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## Q-SINGLE VALUED NEUTROSOPHIC SOFT SETS

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**Abstract** - In this paper, we have introduced the concept of Q-single value neutrosophic soft set, multi Q-single valued neutrosophic set and defined some basic results and related properties. We have also defined the idea of Q-single valued neutrosophic soft set, which is the generalizations of Q-fuzzy set, Q-intuitionistic fuzzy set, multi Q-fuzzy set, Multi Q-intuitionistic fuzzy set, Q-fuzzy soft set, Q-intuitionistic fuzzy soft set. We have also defined and discussed some properties and operations of Q-single valued neutrosophic soft set.

**Keywords** - Neutrosophic set, single valued neutrosophic set, Q-single valued neutrosophic set, Multi Q-Single valued neutrosophic set, Q-single valued neutrosophic soft set.

### 1 Introduction

In 1965 L. A. Zadeh was the first person who presented theory of fuzzy set [23], whose fundamental component is just a degree of membership. After the introduction of fuzzy sets, the idea of intuitionistic fuzzy sets (IFS) was given by K. Atanassov in 1986 [6], whose basic components are the grade of membership and the grade of non-membership under the restrictions that the sum of the two degrees does not surpass one. Atanassov's IFS is more suitable mathematical tool to handle real life application. But in some cases Atanassov's IFS is difficult to apply because in IFS we cannot define degree of indeterminacy independently. To surmount this difficulty Samarandache introduced the concept of neutrosophic sets [21], which not only generalized the Zadeh's fuzzy set and Atanassov's IFS but also generalized Gau's vague sets [17] philosophically. In neutrosophic set degree of indeterminacy is defined independently. Neutrosophic set contains degree of truth-membership function  $T_A(a): X \rightarrow ]0^-, 1^+[$ , indeterminacy membership function  $I_A(a): X \rightarrow ]0^-, 1^+[$  and falsity membership function  $F_A(a): X \rightarrow ]0^-, 1^+[$ , with for

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each  $a \in X$  and satisfy the condition  $0^- \leq \text{Sup}T_A(a) + \text{Sup}I_A(a) + \text{Sup}F_A(a) \leq 3^+$ . In neutrosophic sets degree of indeterminacy is defined independently but it is hard to sue it in real life and engineering problem, because it contains non-standard intervals. It is important to overcome this practical difficulty. Wang et al [22] present the concept of Single Valued Neutrosophic Sets (SVNS) to overcome this difficulty. SVNS is a primary class of neutrosophic sets. SVNS is easy to apply in real life and engineering problems because it contains a single points in the standard unit interval  $]0^-, 1^+[$  instead of non-standard intervals of  $]0^-, 1^+[$ .

Firstly Molodstov [20] introduced the concept of soft set theory. Maji et al. [18] gave some operations and primary properties on theory of soft set. Ali et al. [7] pointed out that the operations defined for soft sets are not correct and due to these operations many mathematical results leads to wrong answers. Fuzzy soft sets theory and fuzzy parameterized soft set theories was studied by Cagman et al [16] . Fuzzy soft set theory was studied by Ahmad, B., and Athar Kharal [5] . F.Adam and N. Hassan [4] introduced the concept of multi Q-fuzzy sets, multi Q-parameterized soft sets and defined some basic properties and operation such as complement, equality, union, intersection F. Adam and N. Hassan [2, 3] also introduced Q-fuzzy soft set, defined some basic operations and defined Q-fuzzy soft aggregation operators that allows constructing more efficient decision making methods. S. Broumi [9, 10] established the notion of Q- intuitionistic fuzzy set (Q-IFS), Q-intuitionistic fuzzy soft set (Q-IFSS) and defined some basic properties with illustrative examples, and also defined some basic operation for Q-IFS and Q-IFSS such as union, intersection, AND and OR operations. Broumi. S. et. al. [8] presented the concept of intuitionistic neutrosophic soft rings by applying intuitionistic neutrosophic soft set to ring theory.

Broumi S. et. al. [8, 11, 12, 13, 14, 15] presented concepts of single valued neutrosophic graphs, interval neutrosophic graphs, on bipolar single valued neutrosophic graphs, and also presented an introduction to bipolar single valued neutrosophic graph.

This article is arranged as proceed, Section 2 contains basic definitions of soft sets, Q-fuzzy sets, multi Q-fuzzy sets, Q-fuzzy soft sets, neutrosophic sets and SVNS are defined. In section 3 Q-SVNS and some basic operations are defined. In section 4 multi Q-SVNS and some basic operations such as union, intersection etc are defined. In section 5 we introduce the concept of Q-single valued neutrosophic soft set(Q-SVNSS) and defined some basic operations and related results are discussed. At the end conclusion and references are given.

## 2 Preliminaries

**2.1. Definition.** [20] Let  $X$  be a universal set ,  $E$  be a set of parameters and  $A \subseteq E$ . A pair  $(F, A)$  is said to be soft set over the universal set  $X$ , if and only if  $F$  is a mapping from  $A$  to the power set of  $X$ .

**2. 2. Definition. [2,3].** Assume  $X$  be a universal set and  $Q \neq \emptyset$ . A  $Q$  –fuzzy subset  $N$  of  $X$  is a function  $X \times X \rightarrow [0,1]$ ." The union of two  $Q$  –fuzzy subsets  $N$  and  $M$  is defined as

$$N \cup M = \{\max(\mu_N(\hat{\theta}, \hat{u}), \mu_M(\hat{\theta}, \hat{u})): \hat{\theta} \in X, \hat{u} \in Q\}$$

The intersection of two  $Q$  –fuzzy subsets  $N$  and  $M$  is defined as

$$N \cap M = \{\min(\mu_N(\hat{\theta}, \hat{u}), \mu_M(\hat{\theta}, \hat{u})): \hat{\theta} \in X, \hat{u} \in Q\}$$

**2. 3. Definition[3].** Let  $I$  be unit interval  $[0,1]$ ,  $k \in Z^+$  (positive integer),  $X$  be universal set and  $Q \neq \emptyset$ . A multi  $Q$  –fuzzy set  $N_Q$  in  $X$  and  $Q$  is a set of ordered sequences,

$$N_Q = \{\max(\mu_j(\hat{\theta}, \hat{u})): \hat{\theta} \in X, \hat{u} \in Q\}$$

Where  $\mu_j: X \times Q \rightarrow I^k$  The function  $\mu_j(\theta, \hat{u})$  is termed as membership function of multi  $Q$ -fuzzy set  $N_Q$ , and  $\sum_{j=0}^k \mu_j(\hat{\theta}, \hat{u}) \leq 1$ , for  $j = 1,2,3, \dots, k$ .  $k$  is the dimension of multi  $Q$  fuzzy set  $N_Q$ . The set of all multi-  $Q$  – fuzzy set of dimension  $k$  in  $X$  and  $Q$  is denoted by  $M^k FQ(X)$ ."

