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Soft β -Separation Axioms in Soft Quad Topological Spaces

Arif Mehmood Khattak¹ <mehdaniyal@gmail.com>
Fahad Jamal² <fahadmath745@gmail.com>
Zaheer Anjum³ <zaheeranjum640@gmail.com>
Younas Khan² <younaswazir5244@gmail.com>
Ishfaq Ahmed² <muhammad.ishfaqtk@gmail.com>

¹Department of Mathematics and Statistics, Riphah International University, Islamabad, Pakistan

²Department of Mathematics, Qurtuba University of Science and Information Technology, Peshawar, Pakistan

³Department of Mathematics, Bannu University of Sciences and Technology, Pakistan

Abstract - The major purpose of this article is to publicize soft β -separations axioms in soft quad topological spaces. We talk over and focus our attention on soft β -separation axioms in soft quad topological spaces with respect to ordinary points and soft points. Moreover study the inherited characteristics at different respects with respect to ordinary points and soft points. Some of their principal properties in soft quad topological spaces are also brought under consideration.

Key words - Soft sets, soft topology, soft β open set, soft β closed set, soft quad topological space, soft βqT_0 structure, soft βqT_1 structure, soft βqT_2 structure, soft βqT_3 structure and soft βqT_4 structure.

1 Introduction

In actual life situation the complexity in economics, engineering, social sciences, medical science etc. we cannot attractively use the outdated classical methods because of different kinds of uncertainties existing in these problems. To finish out these complexity, some types of theories were put forwarded like theory of Fuzzy set, intuitionistic fuzzy set, rough set and bi polar fuzzy sets, inwhich we can safely use mathematical methods for dealing with uncertainties. But, all these theories have their inherent worries. To bury these difficulties in the year 1999, Russian scholar Molodtsov [4] introduced the idea of soft set as a new mathematical methods to deal with uncertainties. Which is free from the above difficulties? Kelly [5] studied bi-topological spaces and discussed different results. Tapi et al. [53] beautifully discussed separation axioms in quad topological spaces. Hameed and Abid [54] discussed separation axioms in Tri-topological spaces.

Recently, in 2011, Shabir and Naz [7] opened the idea of soft topological space and discussed different results with respect to ordinary points. They beautifully defined soft topology as a collection of τ of soft sets over X . they also defined the basic idea of soft topological spaces such as open set and closed soft sets, soft nbd of a point, soft separation axioms, soft regular and soft normal spaces and published their several performances. Soft separation axioms are also discussed at detail. Aktas and Cagman [9] discussed Soft sets and soft groups. Chen [10] discovered the parameterization reduction of soft sets and its applications. Feng et al. [11]. Studied Soft semi rings and its applications. In the recent years, many interesting applications of soft sets theory and soft topology have been discussed at great depth [12,13,14,,15,16,17,18,19,20,21,22]. Kandil et al. [25] explained Soft connectedness via soft ideal developed soft set theory. Kandil et al. [27] launched Soft regularity and normality based on semi open soft sets and soft ideals.

In [28,29,30,31,32,33,34,35,36] discussion is launched soft semi hausdorff spaces via soft ideals, semi open and semi closed sets, separation axioms ,decomposition of some type supra soft sets and soft continuity are discussed. Hussain and Ahmad [51] defined soft points, soft separation axioms in soft topological spaces with respect to soft points and used it in different results, Kandil et al. [52] studied Soft semi separation axioms and some types of soft functions and their characteristics.

In this present paper, concept of soft β -separation axioms in Soft quad topological spaces is introduced with respect to ordinary and soft points.

Many mathematicians threw light on soft separation axioms in soft topological spaces at full length with respect to soft open set, soft b-open set, soft semi-open, soft α -open set and soft β -open set. They also worked over the hereditary properties of different soft topological structures in soft topology. In this present article bridge is built over the gap that exists in soft quad-topology related to soft βqT_0 , soft βqT_1 , soft βqT_2 , soft βqT_3 and soft βqT_4 structures. Some propositions in soft quid topological spaces are discussed with respect to ordinary points and soft points. When we talk about distances between the points in soft topology then the concept of soft separation axioms is automatically taking birth. That is why these structures are catching our attentions. It is hoped that these results will be the driving force for the future study on soft quad topological spaces to achieve general framework for the practical applications and to solve the most complicated problems containing doubts in economics, engineering, medical, environment and in general mechanic systems of various varieties. In future these beautiful soft topological structures may be extended to soft n-topological spaces provided n is even.

