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Risk-Based DEA Efficiency and SSD Efficiency of OECD Members Stock Indices

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ABSTRACT

A stock market index gives some illustrative information regarding the financial market. In this study, we are interested in stock indices efficiency of OECD member countries. We use Data Envelopment Analysis (DEA) methodology and Second Order Stochastic Dominance (SSD) Criteria as an efficiency metrics. DEA is a linear programming based technique for measuring the relative efficiency of homogenous decision making units by their input-output rates. In the Risk-Based DEA, traditional and modern risk measures are used as inputs of the model and the mean return as an output. We consider Conditional Value at Risk (CVaR) as a modern risk measure of financial asset returns. Another approach for the efficiency is Stochastic Dominance (SD) rule that takes into account the entire distribution of return, rather than the return distribution characteristics. There are several papers show that SSD constraints related to the CVaR constraints in an optimization model. Therefore, we compare Risk-Based DEA results with optimization problem with SSD constraints in the empirical study. We also test SSD efficiency of stock index pairs. The results are valuable for the asset managers who need to evaluate the performance of a stock index among others.

Keywords: Data Envelopment Analysis, Stochastic Dominance, Conditional Value at Risk, Index Efficiency

Risk-Tabanlı VZA ve Stokastik Baskınlık Kriteri ile OECD Üyelerinin Hisse Senedi Endekslerinin Etkinliği

ÖZ

Bir hisse senedi endeksi finansal piyasalara ilişkin bazı tanımlayıcı bilgiler vermektedir. Bu çalışmada, biz OECD ülkelerinin hisse senetleri etkinliğiyle ilgilenmekteyiz. Etkinlik ölçüsü olarak Veri Zarflama Analizi (VZA) ve İkinci Dereceden Stokastik Baskınlık (İDSB) Kriterini kullanmaktayız. VZA benzer karar verme birimlerinin göreli etkinliğinin ölçümü için bir doğrusal programlama tekniğidir. Risk Tabanlı VZA'da geleneksel ve modern risk ölçüleri modelin girdileri olarak ve ortalama getiri ise çıktı olarak kullanılır. Finansal yatırım getirilerinin modern bir risk ölçüsü olarak Koşullu Riske Maruz Değeri (RMD) dikkate almaktadyız. Etkinlik için bir başka yaklaşım ise getiri dağılımının spesifik karakteristiklerindense dağılımın tamamını dikkate alan Stokastik Baskınlık kuralıdır. Bir optimizasyon modelinde Koşullu RMD kısıtları ile İDSB kısıtlarının ilişkili olduğunu gösteren pek çok çalışma bulunmaktadır. Bu bağlamda, biz Risk Tabanlı VZA ile İDSB kısıtlı optimizasyon problemlerinin çözümlerini uygulamalı olarak bu çalışmada karşılaştırmaktayız. Ayrıca endeks çiftlerinin İDSB etkinliklerini de test etmekteyiz. Sonuçlar bir endeksin diğer endekler arasında getiri-riskleri açısından nasıl bir perfomansa sahip olduğunu göstermesi açısından yatırım yöneticileri için değerlidir.

Anahtar Kelimeler: Veri Zarflama Analizi, Stokastik Baskınlık, Koşullu Riske Maruz Değer, Endeks Etkinliği

1. Introduction

According to the decision-making theory in finance, an investor wants to maximize return under tolerable risk level or minimize risk to obtain desirable return value. To provide this balance portfolio selection problem is modeled and solved by using various decision making tools. Optimal investment decision can be made by identifying the set of efficient portfolios with respect to a chosen class of risk averse investor's utility functions. Markowitz (1952) Mean-Variance Portfolio Theory is an essential study about portfolio selection and based on several distribution characteristics, such as mean and variance of return distribution. When the two parameters of the normal distribution are unknown the Mean-Variance model provide the set of efficient portfolios by using estimates of mean and covariance matrix of return distribution. The theory works quite well when the return distributions are close to normal. A weakness of this approach is the assumption that the investment of a financial instrument has a specific probability distribution. Adding some other modern risk measures, for instance CVaR, to Mean-Variance model, the portfolio analysis has gained enriched perspective in terms of the portfolio return risk. However due to the restrictions on the Mean-Variance models, new optimal selection rules that consider the increasing utility functions are considered by the researchers to analyze the portfolio efficiencies.