**2. 4. Definition[4].** Let  $X$  be a universal set,  $E$  be the set of parameters,  $Q \neq \emptyset$ . Let  $M^k FQ(X)$  is the power set of all multi  $Q$  –fuzzy subsets of  $X$  with dimension  $k = 1$ . Let  $D \subseteq E$ . A pair  $(F_Q, D)$  is referred as  $Q$  –fuzzy soft set (in short  $QF$  –soft set )over  $X$  where  $F_Q$ , is defined by

$$F_Q: D \rightarrow M^k FQ(X) \text{ such that } (F_Q(\hat{\theta})) = \emptyset \text{ if } \hat{\theta} \notin D$$

Here a  $Q$  –fuzzy soft set can be represented by the set of ordered pairs

$$(F_Q, D) = \{\hat{\theta}, F_Q(\hat{\theta}): \hat{\theta} \in X, F_Q(\hat{\theta}) \in M^k FQ(X)\}$$

The set of all  $Q$  –fuzzy soft sets over  $X$  will be denoted by  $QFS(X)$

**2. 5. Definition. [22]** Let  $X$  be a space of points (objects), with a generic element in  $X$  denoted by  $\hat{\theta}$ . A SVN  $N$  in  $X$  has the features truth-membership function  $T_N$ , indeterminacy-membership function  $I_N$ , and falsity-membership function  $F_N$ . For each

point  $\tilde{\theta}$  in  $X, T_N(\tilde{\theta}), I_N(\tilde{\theta}), F_N(\tilde{\theta}) \in [0,1]$ .

Mathematically single valued neutrosophic is expressed as follows:

$$N = \{(\tilde{\theta}, (T_N(\tilde{\theta}), I_N(\tilde{\theta}), F_N(\tilde{\theta}))) | \tilde{\theta} \in X\}$$

### 3 Q –Single Valued Neutrosophic Sets

**3.1. Definition.** Let  $X$  be a universal set and  $Q \neq \emptyset$ . A  $Q$  –SVNS  $\tilde{N}_Q$  in  $X$  and  $Q$  is an object of the form

$$\tilde{N}_Q = \{(\tilde{\theta}, \hat{u}), \mu_{\tilde{N}_Q}(\tilde{\theta}, \hat{u}), \nu_{\tilde{N}_Q}(\tilde{\theta}, \hat{u}), \lambda_{\tilde{N}_Q}(\tilde{\theta}, \hat{u}) : \tilde{\theta} \in X, \hat{u} \in Q\}$$

Where  $\mu_{\tilde{N}_Q} : X \times Q \rightarrow [0,1]$ ,  $\nu_{\tilde{N}_Q} : X \times Q \rightarrow [0,1]$ ,  $\lambda_{\tilde{N}_Q} : X \times Q \rightarrow [0,1]$ , are respectively truth-membership, indeterminacy-membership and falsity membership functions for every  $\tilde{\theta} \in X$  and  $\hat{u} \in Q$  and satisfy the condition  $0 \leq \mu_{\tilde{N}_Q}(\tilde{\theta}, \hat{u}) + \nu_{\tilde{N}_Q}(\tilde{\theta}, \hat{u}) + \lambda_{\tilde{N}_Q}(\tilde{\theta}, \hat{u}) \leq 3$ .

**3.2. Example.** Let  $X = \{p_1, p_1, p_3\}$  and  $Q = \{\hat{u}, \hat{v}\}$ , then  $Q$  –SVNS  $\tilde{N}_Q$  is defined below,

$$\begin{aligned} \tilde{N}_Q = \{ < (p_1, \hat{u}), (0.4, 0.3, 0.5), (p_1, \hat{v}), (0.2, 0.4, 0.6), (p_2, \hat{u}), (0.6, 0.1, 0.3), \\ & (p_2, \hat{v}), (0.7, 0.2, 0.1), (p_3, \hat{u}), (0.3, 0.6, 0.4), (p_3, \hat{v}), (0.5, 0.4, 0.6) > \} \end{aligned}$$

Now we define some basic operations for  $Q$  –SVNS.

**3.3. Definition.** Let  $X$  be a universal set,  $Q \neq \emptyset$  and  $\tilde{N}_Q$  be a  $Q$  –SVNS. The complement of  $\tilde{N}_Q$  is denoted and defined as follows

$$\tilde{N}_Q^c = \{(\tilde{\theta}, \hat{u}), \lambda_{\tilde{N}_Q}(\tilde{\theta}, \hat{u}), 1 - \nu_{\tilde{N}_Q}(\tilde{\theta}, \hat{u}), \mu_{\tilde{N}_Q}(\tilde{\theta}, \hat{u}) : \tilde{\theta} \in X, \hat{u} \in Q\}$$

**3.4. Definition.** Let  $\tilde{A}_Q$  and  $\tilde{N}_Q$  be two  $Q$  –SVNS. Then the union and intersection is denoted and defined by

$$\begin{aligned} \tilde{A}_Q \cup \tilde{N}_Q = \{ & (\tilde{\theta}, \hat{u}), \max(\mu_{\tilde{A}_Q}(\tilde{\theta}, \hat{u}), \mu_{\tilde{N}_Q}(\tilde{\theta}, \hat{u})), \min(\nu_{\tilde{A}_Q}(\tilde{\theta}, \hat{u}), \nu_{\tilde{N}_Q}(\tilde{\theta}, \hat{u})), \\ & \min(\lambda_{\tilde{A}_Q}(\tilde{\theta}, \hat{u}), \lambda_{\tilde{N}_Q}(\tilde{\theta}, \hat{u})) : \tilde{\theta} \in X, \hat{u} \in Q \} \end{aligned}$$

$$\tilde{A}_Q \cap \tilde{N}_Q = \{(\tilde{\theta}, \hat{u}), \min(\mu_{\tilde{A}_Q}(\tilde{\theta}, \hat{u}), \mu_{\tilde{N}_Q}(\tilde{\theta}, \hat{u})), \max(\nu_{\tilde{A}_Q}(\tilde{\theta}, \hat{u}), \nu_{\tilde{N}_Q}(\tilde{\theta}, \hat{u})),$$

$$\max(\lambda_{\tilde{A}_Q}(\tilde{\theta}, \hat{u}), \lambda_{\tilde{N}_Q}(\tilde{\theta}, \hat{u}))\}$$

