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CUBIC HYPER *KU*-IDEALS

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Abstract – It is known that, the concept of hyper KU-algebras is a generalization of KU-algebras. In this paper, we define cubic (strong, weak,s-weak) hyper KU-ideals of hyper KU-algebras and related properties are investigated.

Keywords **–** *KU-algebra, hyper KU-algebra, cubic* (strong, weak,s-weak) *hyper KU-ideal.*

1. Introduction

Prabpayak and Leerawat [10,11] introduced a new algebraic structure which is called KU-algebras. They studied ideals and congruences in KU-algebras. Also, they introduced the concept of homomorphism of KU-algebra and investigated some related properties. Moreover, they derived some straightforward consequences of the relations between quotient KU-algebras and isomorphism. Mostafa et al. [7]introduced the notion of fuzzy KU-ideals of KU-algebras and then they investigated several basic properties which are related to fuzzy KU-ideals .The hyper structure theory (called also multi-algebras) is introduced in 1934 by Marty [6] at the 8th congress of Scandinvian Mathematiciens. Around the 40's, several authors worked on hyper groups, especially in France and in the United States, but also in Italy, Russia and Japan. Hyper structures have many applications to several sectors of both pure and applied sciences, since then numerous mathematical papers [2,3,4,8] have been written investigating the algebraic properties of the hyper BCK / BCI- KU algebras. Jun and Xin [3] considered the fuzzification of the notion of a (weak, strong, reflexive) hyper BCK-ideal, and investigated the relations among them. In [8], Mostafa et al. applied the hyper structures to KU- algebras and introduced the concept of a

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hyper KU-algebra which is a generalization of a KU-algebra, and investigated some related properties. They also introduced the notion of a hyper KU-ideal, a weak hyper KU-ideal and gave relations between hyper KU-ideals and weak hyper KU-ideals. Mostafa et al [9] the bipolar fuzzy set theory to the (s-weak-strong) hyper KU-ideals in hyper KU-algebras are applied and discussed. In this paper, we define cubic (strong, weak, s-weak) hyper KU-ideals of hyper KU-algebras and related properties are investigated.

2. Preliminaries

Let H be a nonempty set and $P^*(H) = P(H) \setminus \{\phi\}$ the family of the nonempty subsets of H. A multi valued operation (said also hyper operation) "$\circ$" on H is a function, which associates with every pair $(x, y) \in H \times H = H^2$ a non empty subset of H denoted $x \circ y$.An algebraic hyper structure or simply a hyper structure is a non empty set H endowed with one or more hyper operations.

We shall use the $x \circ y$ instead of $x \circ \{y\}$, $\{x\} \circ y$ or $\{x\} \circ \{y\}$.

Definition2.1[8]. Let H be a nonempty set and "$\circ$" a hyper operation on H, such that $\circ : H \times H \to P^*(H)$. Then H is called a hyper KU-algebra if it contains a constant "0" and satisfies the following axioms: for all $x, y, z \in H$

(HKU_1) $[(y \circ z) \circ (x \circ z)] << x \circ y$
(HKU_2) $x \circ 0 = \{0\}$
(HKU_3) $0 \circ x = \{x\}$
(HKU_4) *if* $x << y$, $y << x$ *implies* $x = y$.

where x << y is defined by $0 \in y \circ x$ and for every $A, B \subseteq H$, $A << B$ is defined by $\forall a \in A, \exists b \in B$ *such that* $a << b$. In such case, we call "<<" the hyper order in H.
Note that if $A, B \subseteq H$, then by $A \circ B$ we mean the subset $\bigcup_{a \in A, b \in B} a \circ b$ *of* H.

Example 2.2. [8] Let $H = \{0,1,2,3\}$ be a set. Define hyper operation $\circ$ on H as follows:

$\circ$	0	1	2	3
0	{0}	{1}	{2}	{3}
1	{0}	{0}	{1}	{3}
2	{0}	{0}	{0}	{0,3}
3	{0}	{0}	{1}	{0,3}

Then $(H, \circ, 0)$ is a hyper KU-algebra.