This paper considers efficiency of indices by using portfolio selection problem based on the Stochastic Dominance (SD) rule and Data Envelopment Analysis (DEA). SD rule takes into account the entire distribution of return, rather than the return distribution characteristics. New approaches are proposed to find out effective portfolios without the normality assumption of return distribution by using this rule. Stochastic dominance produces an ordered portfolio return series and identifies a portfolio dominating some other portfolios (Levy, 2006). Several applications of stochastic dominance theory to portfolio selection are considered by Hadar and Russell (1971), Whitmore and Findlay (1978), Dentcheva and Ruszczynski (2006), Roman et al. (2006) and Fidan Keçeci et al (2016). Dentcheva and Ruszczynski (2006), Rudolf and Ruszczynski (2008) and Fabian et al. (2011a, 2011b) considered the efficient methods to solve optimization problem with SSD constraints. Roman et al. (2006) developed a portfolio optimization algorithm for SSD efficient portfolios by using SSD with a multi-objective representation of a problem with CVaR in objective. Kuosmanen (2004), Kopa and Chovanec (2008) described SSD portfolio efficiency measure for diversification. In Branda and Kopa (2012), they deal with the efficiency of world stock indices comparing three approaches: Mean-Variance, SD efficiency and DEA.

DEA is another approach to measure efficiency and estimates the technical efficiency of decision making units (DMUs) over inputs and outputs values of DMUs (Charnes et al. 1978). DEA is commonly used to evaluate the relative efficiency of a number of DMUs (Edirisinghe and Zhang, 2008). General form of this efficiency is a ratio of weighted sum of outputs of weighted sum of inputs. There are several papers applied DEA models to portfolio performance Murthi et al. (1997), Basso and Funari (2001), Daraio and Simar (2006) and Edirisinghe and Zhang (2008). In this aspect, if we set inputs as portfolio risk measures and the output as return, DEA ratio generalizes return-risk ratios for comparing investment funds (Murthi et al. 1997). In the study of Murthi et. al. (1997) standard deviation of returns, expense ratio, load and turnover are used as inputs and mean gross return as output while proposing a DEA efficiency index. For a comprehensive survey the reader can see (Lozano and Gutiérrez, 2008).

In our DEA model, one type of risk measure, CVaR at different confidence levels, is considered. Kopa and Chovanec (2008) introduced a SSD portfolio inefficiency measure consistence with CVaR and they used this result to describe a set of SSD efficient portfolios. Therefore, in terms of the inputs of DEA model, CVaR based DEA model is comparable with SSD portfolio efficiency measure.

In this empirical study we apply these two approaches to efficiency analysis of OECD member's stock indices. We consider 22 stock indices and compare the efficient ones that are classified according to different approaches: DEA efficiency, SSD pairwise comparisons and SSD portfolio efficiency. In the next section we mention about modern risk measures, which we use in DEA efficiency model; in the third section we introduce DEA model and fourth section provides the portfolio optimization model with SSD constraints. Additionally, rolling window analysis is taken into consideration to check the stability of efficient indices. Finally, rolling window results obtained by the two approaches are reported comparatively.

2. Modern Risk Measures in Financial Returns

In this section, we mention the calculation of the CVaR that are used as input in the optimization models to measure the efficiencies.

To show how to risk measures can be computed based on discretely distributed returns, we consider a random vector $\boldsymbol{R} = (r_1, r_2, \dots, r_n)'$ of returns of n indices with a discrete probability distribution described by N assuming returns as equally probable scenarios. Following Rockafellar and Uryasev (2002), VaR as a smallest value such that probability that losses exceed or equal to this value is greater or equal to α confidence level. Let R be a random variable and F_R be its distribution function.

$$F_R(x) = P(R \leq x)$$

For a fixed level α we define Value at Risk; $VaR_\alpha(R) = F_R^{-1}(\alpha)$ (Pflug, 2000).