**3.5. Definition.** Let  $\tilde{A}_Q$  and  $\tilde{N}_Q$  be two  $Q$  –SVNSs over two non-empty universal sets  $G$  and  $H$  respectively and  $Q$  be any non-empty set. Then the product of  $\tilde{A}_Q$  and  $\tilde{N}_Q$  is denoted by  $\tilde{A}_Q \times \tilde{N}_Q$  and defined as

$$\tilde{A}_Q \times \tilde{N}_Q = \{ \langle ((\tilde{\theta}, b), \hat{u}), \mu_{\tilde{A}_Q \times \tilde{N}_Q}((\tilde{\theta}, b), \hat{u}), \nu_{\tilde{A}_Q \times \tilde{N}_Q}((\tilde{\theta}, b), \hat{u}), \lambda_{\tilde{A}_Q \times \tilde{N}_Q}((\tilde{\theta}, b), \hat{u}) \rangle : \tilde{\theta} \in G, b \in H, \hat{u} \in Q \}$$

Where

$$\begin{aligned} \mu_{\tilde{A}_Q \times \tilde{N}_Q}((\tilde{\theta}, b), \hat{u}) &= \min\{\mu_{\tilde{A}_Q}(\tilde{\theta}, \hat{u}), \mu_{\tilde{N}_Q}(b, \hat{u})\} \\ \nu_{\tilde{A}_Q \times \tilde{N}_Q}((\tilde{\theta}, b), \hat{u}) &= \max\{\nu_{\tilde{A}_Q}(\tilde{\theta}, \hat{u}), \nu_{\tilde{N}_Q}(b, \hat{u})\} \\ \lambda_{\tilde{A}_Q \times \tilde{N}_Q}((\tilde{\theta}, b), \hat{u}) &= \max\{\lambda_{\tilde{A}_Q}(\tilde{\theta}, \hat{u}), \lambda_{\tilde{N}_Q}(b, \hat{u})\} \end{aligned}$$

For all  $\tilde{\theta}, b$  in  $G$  and  $\hat{u} \in Q$ .

**3.6. Definition.** Let  $\tilde{A}_Q$  a  $Q$  –single valued neutrosophic subset in a set  $G$ , the strongest  $Q$  –single valued neutrosophic relation on  $G$ , that is a  $Q$  –single valued neutrosophic relation on  $\tilde{A}_Q$  is  $H$  given by

$$\begin{aligned} \mu_H((\tilde{\theta}, b), \hat{u}) &= \min\{\mu_{\tilde{A}_Q}(\tilde{\theta}, \hat{u}), \mu_{\tilde{N}_Q}(b, \hat{u})\} \\ \nu_H((\tilde{\theta}, b), \hat{u}) &= \max\{\nu_{\tilde{A}_Q}(\tilde{\theta}, \hat{u}), \nu_{\tilde{N}_Q}(b, \hat{u})\} \\ \lambda_H((\tilde{\theta}, b), \hat{u}) &= \max\{\lambda_{\tilde{A}_Q}(\tilde{\theta}, \hat{u}), \lambda_{\tilde{N}_Q}(b, \hat{u})\} \end{aligned}$$

For all  $\tilde{\theta}, b$  in  $G$  and  $\hat{u} \in Q$ .

### 4. Multi $Q$ –Single Valued Neutrosophic Sets

**4.1. Definition.** Let  $X$  be a non-empty set and  $Q$  be any non-empty set,  $l$  be any positive integer and  $I$  be a unit interval  $[0,1]$ . A multi  $Q$  –SVNS  $\tilde{A}_Q$  in  $X$  and  $Q$  is a set of ordered sequences

$$\tilde{A}_Q = \{(\tilde{\theta}, \hat{u}), \mu_j(\tilde{\theta}, \hat{u}), \nu_j(\tilde{\theta}, \hat{u}), \lambda_j(\tilde{\theta}, \hat{u}) : \tilde{\theta} \in X, \hat{u} \in Q \text{ for all } j = 1, 2, \dots, l\}$$

Where  $\mu_j: X \times Q \rightarrow I^K$ ,  $\nu_j: X \times Q \rightarrow I^K$ ,  $\lambda_j: X \times Q \rightarrow I^K$ , for all  $j = 1, 2, \dots, l$

and are respectively truth-membership, indeterminacy-membership and falsity membership functions for each  $\tilde{\theta} \in X$  and  $\hat{u} \in Q$  and satisfy the condition

$$0 \leq \mu_j(\tilde{\theta}, \hat{u}) + \nu_j(\tilde{\theta}, \hat{u}) + \lambda_j(\tilde{\theta}, \hat{u}) \leq 3, \text{ for all } j = 1, 2, \dots, l$$

The functions  $\mu_j(\tilde{\theta}, \hat{u}), \nu_j(\tilde{\theta}, \hat{u}), \lambda_j(\tilde{\theta}, \hat{u})$  for all  $j = 1, 2, \dots, l$  are called the "truth-membership, indeterminacy-membership and falsity-membership" functions respectively of the multi  $Q$  –SVNS  $\tilde{A}_Q$  and satisfy the condition

$$0 \leq \mu_j(\tilde{\theta}, \hat{u}) + \nu_j(\tilde{\theta}, \hat{u}) + \lambda_j(\tilde{\theta}, \hat{u}) \leq 3, \text{ for all } j = 1, 2, \dots, l$$

$l$  is called the dimension of the  $Q$  –SVNS  $\tilde{A}_Q$ . The set of all  $Q$  –SVNS is denoted by  $Z^k QSVN(X)$ .

