \to Y$ be a function. Then the following are equivalent.*

1. *f is continuous.*
2. *f is sg^*-continuous and slc-continuous.*
3. *f is sg^*-continuous and λ-sg^*-continuous.*

Proof. It follows from Theorem 3.28(1).

Remark 4.10. *The following Examples show that the concepts of the following are independent of each other.*

1. *αg-continuity and αlc*-continuity.*
2. *αg-continuity and λ-α-continuity.*
3. *gs-continuity and slc*-continuity.*

4. *gs-continuity and λ-s-continuity.*
5. *gp-continuity and plc*-continuity.*
6. *gp-continuity and λ-p-continuity.*
7. *gsp-continuity and βlc^*-continuity.*
8. *gsp-continuity and λ-β-continuity.*
9. *gb-continuity and blc*-continuity.*
10. *gb-continuity and λ-b-continuity.*
11. *αg^*-continuity and αlc-continuity.*
12. *αg^*-continuity and λ-αg^*-continuity.*
13. *sg*-continuity and slc-continuity.*
14. *sg*-continuity and λ-sg*-continuity.*

Example 4.11. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{b\}, \{a, b\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is αg-continuous but it is neither αlc^*-continuous nor λ-α-continuous.*

Example 4.12. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{c\}, \{b, c\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is both αlc^*-continuous and λ-α-continuous but not αg-continuous.*

Example 4.13. *Let X, Y, τ and σ be as in Example 4.11. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is αg^*-continuous but it is neither αlc-continuous nor λ-αg^*-continuous.*

Example 4.14. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{c\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is both αlc-continuous and λ-αg^*-continuous but not αg^*-continuous.*

Example 4.15. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{c\}, \{a, c\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is gs-continuous but it is neither slc*-continuous nor λ-s-continuous.*

Example 4.16. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is both slc*-continuous and λ-s-continuous but not gs-continuous.*

Example 4.17. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{c\}, \{b, c\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is sg*-continuous but it is neither slc-continuous nor λ-sg*-continuous.*

Example 4.18. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is both slc-continuous and λ-sg*-continuous but not sg*-continuous.*

Example 4.19. *Let* $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{b\}, X\}$ *and* $\sigma = \{\emptyset, \{c\}, \{b, c\}, Y\}$. *Then the identity function* $f : (X, \tau) \to (Y, \sigma)$ *is gp-continuous but it is neither plc*-continuous nor* λ*-p-continuous.*

Example 4.20. *Let* $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{b\}, X\}$ *and* $\sigma = \{\emptyset, \{a, c\}, Y\}$. *Then the identity function* $f : (X, \tau) \to (Y, \sigma)$ *is both plc*-continuous and* λ*-p-continuous but not gp-continuous.*

Example 4.21. *In Example 4.19, f is gsp-continuous but it is neither* β*lc*-continuous nor* λ*-*β*-continuous*

Example 4.22. *In Example 4.18, f is both* β*lc*-continuous and* λ*-*β*-continuous but not gsp-continuous.*

Example 4.23. *In Example 4.20, f is gb-continuous but it is neither blc*-continuous nor* λ*-b-continuous.*

Example 4.24. *Let X, Y and* τ *be as in Example 4.15 and* $\sigma = \{\emptyset, \{b, c\}, Y\}$. *Then the identity function* $f : (X, \tau) \to (Y, \sigma)$ *is both blc*-continuous and* λ*-b-continuous but not gb-continuous.*

5 Conclusion

Topology is an area of Mathematics concerned with the properties of space that are preserved under continuous deformations including stretching and bending, but not tearing. By the middle of the 20th century, topology had become a major branch of Mathematics.

Topology as a branch of Mathematics can be formally defined as the study of qualitative properties of certain objects that are invariant under a certain kind of transformation especially those properties that are invariant under a certain kind of equivalence and it is the study of those properties of geometric configurations which remain invariant when these configurations are subjected to one-to-one bicontinuous transformations or homeomorphisms. Topology operates with more general concepts than analysis. Differential properties of a given transformation are nonessential for topology but bicontinuity is essential. As a consequence, topology is often suitable for the solution of problems to which analysis cannot give the answer.

Though the concept of topology has been identified as a difficult territory in Mathematics, we have taken it up as a challenge and cherishingly worked out this research study. It can also further up the understanding of basic structure of classical mathematics and offers new methods and results in obtaining significant results of classical mathematics. Moreover it also has applications in some important fields of Science and Technology.

In this paper, we obtained new classes of sets by using λ-closed sets in topological spaces and studied their basic properties; and their connections with other kind of topological sets. Moreover new decompositions of topological functions are obtained.

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