Depending on the VaR definition, CVaR can be explained with the weighted average of VaR and expected losses strictly exceeding VaR[1].

CVaR equals to conditional expectation of $R, (R \geq VaR_\alpha)$. Therefore CVaR can be written by

$$CVaR_\alpha(R) = E\left(R \middle| \mathrm{R} \geq VaR_\alpha(R)\right).$$

We consider CVaR levels as inputs of the Risk-Based DEA model.

3. Data Envelopment Analysis

Data Envelopment Analysis (DEA) is one of the popular decision making tools, was introduced in 1978 (Charnes et al. 1978). DEA is a linear programming based method that evaluates the efficiency of decision making units' (DMU). The main assumption

[1] The expected losses strictly exceeding VaR is called Mean Excess Loss and Expected Shortfall. The expected losses, which are equal to or exceed VaR are called with Tail VaR.

of DEA methodology is that homogenous DMU's can be compared. In other words, the DMUs should use same kind of inputs and produce same outputs. In this study, financial stock indices correspond DMUs.

The efficiency score of any DMU is the ratio of the weighted sum of the outputs to the weighted sum of the inputs. This score is calculated relatively overall other DMUs.

DEA efficiency of unit o is evaluated using the following mathematical model:

Parameters:

m: the number of inputs,

s: the number of outputs,

n: the number of DMUs,

x_{ij}: amount of input i for unit j,

y_{rj}: amount of output r for unit j,

v_i: weight assigned to input i,

u_j: weight assigned to output j, respectively;

$$\max \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_r x_{io}}$$
$$s.t.\ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_r x_{ij}} \leq 1 \quad j = 1, 2, \ldots, n \tag{1}$$
$$u_r, v_i \geq 0$$

The optimization model (1) is solved for every DMU, if the objective is equal to 1; the related DMU (i.e. DMU o) is efficient. This mathematical model (1) can be rewritten as a linear programming model while adding one assumption as a constraint to optimization problem (Charnes et al. 1978):

$$\max \sum_{r=1}^{s} u_r y_{ro}$$
$$s.t.\ \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_r x_{ij} \leq 1 \tag{2}$$
$$\sum_{r=1}^{s} v_i x_{io}$$
$$u_r, v_i \geq 0$$

DEA as a nonparametric analysis technique permits to rank the indices of OECD member countries based on their efficiency scores. In the comparison of DEA efficiencies of OECD member countries; the risk measures of the stock indices are used as inputs and the mean gross return as the output of the model. Generally we can choose different risk measures as inputs, in this study we select standard deviation as a traditional and CVaR at several confidence levels as a modern risk measures.

Constant Returns to Scale (CRS) DEA model is preferred according to literature search; in one hand Lamb and Tee (2012) showed that Non-Increasing Returns to Scale (NRS) model is appropriate to measure the efficiency of index under risk and return. On the other hand NRS results are identical with CRS results. In our study input oriented CRS model is used as in the study of Branda and Kopa (2012).

4. Stochastic Dominance (SD)

Two parameters of the normal distribution, expected portfolio return rate and variance of the portfolio return rate, are identified as two portfolio performance measures respectively by Markowitz (1959) Mean-Variance approach. However, many work revealed that asset returns do not represent normal distribution. Asset manager needs a distribution of portfolio return to forecast the portfolio risk. If a distribution function fits the asset return series well, also provides consistent estimation of return risk.

Therefore, nonparametric measures are another possible approach for measuring portfolio risk. In this approach the portfolio selection problem is related with the distribution-free stochastic dominance. Series applications of stochastic dominance theory to the portfolio problem are represented (Whitmore and Findlay, 1978). Recently new approaches have been developed showing that dominance constrained portfolio optimization problem can be efficiently solved. Dentcheva and Ruszczynski (2006) introduced a new portfolio optimization model involving SD constraints on the portfolio return rate.