Proposition 2.3. [8]Let H be a hyper KU-algebra. Then for all $x, y, z \in H$, the following statements hold:
(P_1) $A \subseteq B$ implies $A << B$, for all nonempty subsets A, B of H.

(P_2) $0 \circ 0 = \{0\}$.
(P_3) $0 << x$.
(P_4) $z << z$.
(P_5) $x \circ z << z$
(P_6) $A \circ 0 = \{0\}$.
(P_7) $0 \circ A = A$.
(P_8) $(0 \circ 0) \circ x = \{x\}$ and $(x \circ (0 \circ x)) = \{0\}$.
(P_9) $x \circ x = \{x\} \Leftrightarrow x = 0$

Lemma 2.4. [8] In hyper KU-algebra $(X, \circ, 0)$, the following hold:

$$x << y \quad \text{imply} \quad y \circ z << x \circ z \text{ for all } x, y, z \in X .$$

Lemma 2.5. [8] In hyper KU-algebra $(X, \circ, 0)$, we have

$$z \circ (y \circ x) = y \circ (z \circ x) \text{ for all } x, y, z \in X .$$

Lemma 2.6. [8] For all $x, y, z \in H$, the following statements hold:

(i) $x \circ y << z \Leftrightarrow z \circ y << x$,
(ii) $0 << A \Rightarrow 0 \in A$,
(iii) $y \in (0 \circ x) \Rightarrow y << x$.

Definition 2.7. [8] For a hyper KU-algebras H, a non-empty subsets $I \subseteq H$, containing 0 are called :

1- A weak hyper KU-ideal of H if $a \circ (b \circ c) \subseteq I$ and $b \in I$ imply $a \circ c \in I$.
2- A hyper KU-ideal of H if $a \circ (b \circ c) << I$ and $b \in I$ imply $a \circ c \in I$.
3- A strong hyper KU-ideal of H if $(\forall x, y \in H)((a \circ (b \circ c) \cap I \neq \phi)$ and $b \in I$ imply $a \circ c \in I$.

Example 2.8. [8] Let $H = \{0, a, b, c\}$ be a set with the following Cayley table

∘	0	a	b	c
0	{0}	{a}	{b}	{c}
a	{0}	{0,a}	{0,b}	{b,c}
b	{0}	{0,b}	{0 }	{a}
c	{0}	{0,b}	{0 }	{0,a}

Then H is a hyper KU-algebra. Take $I = \{0, b\}$, then I is a weak hyper ideal, however, not a weak hyper KU-ideal of H as $b \circ (b \circ c) \subseteq I,\ b \in I,\ but\ b \circ c = a \notin I$.

Example 2.9. [8] Let $H = \{0, a, b\}$ be a set with the following Cayley table:

$\circ$	**0**	**a**	**b**
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{0\}$	$\{0,a\}$	$\{b\}$
b	$\{0\}$	$\{b\}$	$\{0,b\}$

Then H is a hyper KU-algebra. Take $I=\{0,b\}$. Then I is a hyper ideal, but not a hyper KU-ideal, since $0\circ(b\circ a)<<I$ and $b\in I$ but $a\notin I$

Here $I=\{0,b\}$ is also a strong hyper ideal but it is not a strong hyper KU-ideal of H, since $0\circ(b\circ a)=\{b\}\cap I\neq\phi$ *and* $b\in I$ *but* $a\notin I$.

Definition 3.10. [1] An interval number is $\tilde{a}=[a_L,a_U]$, where $0\le a_L\le a_U\le 1$.