**4. 2. Example.** Let  $X = \{p_1, p_2, p_3\}$  be a universal set and  $Q = \{\hat{u}, v\}$  be a non-empty set and  $l = 2$  be a positive integer. If  $\tilde{A}_Q: X \times Q \rightarrow I^2$ , Then the set

$$\tilde{A}_Q = \{ < ((p_1, \hat{u}), (0.2, 0.3, 0.6), (0.6, 0.2, 0.3)), ((p_1, \hat{v}), (0.5, 0.1, 0.3), (0.4, 0.4, 0.5)), ((p_2, \hat{u}), (0.4, 0.3, 0.5), (0.6, 0.1, 0.3)), ((p_2, \hat{v}), (0.7, 0.2, 0.1), (0.2, 0.4, 0.8)) > \}$$

is a multi  $Q$  –SVNS in  $X$  and  $Q$ .

**4. 3. Remark.** Note that if  $\nu_j(\tilde{\theta}, \hat{u}) = 0$  and  $\lambda_j(\tilde{\theta}, \hat{u}) = 0$  then multi  $Q$  –SVNS reduces to multi  $Q$  –fuzzy set.

**4. 4. Definition.** Let  $\tilde{A}_Q$  be a  $Q$  –SVNS. The the complement of  $\tilde{A}_Q$  is denoted and defined as follows

$$\tilde{A}_Q^c = \{(\tilde{\theta}, \hat{u}), \lambda_j(\tilde{\theta}, \hat{u}), 1 - \nu_j(\tilde{\theta}, \hat{u}), \mu_j(\tilde{\theta}, \hat{u}): \tilde{\theta} \in X \text{ and } \hat{u} \in Q, \text{ for all } j = 1, 2, \dots, l\}$$

**4. 5. Definition.** Let  $\tilde{A}_Q$  and  $A_Q$  and  $B_Q$  be two  $Q$  –SVNSs, and  $l$  be a positive integer such that

$$A = \{(\tilde{\theta}, \hat{u}), \mu_j(\tilde{\theta}, \hat{u}), \nu_j(\tilde{\theta}, \hat{u}), \lambda_j(\tilde{\theta}, \hat{u}): \tilde{\theta} \in X \text{ and } \hat{u} \in Q \text{ for all } j = 1, 2, \dots, l\} \text{ and}$$

$$B = \{(\tilde{\theta}, \hat{u}), \mu_j^*(\tilde{\theta}, \hat{u}), \nu_j^*(\tilde{\theta}, \hat{u}), \lambda_j^*(\tilde{\theta}, \hat{u}): \tilde{\theta} \in X \text{ and } \hat{u} \in Q \text{ for all } j = 1, 2, \dots, l\}$$

Then we define the following basic operations for  $Q$  –SVNSs.

1.  $A \subset B$  iff  $\mu_j(\hat{\theta}, \hat{u}) \leq \mu_j^*(\hat{\theta}, \hat{u}), \nu_j(\hat{\theta}, \hat{u}) \geq \nu_j^*(\hat{\theta}, \hat{u})$  and  $\lambda_j(\hat{\theta}, \hat{u}) \geq \lambda_j^*(\hat{\theta}, \hat{u})$  for all  $j = 1, 2, \dots, l$ .
2.  $A = B$  iff  $\mu_j(\hat{\theta}, \hat{u}) = \mu_j^*(\hat{\theta}, \hat{u}), \nu_j(\hat{\theta}, \hat{u}) = \nu_j^*(\hat{\theta}, \hat{u})$  and  $\lambda_j(\hat{\theta}, \hat{u}) = \lambda_j^*(\hat{\theta}, \hat{u})$  for all  $j = 1, 2, \dots, l$ .
3.  $A \cup B = \{(\hat{\theta}, \hat{u}), \max(\mu_j(\hat{\theta}, \hat{u}), \mu_j^*(\hat{\theta}, \hat{u})), \min(\nu_j(\hat{\theta}, \hat{u}), \nu_j^*(\hat{\theta}, \hat{u})), \min(\lambda_j(\hat{\theta}, \hat{u}), \lambda_j^*(\hat{\theta}, \hat{u}))\}$
4.  $A \cap B = \{(\hat{\theta}, \hat{u}), \min(\mu_j(\hat{\theta}, \hat{u}), \mu_j^*(\hat{\theta}, \hat{u})), \max(\nu_j(\hat{\theta}, \hat{u}), \nu_j^*(\hat{\theta}, \hat{u})), \max(\lambda_j(\hat{\theta}, \hat{u}), \lambda_j^*(\hat{\theta}, \hat{u}))\}$

### 5. Q –Single Valued Neutrosophic Soft Sets

In this section we introduce the concept of  $Q$  –SVNSSs by combining soft sets and  $Q$  –SVNS. We also define some basic operations and properties of  $Q$  –SVNSSs.

**5.1. Definition.** Let  $X$  be a universal set,  $Q$  be any non-empty set and  $E$  be the set of parameters. Let  $Z^1QSVN(X)$  denote the set of all multi  $Q$  –single valued neutrosophic subsets of  $X$  with dimension  $l = 1$ . Let  $K \subset E$ . A pair  $(F_Q, K)$  is called  $Q$  –SVNSS over  $X$  where  $F_Q$  is a mapping given

$$F_Q: K \rightarrow Z^1QSVN(X) \text{ such that } (F_Q, (\hat{\theta})) = \emptyset \text{ if } \hat{\theta} \notin K$$

A  $Q$  –SVNSS can be represented by the set of ordered pairs

$$(F_Q, K) = \{(\hat{\theta}, F_Q(\hat{\theta})) : \hat{\theta} \in X, F_Q(\hat{\theta}) \in Z^1QSVN(X)\}$$