4.1. Definition of the SSD Criteria

For two integrable random variables X and Y, X dominates Y in the second order if

$$\int_{-\infty}^{n} F_X(t)\,dt \le \int_{-\infty}^{n} F_Y(t)\,dt \quad \forall \eta \in \mathbb{R} \tag{3}$$

and it is denoted by $X \succcurlyeq 2\ Y$ (Hadar and Russel, 1969). By using the shortfalls of a random variable X for each target value η, the second order stochastic dominance relation is defined as follows

$$E\left(\left[\eta - X\right]_+\right) \le E\left(\left[\eta - Y\right]_+\right), \quad \forall\ \eta \in \mathbb{R} \tag{4}$$

where, $[\eta - X]_+ = max(0, \eta - X)$. Assuming that η is discrete set of scenarios, relation (4) corresponds to finite number of inequalities

$$E\left(\left[\eta_i - X\right]_+\right) \le E\left(\left[\eta_i - Y\right]_+\right), \quad i = 1, 2, \ldots, N \tag{5}$$

for N number of scenarios. Every single one of these inequalities above, which can be a constraint in an optimization problem, expresses that X dominates Y in the second order for each $\eta_1, \eta_2, \ldots, \eta_N$ respectively (Ogryczak and Ruszczynski, 1999).

4.2. SSD Pairwise Efficiency

Using SSD criteria we can test stock index pairs efficiency. If the Y index has a discrete distribution with realization $Y = y_1, y_2, \dots, y_N$, the X index dominates the Y index in the second order

$$E\left(\left[y_i - X\right]_+\right) \le E\left(\left[y_i - Y\right]_+\right), \quad i = 1, 2, \dots, N \tag{6}$$

Branda and Kopa (2012) determined efficiency between stock indices checking against their sorted returns in ascending order. Then they defined SSD portfolio inefficiency measure (Kopa and Chovanec, 2008) via CVaR constraints that is related to the average of a certain percentages of sorted returns. In our empirical study, we implement SSD pairwise efficiency directly as it is in its definition. However, the optimization problem (5) can be directly considered without a transformation to dual risk space (CVaR) in constraints of the problem.

4.3. SSD Portfolio Efficiency

For N number of scenarios and n number of indices,

$R(w) = (R_1, R_{2,}, \dots, R_N)$, series of random return rates of new portfolio and depends on the weights of the indices by $(w_1, w_{2,}, \dots, w_N)$ in the portfolio. The benchmark index Y has a discrete distribution with realization $y_i, i = 1,2, \dots, N$.

We want to find a new portfolio having maximum return and dominating second order stochastically the benchmark index. The objective function of the optimization problem is expected portfolio return. Therefore, optimization problem with SSD constraints is given in the (7);

$$\begin{aligned} &\max_w \sum_{j=1}^{n} \overline{r}_j \cdot w_j \\ &s.t. \\ &\quad R(w) \ge_2 Y \\ &\quad w \in W \\ &W = \left\{ w \in \mathbb{R}^n : w_j \ge 0, \quad j = 1, 2, \dots, n \right\} \end{aligned} \tag{7}$$

The optimization model (7) finds the portfolio having maximum rate of return, which dominates second order, stochastically the existing one. Therefore the return rate of new portfolio $R(w)$ is a more preferable portfolio than benchmark index Y.