Let D[0, 1] denote the family of all closed subintervals of [0, 1], i.e.,

$$D[0,1]=\{\tilde{a}=[a_L,a_U]:a_L\le a_U \quad for\ a_L,a_U\in I\}.$$

We define the operations ≤ , ≥ , = ,rmin and rmax in case of two elements in D[0, 1]. We consider two elements $\tilde{a}=[a_L,a_U]$ and $\tilde{b}=[b_L,b_U]$ in D[0, 1]. Then

1- $\tilde{a}\le\tilde{b}$ *iff* $a_L\le b_L, a_U\le b_U$;
2- $\tilde{a}\ge\tilde{b}$ *iff* $a_L\ge b_L, a_U\ge b_U$;
3- $\tilde{a}=\tilde{b}$ *iff* $a_L=b_L, a_U=b_U$;
4- $rmim\{\tilde{a},\tilde{b}\}=[\min\{a_L,b_L\},\min\{a_U,b_U\}]$;
5- $r\max\{\tilde{a},\tilde{b}\}=[\max\{a_L,b_L\},\max\{a_U,b_U\}]$

Here we consider that $\tilde{0}=[0,0]$ as least element and $\tilde{1}=[1,1]$ as greatest element.
Let $\tilde{a}_i\in D[0,1]$, where $i\in\Lambda$. We define

$$r\inf_{i\in\Lambda}\tilde{a}_i=\left[\inf_{i\in\Lambda}(a_i)_L,\inf_{i\in\Lambda}(a_i)_U\right] \text{ and } r\sup_{i\in\Lambda}\tilde{a}_i=\left[\sup_{i\in\Lambda}(a_i)_L,\sup_{i\in\Lambda}(a_i)_U\right]$$

An interval valued fuzzy set (briefly, i-v-f-set) $\tilde{\mu}$ on a set X is defined as

$$\tilde{\mu}=\left\{\left\langle x\ ,\ [\mu^L(x),\mu^U(x)],\ x\in X\right\rangle\right\}$$

where $\tilde{\mu}:X\to D[0,1]$ and $\mu^L(x)\le\mu^U(x)$, for all $x\in X$. A cubic fuzzy set A over a set X (see [5]) is an object having the form $A=\{(x,\tilde{\mu}_A(x),\lambda_A(x))\mid x\in X\}$, where $\tilde{\mu}_A(x)\subseteq D[0,1]$ and $\lambda_A(x)\in[0,1]$ Jun et al. [5], introduced the concept of cubic sets defined on a non-empty set X as objects having the form: $A=\{\langle x,\tilde{\mu}_A(x),\lambda_A(x)\rangle : x\in X\}$, which is briefly denoted by $A=\langle\tilde{\alpha}_A,\lambda_A\rangle$, where the functions $\tilde{\alpha}_A : X \to D[0,1]$ and $\lambda_A : X \to [0,1]$.

3. Cubic Hyper KU–ideals

Now some fuzzy logic concepts are reviewed .A fuzzy set μ in a set H is a function $\mu : H \to [0,1]$. A fuzzy set μ in a set H is said to satisfy the inf (resp. sup) property if for any subset T of H there exists $x_0 \in T$ such that $\mu(x_0)=\inf_{x\in T}\mu(x)$ (resp. $\mu(x_0)=\sup_{x\in T}\mu(x)$). For a fuzzy set μ in X and a $\in$ [0, 1] the set U(μ ; a):={x $\in$ H , μ (x) $\geq$ a}, which is called a level set of μ .

Definition 3.1. A fuzzy set μ in H is said to be a fuzzy hyper KU-subalgebra of H if it satisfies the inequality: $\inf_{z\in x\circ y}\mu(z)\geq\min\{\mu(x),\mu(y)\}\ \forall x,y\in H$.

Proposition 3.2. Let μ be a fuzzy hyper KU-sub-algebra of H. Then $\mu(0)\geq\mu(x)$ for all x $\in H$.

Proof. Using Proposition **2.3** (P_9), we see that 0 $\in$ x ∘ x for all x $\in H$. Hence

$$\inf_{0\in x\circ x}\mu(0)\geq\min\{\mu(x),\mu(x)\}=\mu(x) \text{ for all } x \in H.$$

Example 3.3. Let $H=\{0,a,b\}$ be a set. Define hyper operation ∘ on H as follows:

∘	0	a	b
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{0\}$	$\{0,a\}$	$\{a,b\}$
b	$\{0\}$	$\{0,a\}$	$\{0,a,b\}$

Then $(H,\circ,0)$ is a hyper KU-algebra. Define a fuzzy set μ : H →[0, 1] by

$$\mu(0)=\mu(a)=\alpha_1>\alpha_2=\mu(b)$$

Then μ is a fuzzy hyper sub-algebra of H . A fuzzy set ν : H →[0, 1] defined by

$$\nu(0) = 0.7,\ \nu(a)=0.5 \text{ and } \nu(b)=0.2$$

is also a fuzzy Hyper sub-algebra of H .