**5.2. Example.** Let  $X = \{p_1, p_2, p_3, p_4\}$  be a universal set,  $E = \{k_1, k_2, k_3, k_4\}$  and  $Q = \{\hat{u}, \hat{v}\}$  be a non-empty set. If  $K = \{k_1, k_2, k_3\} \subset E$ ,

$$F_Q(k_1) = \{((p_1, \hat{u}), (0.3, 0.4, 0.6)), ((p_1, \hat{v}), (0.2, 0.3, 0.5)), ((p_2, \hat{u}), (0.6, 0.2, 0.4))\}$$

$$F_Q(k_2) = \{((p_1, \hat{u}), (0.5, 0.3, 0.4)), ((p_1, \hat{v}), (0.4, 0.1, 0.7)), ((p_3, \hat{u}), (0.8, 0.1, 0.2))\}$$

$$F_Q(k_3) = \{((p_1, \hat{u}), (0.9, 0.1, 0.1)), ((p_1, \hat{v}), (0.8, 0.2, 0.3)), ((p_3, \hat{v}), (0.4, 0.3, 0.6))\}$$

Then

$$(F_Q, K) = \{(\mathbf{k}_1, ((\mathbf{p}_1, \hat{u}), (0.3, 0.4, 0.6)), ((\mathbf{p}_1, \hat{v}), (0.2, 0.3, 0.5)), ((\mathbf{p}_2, \hat{u}), (0.6, 0.2, 0.4)), \mathbf{k}_2, ((\mathbf{p}_1, \hat{u}), (0.5, 0.3, 0.4)), ((\mathbf{p}_1, \hat{v}), (0.4, 0.1, 0.7)), ((\mathbf{p}_3, \hat{u}), (0.8, 0.1, 0.2)), \mathbf{k}_3, ((\mathbf{p}_1, \hat{u}), (0.9, 0.1, 0.1)), ((\mathbf{p}_1, \hat{v}), (0.8, 0.2, 0.3)), ((\mathbf{p}_3, \hat{v}), (0.4, 0.3, 0.6))\}$$

is a  $Q$  –SVNSS.

**5.3. Definition.** Let  $(F_Q, K) \in QSVNSS(X)$ . If  $F_Q(\hat{\theta}) = \emptyset$  for all  $\hat{\theta} \in E$  then  $(F_Q, K)$  is called a null  $Q$  –SVNSS denoted by  $(\emptyset, K)$ .

**5.4. Example.** Let  $X, E$  and  $Q$  be defined in the above example 5.2 then

$$(\emptyset, K) = \{(\mathbf{k}_1, ((\mathbf{p}_1, \hat{u}), (0, 1, 1)), ((\mathbf{p}_1, \hat{v}), (0, 1, 1)), ((\mathbf{p}_2, \hat{u}), (0, 1, 1)), \mathbf{k}_2, ((\mathbf{p}_1, \hat{u}), (0, 1, 1)), ((\mathbf{p}_1, \hat{v}), (0, 1, 1)), ((\mathbf{p}_3, \hat{u}), (0, 1, 1)), \mathbf{k}_3, ((\mathbf{p}_1, \hat{u}), (0, 1, 1)), ((\mathbf{p}_1, \hat{v}), (0, 1, 1)), ((\mathbf{p}_3, \hat{v}), (0, 1, 1))\}$$

**5.5. Definition.** Let  $(F_Q, K) \in QSVNSS(X)$ , If  $F_Q(\hat{\theta}) = X$  for all  $\hat{\theta} \in E$  then  $(F_Q, K)$  is called a null  $Q$  –SVNSS denoted by  $(X, K)$ .

**5.6. Example.** Let  $X, E$  and  $Q$  be defined in the above example 5.2 then

$$(X, K) = \{(\mathbf{k}_1, ((\mathbf{p}_1, \hat{u}), (1, 0, 0)), ((\mathbf{p}_1, \hat{v}), (1, 0, 0)), ((\mathbf{p}_2, \hat{u}), (1, 0, 0)), \mathbf{k}_2, ((\mathbf{p}_1, \hat{u}), (1, 0, 0)), ((\mathbf{p}_1, \hat{v}), (1, 0, 0)), ((\mathbf{p}_3, \hat{u}), (1, 0, 0)), \mathbf{k}_3, ((\mathbf{p}_1, \hat{u}), (1, 0, 0)), ((\mathbf{p}_1, \hat{v}), (1, 0, 0)), ((\mathbf{p}_3, \hat{v}), (1, 0, 0))\}$$

**5.7. Definition.** Let  $(F_Q, K), (G_Q, L) \in QSVNS(X)$ . Then  $(F_Q, K)$  is  $Q$  –SVNSS subset of  $(G_Q, L)$ , denoted by  $(F_Q, K) \subset (G_Q, L)$  if  $K \subset L$  and  $F_Q(\hat{\theta}) \subset G_Q(\hat{\theta})$  for all  $\theta \in X$ .

**5.8. Proposition.** Let  $(F_Q, K), (G_Q, L), (M_Q, N) \in QSVNS(X)$ . Then

1.  $(F_Q, K) \subset (G_Q, E)$
2.  $(\emptyset, K) \subset (G_Q, L)$
3.  $(F_Q, K) \subset (G_Q, L)$  and  $(G_Q, L) \subset (M_Q, N)$  then  $(F_Q, K) \subset (M_Q, N)$ .
4. If  $(F_Q, K) = (G_Q, L)$  and  $(G_Q, L) = (M_Q, N)$  then  $(F_Q, K) = (M_Q, N)$

*Proof* : Straightforward.



**5. 9. Definition.** Let  $(F_Q, K) \in QSVNS(X)$ , Then the complement of  $Q - SVNSS$  set is written as  $(F_Q, K)^c$  and is defined by  $(F_Q, K)^c = (F_Q^c, \neg K)$  where

$$F_Q^c: \neg K \rightarrow QSVNS(X)$$

is the mapping given by  $F_Q^c(e)$   $Q -$ single valued neutrosophic complement for each  $e \in K$ .