Since benchmark index Y has a discrete distribution with realization $y_i, i = 1,2, \dots, N.$, the relation (7) is equivalent to (Dentcheva and Ruszczynski, 2006)

$$E\left(\left[y_i - R(w)\right]_+\right) \le E\left(\left[y_i - Y\right]_+\right), \quad i = 1, 2, \dots, N \tag{8}$$

for N number of scenarios. If we rewrite the optimization problem more precise:

$$
\begin{aligned}
&\max_w \sum_{j=1}^{n} \bar{r}_j \cdot w_j \\
&s.t. \\
&\quad E\left(\left[y_i - R(w)\right]_+\right) \le E\left(\left[y_i - Y\right]_+\right), \quad i = 1, 2, \dots, N \\
&\quad w \in W \\
&W = \left\{ w \in \mathbb{R}^n : w_j \ge 0, \quad j = 1, 2, \dots, n \right\}
\end{aligned}
\tag{9}
$$

In here;

$$R(w) = (R_1, R_2, \dots, R_N) = \sum_{j=1}^{n} w_j r_{ij}, \;\; i = 1,2, \dots, N$$

i: $1, \dots, N$ (number of scenarios)

j: $1, \dots, n$ (number of indices)

w_j: weights of the the index j in the portfolio

r_{ij}: return rate of index j in scenarios i

$\bar{r}_j$: mean return rate of the index j

p_i : scenarios i, we assume that all the scenarios have equal probability ($p = 1/N$)

There are many efficient implementations of portfolio optimization problems with SSD constraints we have mentioned in this paper before. We use the same optimization model with SSD and solution process in developed in Fidan Keçeci et al (2016) to test SSD portfolio efficiency.

5. Efficiency Analysis of OECD Countries Stock Indices

The data consist of OECD countries financial stock indices. We consider the index returns, r_{ij}, on a weekly basis, were calculated using the natural logarithm of the ratio of the index values, fi,

$$r_{ij} = \ln\left(\frac{f_i}{f_{i-1}}\right)$$

We assumed weekly returns as equally probable scenarios. Currently the number of OECD members is 35, we consider 22 of them due to accessibility of their index values. The data were downloaded from www.investing.com and includes 525 weekly index values between March 4, 2007 and March 19, 2017. The descriptive statistics of the indices, for the 525 weeks of the whole time period are given with the Table 1.

Index	Min	Max	Mean	Std Dev
ASX All Ordinaries	-0.1771	0.0810	0	0.0242
BEL 20 Historical Data	-0.2611	0.0907	-0.0002	0.0301
S&P/TSX Composite	-0.1754	0.1282	0.0003	0.0253
IGPA General	-0.1761	0.1126	0.0011	0.0213
PX	-0.3045	0.1557	-0.0010	0.0327
OMX Copenhagen All Shares	-0.2107	0.1036	0.0014	0.0288
Tallinn SE General	-0.1542	0.1597	0.0005	0.0287
OMX Helsinki	-0.1798	0.1038	-0.0002	0.0318
CAC All Shares	-0.2461	0.1124	0.0001	0.0302
DAX	-0.2435	0.1494	0.0011	0.0328
Athens General-Composite	-0.2254	0.1756	-0.0036	0.0487
ICEX Main	-1.0913	0.0755	-0.0032	0.0568
KOSPI	-0.2293	0.1703	0.0008	0.0289
Riga General	-0.1392	0.1310	0.0002	0.0268
IPC	-0.1793	0.1858	0.0011	0.0291
AEX	-0.2875	0.1248	0.0001	0.0318
Oslo OBX	-0.2478	0.1683	0.0010	0.0355
PSI 20	-0.2057	0.0851	-0.0018	0.0311
Blue-Chip SBITOP	-0.1925	0.0925	-0.0013	0.0269
IBEX 35	-0.2383	0.1110	-0.0006	0.0357
OMX Stockholm	-0.2305	0.1099	0.0008	0.0297
BIST 100	-0.1927	0.1576	0.0015	0.0377

Table 1. Descriptive Statistics

We represent our results via rolling window approach. Rolling window analysis is considered to check the stability of efficient indices during time. From March 4, 2007 to March 19, 2017, we have 525 weekly returns of 22 stock indices. The optimization problems are solved for the time periods including 105 weeks. Therefore, from totally 421 time windows, we have Risk-Based DEA problem and optimization problem with SSD-constraints solutions. Optimization problems of rolling windows are solved in MATLAB R2012b environment.