Definition3.4. Let X be nonempty set .A cubic set A in X is structure

$$A=\{\langle x,\tilde{\mu}_A(x),\lambda_A(x)\rangle, x\in X\}$$

which is briefly denoted by $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$, where $\tilde{\mu}_A(x) = [\mu_A^L \ , \mu_A^U \ ]$ is an interval value fuzzy set in X and λ_A is an fuzzy set in X.

Definition3.5. For a hyper KU-algebra H , a cubic $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$" in H is called:

(I) Cubic hyper ideal of H ,if K_1: $x << y$ *implies* $\tilde{\mu}(x) \geq \tilde{\mu}(y)$,

$$\lambda_A(x) \leq \lambda_A(y) \, , \tilde{\mu}_A(z) \geq r\min\left\{ \inf_{u \in (y \circ z)} \tilde{\mu}_A(u) \ , \ \tilde{\mu}_A(y) \right\}$$

and

$$\lambda_A(z) \leq \max\left\{ \sup_{a \in (y \circ z)} \mu(a), \mu(y) \right\}$$

(II) Cubic weak hyper ideal of H" if, for any x; y; z ∈ H

$$\tilde{\mu}_A(0) \geq \tilde{\mu}_A(z) \geq r\min\left\{ \inf_{u \in (y \circ z)} \tilde{\mu}_A(u), \tilde{\mu}_A(y) \right\}$$

$$\lambda_A(0) \leq \lambda_A(z) \leq \max\left\{ \sup_{a \in (y \circ z)} \lambda_A(a), \lambda_A(y) \right\}$$

(III) Cubic strong hyper ideal of H" if, for any x; y; z ∈ H

$$\inf_{u \in (y \circ z)} \tilde{\mu}_A(u) \geq \tilde{\mu}_A(z) \geq r\min\left\{ \inf_{u \in (y \circ z)} \tilde{\mu}_A(u), \tilde{\mu}_A(y) \right\}$$

$$\sup_{u \in (y \circ z)} \lambda_A(u) \leq \lambda_A(z) \leq \max\left\{ \sup_{u \in (y \circ z)} \lambda_A(u), \lambda_A(y) \right\}$$

Definition3.6. For a hyper KU-algebra H , a cubic $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$" in H is called:

(I) Cubic hyper KU-ideal of H , if K_1: $x << y$ *implies* $\tilde{\mu}(x) \geq \tilde{\mu}(y)$,

$$\lambda_A(x) \leq \lambda_A(y) \, , \tilde{\mu}_A(x \circ z) \geq r\min\left\{ \inf_{u \in (x \circ (y \circ z)} \tilde{\mu}_A(u) \ , \ \tilde{\mu}_A(y) \right\}$$

and

$$\lambda_A(x \circ z) \leq \max\left\{ \sup_{a \in x \circ (y \circ z)} \mu(a), \mu(y) \right\}$$

(II) Cubic weak hyper KU-ideal of H" if, for any x; y; z ∈ H

$$\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x \circ z) \geq r\min\left\{ \inf_{u \in (x \circ (y \circ z)} \tilde{\mu}_A(u), \tilde{\mu}_A(y) \right\}$$

$$\lambda_A(0) \le \lambda_A(x \circ z) \le \max\left\{ \sup_{a \in x \circ (y \circ z)} \lambda_A(a), \lambda_A(y) \right\}$$