**5. 10. Proposition.** Let  $(F_Q, K) \in QSVNS(X)$ , Then

1.  $((F_Q, K)^c)^c = (F_Q, K)$
2.  $(\emptyset, K)^c = (X, E)$
3.  $(X, E)^c = (\emptyset, E)$

*Proof.* 1. Let  $k \in K$ . Then

$$(F_Q, K) = F_Q(k) = \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \nu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$(F_Q, K)^c = (F_Q(k))^c = \{(\mathbf{p}_1, \hat{u}), (\lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 1 - \nu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}))\}$$

$$((F_Q, K)^c)^c = \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 1 - (1 - \nu_{F_Q(k)}(\mathbf{p}_1, \hat{u})), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u}))\}$$

$$((F_Q, K)^c)^c = \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \nu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u}))\}$$

$$((F_Q, K)^c)^c = (F_Q, K)$$

2. Let  $(\emptyset, K) = (F_Q, K)$ , Than for all  $k \in K$

$$F_Q(k) = \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \nu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$= \{(\mathbf{p}_1, \hat{u}), (0, 1, 1) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$(\emptyset, K)^c = (F_Q, K)^c = (F_Q(k))^c = \{(\mathbf{p}_1, \hat{u}), (1, 1 - 1, 0) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$= \{(\mathbf{p}_1, \hat{u}), (1, 0, 0) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$= (X, E)$$

3. Let  $(X, E) = (F_Q, E)$ , Then for all  $k \in K$

$$F_Q(k) = \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \nu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$= \{(\mathbf{p}_1, \hat{u}), (1,0,0) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$\begin{aligned} (X, E)^c &= (F_Q, E)^c = (F_Q(k))^c = \{(\mathbf{p}_1, \hat{u}), (0,1 - 0,1) : \hat{u} \in Q, \mathbf{p}_1 \in X\} \\ &= \{(\mathbf{p}_1, \hat{u}), (0,1,1) : \hat{u} \in Q, \mathbf{p}_1 \in X\} \\ &= (\emptyset, E) \end{aligned}$$

**5.11. Definition.** Let  $(F_Q, K)$  and  $(G_Q, L) \in QSVNS(X)$ . Then the union of two  $Q$ -SVNSSs  $(F_Q, K)$  and  $(G_Q, L)$  is the  $Q$ -SVNSS,  $(M_Q, N)$  written as  $(F_Q, K) \cup (G_Q, L) = (M_Q, N)$  where  $N = K \cup L$  for all  $l \in N$  and

$$(M_Q, N) = \begin{cases} F_Q(l) & \text{if } l \in K - L \\ G_Q(l) & \text{if } l \in L - K \\ F_Q(l) \cup G_Q(l) & \text{if } l \in K \cap L \end{cases}$$

**5.12. Example.** Let  $X = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_5\}$  be a universal set,  $E = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5\}$  be a set of parameters and  $Q = \{\hat{u}, \hat{v}, w\}$  be a non-empty set. Let  $N = \{\mathbf{a}_1, \mathbf{a}_3, \mathbf{a}_4\} \subset E$ , and  $M = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$

$$\begin{aligned} (F_Q, N) &= \{(\mathbf{a}_1, ((\mathbf{p}_1, \hat{u}), (0.3,0.4,0.5)), ((\mathbf{p}_1, \hat{v}), (0.5,0.3,0.4)), ((\mathbf{p}_1, w), (0.6,0.1,0.2))) \\ &(\mathbf{a}_3, ((\mathbf{p}_1, \hat{u}), (0.2,0.3,0.4)), ((\mathbf{p}_1, \hat{v}), (0.4,0.2,0.3)), ((\mathbf{p}_1, w), (0.6,0.2,0.4)), ((\mathbf{p}_3, \hat{u}), (0.7,0.1,0.2)), \\ &((\mathbf{p}_3, \hat{v}), (0.8,0.2,0.2)), ((\mathbf{p}_3, w), (0.2,0.4,0.6)))\}, (\mathbf{a}_4, \{((\mathbf{p}_2, \hat{u}), (0.6,0.2,0.1)), ((\mathbf{p}_2, \hat{v}), (0.4,0.2,0.5)) \\ &, ((\mathbf{p}_2, w), (0.5,0.4,0.4))\}, \end{aligned}$$

and

$$\begin{aligned} (G_Q, M) &= \{(\mathbf{a}_1, ((\mathbf{p}_1, \hat{u}), (0.4,0.3,0.5)), ((\mathbf{p}_1, \hat{v}), (0.3,0.3,0.4)), ((\mathbf{p}_1, w), (0.4,0.2,0.3))), \\ &(\mathbf{a}_2, ((\mathbf{p}_2, \hat{u}), (0.4,0.5,0.2)), ((\mathbf{p}_2, \hat{v}), (0.7,0.1,0.1)), ((\mathbf{p}_2, w), (0.6,0.2,0.3)), \\ &(\mathbf{a}_3, \{((\mathbf{p}_1, \hat{u}), (0.4,0.3,0.5)), ((\mathbf{p}_1, \hat{v}), (0.2,0.2,0.4)), ((\mathbf{p}_1, w), (0.4,0.1,0.4)) \\ &, ((\mathbf{p}_3, \hat{v}), (0.6,0.1,0.2)), ((\mathbf{p}_3, w), (0.7,0.2,0.3))\}, \end{aligned}$$

Then

$$\begin{aligned} (K_Q, L) &= \{(\mathbf{a}_1, \{((\mathbf{p}_1, \hat{u}), (0.4,0.3,0.5)), ((\mathbf{p}_1, \hat{v}), (0.5,0.3,0.4)), ((\mathbf{p}_1, w), (0.6,0.1,0.2))\}), \\ &\mathbf{a}_2, ((\mathbf{p}_2, \hat{u}), (0.4,0.5,0.2)), ((\mathbf{p}_2, \hat{v}), (0.7,0.1,0.1)), ((\mathbf{p}_2, w), (0.6,0.2,0.3)), \\ &\mathbf{a}_3, ((\mathbf{p}_1, \hat{u}), (0.4,0.3,0.4)), ((\mathbf{p}_1, \hat{v}), (0.4,0.2,0.3)), ((\mathbf{p}_1, w), (0.6,0.1,0.4)), (\mathbf{p}_3, \hat{u}), (0.8,0.1,0.1), \\ &(\mathbf{p}_3, \hat{v}), (0.8,0.1,0.2), \end{aligned}$$

$$(\mathbf{p}_3, w), (0.7, 0.2, 0.3))\}, \mathbf{a}_4, \{((\mathbf{p}_2, \hat{u}), (0.6, 0.2, 0.1)), ((\mathbf{p}_2, \hat{v}), (0.4, 0.2, 0.5)), ((\mathbf{p}_2, w), (0.5, 0.4, 0.4))\}.$$