In the Risk-Based DEA model we chose CVaR as inputs at 0.75, 0.90, 0.95, 0.99 levels. Addition to CVaR inputs, another DEA model is considered with the standard deviation of the stock index returns. As we mentioned before weekly mean gross returns are used as the output in both DEA models. The Risk-Based DEA problems are solved by "linprog" function in MATLAB environment.

Optimization problem with SSD constraints aims to have a portfolio with maximum expected return. If there is no such a portfolio, which second order stochastically dominates the benchmark index, then the benchmark index is SSD portfolio efficient. To solve optimization problems with SSD constraints (9) we have used the "PSG riskprog" subroutine for MATLAB environment. The calculations were performed on a PC having a 2.5-GHz CPU and 8 GByte of RAM. The optimization problem (5) can be directly solved with PSG software without additional coding. More detailed about the implementation of problem can be found at Fidan Keçeci et al. (2016). They presented numerical aspects of portfolio optimization with SSD constraints and they showed that their algorithm works quite efficiently. They used Portfolio Safeguard (PSG) optimization package of AORDA.com, which has precoded functions for optimization with SSD constraints[2].

Firstly, from rolling windows we show the results only for five windows dividing the whole period into five equal time intervals, in Table 2 and Table 3. After that, rolling windows analysis results are listed completely in tables except period information. Our study differs from Branda and Kopa (2012) in one way; they investigated the efficiency for two time periods (before crisis at 2008, during crisis at 2008) but we examine 421 time periods with the help of rolling window analysis. Consequently, our study is an expanded form of theirs.

Table 2 represents the efficiency results that are obtained by the Risk-Based DEA only with CVaR and optimization model with SSD constraints. According to results of two approaches, there is only one efficient index for the first three periods. In the Period4 the models can find an index as efficient commonly. We see that in the last two periods, there are two DEA efficient indices while only there is one SSD portfolio efficient index.

In addition to CVaR, standard deviations of the index returns are added to the Risk-Based DEA model. Efficiency results of the new model are represented in the Table 3.

Index	Period1	Period2	Period3	Period4	Period5
IGPA General	DEA	DEA	-	-	SSD
OMX Copenhagen All Shares	-	SSD	-	-	-
Tallinn SE General	-	-	-	DEA-SSD	DEA
ICEX Main	-	-	DEA	DEA	-
Riga General	SSD	-	-	-	DEA
BIST 100	-	-	SSD	-	-

Table 2. Efficient Indices, Risk-BasedDEA (only with CVaR) and SSD portfolio

Index	Period1	Period2	Period3	Period4	Period5
IGPA General	DEA	DEA	-	-	SSD
OMX Copenhagen All Shares	-	SSD	-	-	-
Tallinn SE General	-	-	-	DEA-SSD	DEA
ICEX Main	-	-	DEA	DEA	-
Riga General	SSD	-	DEA	-	DEA
BIST 100	-	-	SSD	-	-

Table 3. Efficient Indices, Risk-Based DEA (with Standard Deviation and CVaR) and SSD portfolio

According to the results, there is only one more index classified as efficient by Risk-Based DEA model. If we compare the results in Table 2 and Table 3, we may say that there is no significant similarity between Risk-Based DEA and SSD portfolio efficiency scores.

In Table 4, we have listed SSD pairwise efficiency numbers for the five time periods. For instance, in Period2 DAX index dominates number of 9 indices among 22, in second order stochastically (SSD-pairwise efficient). Another index Athens has only one time SSD pairwise efficiency in the all five periods.

[2] American Optimal Decision (www.aorda.com), Gainesville, FL 32611, USA.