(III) Cubic strong hyper KU-ideal of H" if, for any x; y; z $\in$ H

$$\inf_{u \in x \circ (y \circ z)} \tilde{\mu}_A(u) \ge \tilde{\mu}_A(z) \ge r\min\left\{ \inf_{u \in x \circ (y \circ z)} \tilde{\mu}_A(u), \tilde{\mu}_A(y) \right\},$$

$$\sup_{u \in x \circ (y \circ z)} \lambda_A(u) \le \lambda_A(z) \le \max\left\{ \sup_{u \in x \circ (y \circ z)} \lambda_A(u), \lambda_A(y) \right\}$$

Example 3.7. Let $H = \{0, a, b\}$ be a set with a binary operation $\circ$ as Example 3.3. Then $(H, \circ, 0)$ is a hyper KU-algebra. . Define $\tilde{\mu}_A(x)$, as follows:

$$\tilde{\mu}_A(x) = \begin{cases} [0.2, 0.9] & if\ x = \{0,1\} \\ [0.1, 0.4] & otherwise \end{cases}$$

H	0	1	2	3
$\lambda_A(x)$	0.2	0.2	0.6	0.7

It is easy to check that $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ is cubic hyper KU-ideal of H .

Example 3.8. Let $H = \{0, a, b\}$ be a set. Define hyper operation $\circ$ on H as follows:

$\circ$	0	a	b
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{0\}$	$\{0\}$	$\{b\}$
b	$\{0\}$	$\{a\}$	$\{0, b\}$

Then (H, $\circ$) is a Hyper KU-algebra. Define a cubic set $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ in H by

$$\tilde{\mu}_A(0) = [0.4, 0.9],\ \tilde{\mu}_A(a) = [0.5, 0.7], \tilde{\mu}_A(b) = [0.2, 0.3]$$

and

H	0	1	2	3
$\lambda_A(x)$	0.2	0.3	0.5	0.7

It is easy to check that $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ is a cubic strong hyper KU-ideal of H

Definition 3.9. A cubic set $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ in H is called a cubic s-weak hyper KU-ideal of H if

(i) $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$, $\lambda_A(0) \leq \lambda_a(x)$ $\forall x \in H$,

(ii) for every $x, y, z \in H$ there exists $a \in x \circ (y \circ z)$ such that

$$\tilde{\mu}_A(x \circ z) \geq r\min\left\{ \inf_{a \in x \circ (y \circ z)} \tilde{\mu}_A(a), \tilde{\mu}_A(y) \right\}$$

(iii) $\lambda_A(x \circ z) \leq \max\left\{ \sup_{a \in x \circ (y \circ z)} \lambda_A(a), \lambda_A(y) \right\}$.

Theorem 3.10. Any cubic (weak, strong) hyper KU-ideal is a cubic (weak, strong) hyper ideal.

Proof. Let $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ be cubic a hyper KU-ideal of *H*, we get for any x; y; z $\in H$,

$$\tilde{\mu}_A(x \circ z) \geq r\min\left\{ \inf_{u \in x \circ (y \circ z)} \tilde{\mu}_A(u), \tilde{\mu}_A(y) \right\} \text{ Put } x = 0$$

we get

$$\tilde{\mu}_A(0 \circ z) \geq r\min\left\{ \inf_{u \in 0 \circ (y \circ z)} \tilde{\mu}_A(u) \ , \ \tilde{\mu}_A(y) \right\}$$

which gives,

$$\tilde{\mu}_A(z) \geq r\min\left\{ \inf_{u \in (y \circ z)} \tilde{\mu}_A(u) \ , \ \tilde{\mu}_A(y) \right\}$$

And

$$\lambda_A(x \circ z) \leq \max\left\{ \sup_{u \in x \circ (y \circ z)} \lambda_A(u), \lambda_A(y) \right\}$$

Take x = 0 , we get

$$\lambda_A(0 \circ z) \leq \max\left\{ \sup_{u \in 0 \circ (y \circ z)} \lambda_A(u) \ , \lambda_A(y) \right\}$$

which gives,

$$\lambda_A(z) \leq \max\left\{ \sup_{u \in (y \circ z)} \lambda_A(u) \ , \ \lambda_A(y) \right\}$$

Ending the proof.