**5. 13. Definition.** Let  $(F_Q, K)$  and  $(G_Q, L) \in QSVNSS(X)$ . Then the intersection of two  $Q$  –SVNSSs,  $(F_Q, K)$  and  $(G_Q, L)$  is the  $Q$  – SVNSS  $(M_Q, N)$  written as  $(F_Q, K) \cap (G_Q, L) = (M_Q, N)$  where  $N = K \cap L$  for all  $l \in N$  and

$$(M_Q, N) = \{e, \min(\mu_{F_Q}(\hat{\theta}, \hat{u}), \mu_{G_Q}(\hat{\theta}, \hat{u})), \max(\nu_{F_Q}(\hat{\theta}, \hat{u}), \nu_{G_Q}(\hat{\theta}, \hat{u})), \max(\lambda_{F_Q}(\hat{\theta}, \hat{u}), \lambda_{G_Q}(\hat{\theta}, \hat{u})) : \hat{\theta} \in X, \hat{u} \in Q \text{ and } j = 1, 2, \dots, l\}$$

**5. 14. Example.** Let  $X = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_5\}$  be a universal set,  $E = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5\}$  be a set of parameters and  $Q = \{\hat{u}, \hat{v}, w\}$  be a non-empty set. Let  $N = \{\mathbf{a}_1, \mathbf{a}_3, \mathbf{a}_4\} \subset E$ , and  $M = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$

$$(F_Q, N) = \{(\mathbf{a}_1, ((\mathbf{p}_1, \hat{u}), (0.3, 0.4, 0.5)), ((\mathbf{p}_1, \hat{v}), (0.5, 0.3, 0.4)), ((\mathbf{p}_1, w), (0.6, 0.1, 0.2))), (\mathbf{a}_3, ((\mathbf{p}_1, \hat{u}), (0.2, 0.3, 0.4)), ((\mathbf{p}_1, \hat{v}), (0.4, 0.2, 0.3)), ((\mathbf{p}_1, w), (0.6, 0.2, 0.4)), ((\mathbf{p}_3, \hat{u}), (0.7, 0.1, 0.2)), ((\mathbf{p}_3, \hat{v}), (0.8, 0.2, 0.2)), ((\mathbf{p}_3, w), (0.2, 0.4, 0.6))), (\mathbf{a}_4, \{((\mathbf{p}_2, \hat{u}), (0.6, 0.2, 0.1)), ((\mathbf{p}_2, \hat{v}), (0.4, 0.2, 0.5)), ((\mathbf{p}_2, w), (0.5, 0.4, 0.4))\},$$

and

$$(G_Q, M) = \{(\mathbf{a}_1, ((\mathbf{p}_1, \hat{u}), (0.4, 0.3, 0.5)), ((\mathbf{p}_1, \hat{v}), (0.3, 0.3, 0.4)), ((\mathbf{p}_1, w), (0.4, 0.2, 0.3))), (\mathbf{a}_2, ((\mathbf{p}_2, \hat{u}), (0.4, 0.5, 0.2)), ((\mathbf{p}_2, \hat{v}), (0.7, 0.1, 0.1)), ((\mathbf{p}_2, w), (0.6, 0.2, 0.3))), (\mathbf{a}_3, \{((\mathbf{p}_1, \hat{u}), (0.4, 0.3, 0.5)), ((\mathbf{p}_1, \hat{v}), (0.2, 0.2, 0.4)), ((\mathbf{p}_1, w), (0.4, 0.1, 0.4)), ((\mathbf{p}_3, \hat{v}), (0.6, 0.1, 0.2)), ((\mathbf{p}_3, w), (0.7, 0.2, 0.3))\},$$

Then

$$(K_Q, L) = \{(\mathbf{a}_1, \{((\mathbf{p}_1, \hat{u}), (0.3, 0.4, 0.5)), ((\mathbf{p}_1, \hat{v}), (0.3, 0.3, 0.4)), ((\mathbf{p}_1, w), (0.4, 0.2, 0.3))\}), (\mathbf{a}_3, \{((\mathbf{p}_1, \hat{u}), (0.2, 0.3, 0.5)), ((\mathbf{p}_1, \hat{v}), (0.2, 0.2, 0.4)), ((\mathbf{p}_1, w), (0.4, 0.2, 0.4)), ((\mathbf{p}_3, \hat{u}), (0.7, 0.2, 0.2)), ((\mathbf{p}_3, \hat{v}), (0.6, 0.2, 0.2)), ((\mathbf{p}_3, w), (0.2, 0.4, 0.6))\})$$

**5. 15 Proposition.** Let  $(F_Q, K), (M_Q, N)$  and  $(G_Q, L) \in QSVNSS(X)$ . Then

1.  $(F_Q, K) \cup (\emptyset, K) = (F_Q, K)$
2.  $(F_Q, K) \cup (X, K) = (X, K)$
3.  $(F_Q, K) \cup (F_Q, K) = (F_Q, K)$

$$4. (F_Q, K) \cup (G_Q, L) = (G_Q, L) \cup (F_Q, K)$$

$$5. (F_Q, K) \cup ((G_Q, L) \cup (M_Q, N)) = ((G_Q, L) \cup (F_Q, K)) \cup (M_Q, N)$$

Proof. 1. We have

$$(F_Q, K) = \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \mathbf{v}_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$(\emptyset, K) = \{(\mathbf{p}_1, \hat{u}), (0, 1, 1) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$(F_Q, K) \cup (\emptyset, K)$$

$$= \{k, \left\{ (\mathbf{p}_1, \theta \hat{u}), \max(\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 0), \min(\mathbf{v}_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 1), \min(\lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 1) \right\}\}$$

$$= \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \mathbf{v}_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$= (F_Q, K)$$

2. Let  $(X, K) = (G_Q, K)$  then

$$(F_Q, K) = \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \mathbf{v}_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$(G_Q, L) = \{(\mathbf{p}_1, \hat{u}), (1, 0, 0) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$(F_Q, K) \cup (G_Q, K)$$

$$= \{k, \left\{ (\mathbf{p}_1, \hat{u}), \max(\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 1), \min(\mathbf{v}_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 0), \min(\lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 0) \right\}\}$$

$$= \{(\mathbf{p}_1, \hat{u}), (1, 0, 0) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$= (G_Q, K) = (X, K)$$

3. Let

$$(F_Q, K) = \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \mathbf{v}_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$(F_Q, K) \cup (F_Q, K)$$

$$= \left\{ k, \left( \left( (\mathbf{p}_1, \hat{u}), \max(\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \mu_{F_Q(k)}(\mathbf{p}_1, \hat{u})), \min(\mathbf{v}_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \mathbf{v}_{F_Q(k)}(\mathbf{p}_1, \hat{u})), \right) \right. \right. \\ \left. \left. \min(\lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X \right) \right\}$$

$$= \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \mathbf{v}_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\}$$

$$= (F_Q, K)$$

4 and 5 can be proved easily in a similar way.