5.1. Rolling Window Results

Table 5 shows the indices that are how many times efficient with respect the different approaches. In the 421 time windows, Risk-Based DEA model classifies ICEX Main index that has the most efficiency with 210 times. Similarly Optimization Model with SSD constraints yields OMX Copenhagen All Shares index as SSD-efficient at most 115 times among others. Moreover, in the same time period only three indices, IGPA General, Tallinn SE General, ICEX Main, are classified as DEA-efficient and SSD-efficient together. This provides an important result for the asset managers who seek

for more consistency in his risk attitude. Additionally, we observed that during the 421 time windows, in every time period there is at least one efficient index and there are at most four efficient indices for both two approaches. In other words, at any time window there are maximum four efficient indices with respect to Risk-Based DEA model. Similarly, there are maximum four SSD portfolio efficient indices together. There is no constantly efficient index during all 421 time windows for any approach. To be able to compare two approaches totally; sum of the numbers of efficient indices are represented in Table 5.

Index	Period1	Period2	Period3	Period4	Period5
ASX All Ordinaries	13	3	10	6	3
BEL 20 Historical Data	1	3	2	8	5
S&P/TSX Composite	8	9	5	5	9
IGPA General	18	11	9	1	18
PX	1	2	1	3	3
OMX Copenhagen All Shares	4	7	9	5	5
Tallinn SE General	1	4	10	5	20
OMX Helsinki	2	3	0	5	2
CAC All Shares	4	3	2	5	4
DAX	5	9	1	4	1
Athens General-Comp.	1	0	0	0	0
ICEX Main	0	0	20	10	7
KOSPI	4	9	2	3	12
Riga General	1	2	7	3	19
IPC	11	11	12	3	8
AEX	1	3	2	5	2
Oslo OBX	1	2	1	3	2
PSI 20	8	1	1	1	0
Blue-Chip SBITOP	6	0	1	4	4
IBEX 35	4	0	1	3	1
OMX Stockholm	5	3	1	8	4
BIST 100	2	1	6	1	1

Table 4. SSD Pairwise Efficiency

The number of Risk-Based DEA efficient indices (641) is more than SSD portfolio efficient indices (483). We may conclude that DEA efficiency model is relatively easier to classify efficiency than SSD efficiency for the data in the related time periods. We also see from the Table 5; an approach classifies some indices as efficient many times, while these indices cannot be classified as efficient by other approach. In this respect, Athens General-Comp index has SSD-efficiency 38 times even though it is only one time classified as SSD-pairwise efficient in Table 4. This result puts emphasize on the importance of rolling window analysis.

Index	DEA Efficient (CVaRs&StdDev)	DEA Efficient (CVaRs)	SSD portfolio Efficient	All Approaches
ASX All Ordinaries	4	-	-	-
S&P/TSX Composite	60	48	-	-
IGPA General	178	147	102	43
OMX Copenhagen All Shares	-	-	115	-
Tallinn SE General	117	96	50	30
Athens General-Comp.	-	-	38	-
ICEX Main	210	210	52	8
KOSPI	-	-	4	-
Riga General	160	133	39	-
IPC	-	-	26	-
Oslo OBX	-	-	3	-
Blue-Chip SBITOP	17	7	-	-
BIST 100	-	-	54	-
Total	746	640	483	-

Table 5. Rolling Window: Number of times efficiency in 421 periods.

6. Conclusion

In this paper we have analyzed the efficiency of OECD countries stock indices using two approaches: Risk-Based DEA and Optimization problem with SSD constraints. We implement rolling window analysis to follow up the persistency of efficient index during time periods. We revealed that efficiency of an index mostly depends on the time period and the approach. Thus, there are three indices exhibit consistent efficiency according to the both approaches we use. This result is important for the asset manager who seeks for more consistency in his risk attitude. However, none of these indices exhibit continuity being efficient during the all of the time windows.

We may conclude that optimization problem with SSD constraint classifies the indices as efficient more strictly than Risk-Based DEA. This is consistent with the results of Branda and Kopa (2012). However, on the contrary of theirs when we plug traditional risk measure, standard deviation of index returns to Risk-Based DEA model, it has seen that total number of the efficient indices has increased.

We also represent SSD-pairwise efficiency to see one to one performances of the stock indices. The SSD-pairwise efficiency provides more specific information about the efficiency of the index. Finally, we anticipate that Risk-Based DEA efficiency classification changes depending on the inputs.

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