Theorem 3.11. Every cubic s-weak hyper KU-ideal of H is a cubic weak hyper KU-ideal.

Proof. Let $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ be a cubic s- weak hyper KU-ideal of *H*, then there exists $a, b \in x \circ (y \circ z)$ such that

$$\tilde{\mu}_A(x\circ z)\geq r\min\left\{\inf_{a\in x\circ(y\circ z)}\tilde{\mu}_A(a),\tilde{\mu}_A(y)\right\}$$

$$\lambda_A(x\circ z)\leq \max\left\{\sup_{b\in x\circ(y\circ z)}\lambda_A(b),\lambda_A(y)\right\}$$

Since $\tilde{\mu}_A(a)\geq \inf_{c\in x\circ(y\circ z)}\tilde{\mu}_A(c)$ and $\lambda_A(b)\leq \sup_{d\in x\circ(y\circ z)}\lambda_A(d)$, it follows that

$$\tilde{\mu}_A(x\circ z)\geq r\min\left\{\inf_{c\in x\circ(y\circ z)}\tilde{\mu}_A(c),\tilde{\mu}_A(y)\right\}$$

$$\lambda_A(x\circ z)\leq \max\left\{\sup_{d\in x\circ(y\circ z)}\lambda_A(d),\lambda_A(y)\right\}$$

Proposition 3.12. Let $A=\langle\tilde{\mu}_A(x),\lambda_A(x)\rangle$ be a cubic weak hyper KU-ideal of *H*. If $A=\langle\tilde{\mu}_A(x),\lambda_A(x)\rangle$ satisfies the inf-sup property, then $A=\langle\tilde{\mu}_A(x),\lambda_A(x)\rangle$ is a cubic s-weak hyper KU -ideal of *H*.

Proof. Since $A=\langle\tilde{\mu}_A(x),\lambda_A(x)\rangle$ satisfies the inf property, there exists $a_0\in x\circ(y\circ z)$,such that $\tilde{\mu}_A(a_0)=\inf_{a_0\in x\circ(y\circ z)}\tilde{\mu}_A(a_0)$. It follows that

$$\tilde{\mu}_A(x\circ z)\geq r\min\left\{\inf_{a\in x\circ(y\circ z)}\tilde{\mu}_A(a)\,,\,\tilde{\mu}_A(y)\right\}$$

And since $A=\langle\tilde{\mu}_A(x),\lambda_A(x)\rangle$ satisfies the sup property, there exists $b_0\in x\circ(y\circ z)$,such that $\lambda_A(b_0)=\sup_{b_0\in x\circ(y\circ z)}\lambda_A(a_0)$ It follows that

$$\lambda_A(x\circ z)\leq \max\left\{\sup_{b\in x\circ(y\circ z)}\lambda_A(b)\,,\,\lambda_A(y)\right\}$$

Proposition 3.13. Let $A=\langle\tilde{\mu}_A(x),\lambda_A(x)\rangle$ be a cubic strong hyper KU-ideal of *H* and let x; y; z $\in$ *H*. Then

(i) $\tilde{\mu}_A(0)\geq\tilde{\mu}_A(x)$, $\lambda_A(0)\leq\lambda_A(x)$
(ii) x << y implies $\tilde{\mu}_A(x)\geq\tilde{\mu}_A(y)$.
(iii) $\tilde{\mu}_A(x\circ z)\geq r\min\{\tilde{\mu}_A(a),\tilde{\mu}_A(y)\},\forall a\in x\circ(y\circ z)$
(v) x << y implies $\lambda_A(x)\leq\lambda_A(y)$
(iv) $\lambda_A(x\circ z)\leq \max\left\{\sup_{b\in x\circ(y\circ z)}\lambda_A(b)\,,\,\lambda_A(y)\right\}$

Proof. (i) Since $0\in x\circ x\ \forall x\in H$, we have

$$\mu(0)\geq\inf_{a\in x\circ x}\mu(a)\geq\mu(x)\,,\ \lambda(0)\leq\sup_{b\in x\circ x}\lambda(b)\leq\lambda(x)$$

which proves (i).