**5. 16. Proposition.** Let  $(F_Q, K), (M_Q, N)$  and  $(G_Q, L) \in QSVNSS(X)$ . Then

1.  $(F_Q, K) \cap (\emptyset, K) = (\emptyset, K)$
2.  $(F_Q, K) \cap (X, K) = (F_Q, K)$
3.  $(F_Q, K) \cap (F_Q, K) = (F_Q, K)$
4.  $(F_Q, K) \cap (G_Q, L) = (G_Q, L) \cap (F_Q, K)$
5.  $(F_Q, K) \cap ((G_Q, L) \cap (M_Q, N)) = ((G_Q, L) \cap (F_Q, K)) \cap (M_Q, N)$

*Proof.* 1. We have

$$\begin{aligned} (F_Q, K) &= \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \nu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\} \\ (\emptyset, K) &= \{(\mathbf{p}_1, \hat{u}), (0, 1, 1) : \hat{u} \in Q, \mathbf{p}_1 \in X\} \\ (F_Q, K) \cap (\emptyset, K) &= \{k, (\{(\mathbf{p}_1, \hat{u}), \min(\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 0), \max(\nu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 1), \max(\lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 1)\})\} \\ &= \{(\mathbf{x}_1, \hat{u}), (0, 1, 1) : \hat{u} \in Q, \mathbf{p}_1 \in X\} \\ &= (\emptyset, K) \end{aligned}$$

2. Let  $(X, K) = (G_Q, L)$  then

$$\begin{aligned} (F_Q, K) &= \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \nu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\} \\ (G_Q, L) &= \{(\mathbf{p}_1, \hat{u}), (1, 0, 0) : \hat{u} \in Q, \mathbf{p}_1 \in X\} \\ (F_Q, K) \cap (G_Q, L) &= \{k, (\{(\mathbf{p}_1, \hat{u}), \min(\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 1), \max(\nu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 0), \max(\lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u}), 0)\})\} \\ &= \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \nu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\} \\ &= (F_Q, K) \end{aligned}$$

3. Let

$$\begin{aligned} (F_Q, K) &= \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \nu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\} \\ (F_Q, K) \cap (F_Q, K) &= \{(\mathbf{p}_1, \hat{u}), (\min(\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \mu_{F_Q(k)}(\mathbf{p}_1, \hat{u})), \\ &\min(\nu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \nu_{F_Q(k)}(\mathbf{p}_1, \hat{u})), \min(\lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\} \\ &= \{(\mathbf{p}_1, \hat{u}), (\mu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \nu_{F_Q(k)}(\mathbf{p}_1, \hat{u}), \lambda_{F_Q(k)}(\mathbf{p}_1, \hat{u})) : \hat{u} \in Q, \mathbf{p}_1 \in X\} \\ &= (F_Q, K) \end{aligned}$$

4 and 5 can be proved easily in a similar way.

**5. 17. Proposition.** Let  $(F_Q, K)$  and  $(G_Q, L) \in QSVNSS(X)$ . Then

1.  $((F_Q, K) \cup (G_Q, L))^c = (F_Q, K)^c \cap (G_Q, L)^c$
2.  $((F_Q, K) \cap (G_Q, L))^c = (F_Q, K)^c \cup (G_Q, L)^c$

*Proof.* Straightforward

**5. 18. Proposition.** Let  $(F_Q, K), (M_Q, N)$  and  $(G_Q, L) \in QSVNSS(X)$ . Then

$$(F_Q, K) \cap ((G_Q, L) \cup (M_Q, N)) = ((F_Q, K) \cap (G_Q, L)) \cup ((F_Q, K) \cap (M_Q, N))$$

$$(F_Q, K) \cup ((G_Q, L) \cap (M_Q, N)) = ((F_Q, K) \cup (G_Q, L)) \cap ((F_Q, K) \cup (M_Q, N))$$

*Proof.* Straightforward.

**5. 19. Definition.** Let  $(F_Q, K), (M_Q, N)$  and  $(G_Q, L) \in QSVNSS(X)$ . Then the "AND" operation of two  $Q - SVNSSs$   $(F_Q, K)$  and  $(G_Q, L)$  is the  $Q - SVNSS$  denoted by  $(F_Q, K) \wedge (G_Q, L)$  and is defined by

$$(F_Q, K) \wedge (G_Q, L) = (M_Q, K \times L)$$

Where  $M_Q(\gamma, \delta) = F_Q(\gamma) \cap G_Q(\delta)$  for all  $\gamma \in K, \delta \in L$  is the intersection of two  $Q - SVNSSs$ .

**5. 20. Definition.** Let  $(F_Q, K), (M_Q, N)$  and  $(G_Q, L) \in QSVNSS(X)$ . Then the "OR" operation of two  $Q - SVNSSs$   $(F_Q, K)$  and  $(G_Q, L)$  is the  $Q - SVNSS$  denoted by  $(F_Q, K) \vee (G_Q, L)$  and is defined by

$$(F_Q, K) \vee (G_Q, L) = (M_Q, K \times L)$$

Where  $M_Q(\gamma, \delta) = F_Q(\gamma) \cup G_Q(\delta)$  for all  $\gamma \in K, \delta \in L$  is the union of two  $Q - SVNSSs$ .

## Conclusion

In this paper we have inaugurated the concept of Q-SVNS, Multi Q-SVNS. We also gave the concept of Q- SVNSS and studied some related properties with associate proofs. The equality, subset, complement, union, intersection, AND or OR operations have been defined on the Q- SVNSS. This new wing will be more useful than Q-fuzzy soft set, Q-intuitionistic fuzzy soft set and provide a substantial addition to existing theories for handling uncertainties, and pass to possible areas of further research and relevant applications.

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