(ii) Let x; y ∈ H be such that x << y. Then $0 \in y \circ x\ \forall x, y \in H$ and so $\inf_{b \in (y \circ x)} \tilde{\mu}_A(b) \leq \tilde{\mu}_A(0)$, it follows from (i) that,

$$\tilde{\mu}_A(x) \geq r\min\left\{ \inf_{a \in (y \circ x)} \tilde{\mu}_A(a), \tilde{\mu}_A(y) \right\} \geq r\min\{\tilde{\mu}_A(0), \tilde{\mu}_A(y)\} = \tilde{\mu}_A(y)$$

$$\lambda_A(x) \leq \max\left\{ \sup_{b \in (y \circ x)} \lambda_A(b), \lambda_A(y) \right\} \leq \max\{\lambda_A(0), \lambda_A(y)\} = \lambda_A(y)$$

(iii) $\tilde{\mu}_A(x \circ z) \geq r\min\left\{ \inf_{a \in x \circ (y \circ z)} \tilde{\mu}_A(a), \tilde{\mu}_A(y) \right\} \geq r\min\{\tilde{\mu}_A(a), \tilde{\mu}_A(y)\}, \forall a \in x \circ (y \circ z)$,

$$\lambda(x \circ z) \leq \max\left\{ \sup_{b \in x \circ (y \circ z)} \lambda(b), \lambda(y) \right\} \leq \max\{\mu(b), \mu(y)\}, \forall b \in x \circ (y \circ z)$$

we conclude that (iii), (v), (iv) are true. Ending the proof.

Proposition 3.14. Every cubic strong hyper KU-ideal is both a cubic s-weak hyper KU-ideal and a cubic hyper KU-ideal.

Proof. Straight forward.

Proposition 3.15. Let $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ be a cubic hyper KU -ideal of H and let $x, y, z \in H$. Then,

(i) $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$, $\lambda_A(0) \leq \lambda_A(x)$

(ii) if $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ satisfies the inf -sup property, then

$$\tilde{\mu}_A(x \circ z) \geq r\min\{\tilde{\mu}_A(a), \tilde{\mu}_A(y)\}, \forall a \in x \circ (y \circ z), \lambda_A(x \circ z) \leq \max\left\{ \sup_{b \in x \circ (y \circ z)} \lambda_A(b), \lambda_A(y) \right\}$$

Proof. (i) Since 0 << x for each $x \in H$; we have $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$, $\lambda_A(0) \leq \lambda_A(x)$ by Definition 3.6(I) and hence (i) holds.

(ii) Since $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ satisfies the inf property, there is $a_0 \in x \circ (y \circ z)$, such that $\tilde{\mu}(a_0) = \inf_{a \in x \circ (y \circ z)} \tilde{\mu}_A(a)$. Hence

$$\tilde{\mu}_A(x \circ z) \geq r\min\left\{ \inf_{a \in x \circ (y \circ z)} \tilde{\mu}_A(a), \tilde{\mu}_A(y) \right\} = r\min\{ \tilde{\mu}_A(a_0), \tilde{\mu}_A(y) \}$$

Since $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ satisfies the sup- property, there is $b_0 \in x \circ (y \circ z)$, such that $\lambda(b_0) = \sup_{b \in x \circ (y \circ z)} \lambda_A(b)$, Hence

$$\lambda_A(x \circ z) \le \max\left\{ \sup_{b \in x\circ(y\circ z)} \lambda_A(b) \ , \lambda_A(y) \right\} = \max\{ \ \lambda_A(b_0) \ , \lambda_A(y) \ \}$$

which implies that (ii) is true. The proof is complete.

Proposition 3.16. Let $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ be a cubic strong hyper KU -ideal of H, then $\tilde{\mu}_A(x \circ z) \ge r\min\left\{ \inf_{a \in x\circ(y\circ z)} \tilde{\mu}_A(a) \ , \tilde{\mu}_A(y) \right\}$ and $\lambda_A(x \circ z) \le \max\left\{ \sup_{b \in x\circ(y\circ z)} \lambda_A(b) \ , \lambda_A(y) \right\}$. $\forall x, y, z \in H$.

Proof. For any $x, y, z \in H$, we have

$$\sup_{a \in x\circ(y\circ z)} \tilde{\mu}_A(a) \ge \inf_{a \in x\circ(y\circ z)} \tilde{\mu}_A(a) \ \text{ and } \inf_{b \in x\circ(y\circ z)} \lambda_A(b) \le \sup_{b \in x\circ(y\circ z)} \lambda_A(b)$$

It follows from the definition, we get

$$\tilde{\mu}_A(x \circ z) \ge r\min\left\{ \sup_{a \in x\circ(y\circ z)} \tilde{\mu}_A(a) \ , \tilde{\mu}_A(y) \right\} \ge r\min\left\{ \inf_{a \in x\circ(y\circ z)} \tilde{\mu}_A(a) \ , \tilde{\mu}_A(y) \right\}$$

and

$$\lambda_A(x \circ z) \le \max\left\{ \inf_{b \in x\circ(y\circ z)} \lambda_A(b) \ , \lambda_A(y) \right\} \le \max\left\{ \sup_{b \in x\circ(y\circ z)} \lambda_A(b) \ , \lambda_A(y) \right\}.$$

Corollary 3.17. (i) Every cubic hyper KU-ideal of *H* is a cubic weak hyper KU-ideal of *H*.
(ii) If $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ is a cubic hyper KU -ideal of H satisfying inf-sup property, then $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ is a cubic s-weak Hyper KU -ideal of *H*.

Proof. Straightforward.

Theorem 3.18 . If $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ is a cubic strong hyper KU-ideal of H , then the set

$$\mu_{t,s} = \{ x \in H, \tilde{\mu}_A(x) \ge \tilde{t}, \lambda_A(x) \le s \}$$

is a strong hyper KU-ideal of H , when $\mu_{t,s} \ne \Phi, \ for\ \tilde{t} \in D[0,1], s \in [0,1]$.

Proof. Let $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ be a cubic strong hyper KU-ideal of H and $\mu_{t,s} \ne \Phi, \ for\ \tilde{t} \in D[0,1], s \in [0,1]$ .Then there $a \in \mu_{t,s}$ and so $\tilde{\mu}_A(a) \ge \tilde{t}, \lambda_A(a) \le s$.

By Proposition 3.13 (i), $\tilde{\mu}_A(0) \ge \tilde{\mu}_A(a) \ge \tilde{t}, \lambda(0) \ge \lambda(a) \le s$ and so $0 \in \mu_{t,s}$. Let $x, y, z \in H$ such that $x \circ (y \circ z) \cap \mu_{t,s} \ne \Phi\ and\ y \in \mu_{t,s}$, Then there exist

$a_0 \in x \circ (y \circ z) \cap \mu_{t,s}$ *and hence* $\tilde{\mu}_A(a_0) \geq \tilde{t}, \lambda_A(a_0) \leq s$ By Definition 3.6(B) (III), we have

$$\tilde{\mu}_A(x \circ z) \geq r\min\left\{\inf_{a \in x \circ (y \circ z)} \tilde{\mu}_A(a), \tilde{\mu}_A(y)\right\} \geq r\min\{\tilde{\mu}_A(a), \tilde{\mu}_A(y)\} \geq r\min\{\tilde{t}, \tilde{t}\} = \tilde{t}$$

and

$$\lambda_A(x \circ z) \leq \max\left\{ \sup_{a \in x \circ (y \circ z)} \lambda_A(a), \lambda_A(y) \right\} = \max\{ \lambda_A(a_0), \lambda_A(y) \} = \max\{ s, s) \} = s$$

So $(x \circ z) \in \mu_{t,s}$. It follows that $\mu_{t,s}$ is a strong hyper KU-ideal of H.

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Conflicts of Interest

State any potential conflicts of interest here or “The author declare no conflict of interest”.

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