2. Preliminaries

The following Definition s which are pre-requisites for present study

Definition 1 [4]. Let X be an initial universe of discourse and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty sub-set of E . A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(X)$

In other words, a set over X is a parameterized family of sub set of universe of discourse X . For $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A)

and if $e \notin A$ then $F(e) = \phi$ that is $F_A = \{F(e) : e \in A \subseteq E, F : A \rightarrow P(X)\}$ the family of all these soft sets over X denoted by $SS(X)_A$

Definition 2 [4]. Let $F_A, G_B \in SS(X)_E$ then F_A is a soft subset of G_B denoted by $F_A \subseteq G_B$, if

1. $A \subseteq B$ and
2. $F(e) \subseteq G(e), \forall e \in A$

In this case F_A is said to be a soft subset of G_B and G_B is said to be a soft super set $F_A, G_B \supseteq F_A$

Definition 3 [6]. Two soft subsets F_A and G_B over a common universe of discourse set X are said to be equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A

Definition 4 [6]. The complement of soft subset (F, A) denoted by $(F, A)^c$ is defined by $(F, A)^c = (F^c, A)$ $F^c \rightarrow P(X)$ is a mapping given by $F^c(e) = U - F(e) \forall e \in A$ and F^c is called the soft complement function of F . Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$

Definition 5 [7]. The difference between two soft subset (G, E) and (F, E) over common of universe discourse X denoted by $(F, E) \setminus (G, E)$ is defined as $F(e) \setminus G(e)$ for all $e \in E$

Definition 6 [7]. Let (G, E) be a soft set over X and $x \in X$ We say that $x \in (F, E)$ and read as x belong to the soft set (F, E) whenever $x \in F(e) \forall e \in E$ The soft set (F, E) over X such that $F(e) = \{x\}, \forall e \in E$ is called singleton soft point and denoted by x , or (x, E)

Definition 7 [6]. A soft set (F, A) over X is said to be Null soft set denoted by $\bar{\emptyset}$ or \emptyset_A if $\forall e \in A, F(e) = \emptyset$

Definition 8 [6]. A soft set (F, A) over X is said to be an absolute soft denoted by \bar{A} or X_A if $\forall e \in A, F(e) = X$

Clearly, we have, $X_A^c = \emptyset_A$ and $\emptyset_A^c = X_A$

Definition 9 [51]. The soft point e_F is said to be in the soft set (G, A) , denoted by $e_F \in (G, A)$ if for the element $e \in A, F(e) \subseteq G(e)$.

Definition 10 [44]. Two soft sets $(G, A), (H, A)$ in $SS(X)_A$ are said to be soft disjoint, written $(G, A) \cap (H, A) = \emptyset_A$ If $G(e) \cap H(e) = \emptyset$ for all $e \in A$.

Definition 11 [51]. The soft point e_G, e_H in X_A are disjoint, written $e_G \neq e_H$ if their corresponding soft sets (G, A) and (H, A) are disjoint.

Definition 12 [6]. The union of two soft sets (F, A) and (G, B) over the common universe of discourse X is the soft set (H, C) , where, $C = A \cup B, \forall e \in C$

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

Written as $(F, A) \cup (G, B) = (H, C)$

Definition 13 [6]. The intersection (H, C) of two soft sets (F, A) and (G, B) over common universe X , denoted $(F, A) \bar{\cap} (G, B)$ is defined as

$$C = A \cap B \text{ and } H(e) = F(e) \cap G(e), \forall e \in C$$

Definition 14 [2]. Let (F, E) be a soft set over X and Y be a non-empty sub set of X . Then the sub soft set of (F, E) over Y denoted by (Y_F, E) , is defined as follow $Y_{F(\alpha)} = Y \cap F(\alpha), \forall \alpha \in E$ in other words

$$(Y_F, E) = Y \cap (F, E).$$

Definition 16 [3]. Let τ be the collection of soft sets over \tilde{X} then τ is said to be a soft topology on \tilde{X} if

1. $\emptyset, \tilde{X} \in \tau$
 2. The union of any number of soft sets in τ belongs to τ
 3. The intersection of any two soft sets in τ belong to τ
- The triplet (\tilde{X}, τ, E) is called a soft topological space.

Definition 17 [1]. Let (\tilde{X}, F, E) be a soft topological space over X , then the member of \mathcal{I} are said to be soft open sets in X .

Definition 18 [1]. Let (X, τ, E) be a soft topological space over X . A soft set (F, A) over \tilde{X} is said to be a soft closed set in X if its relative complement $(F, E)^c$ belong to \mathcal{I} .

Definition 20 [51]. Let (\tilde{X}, τ, E) be a soft topological space and $(F, E) \subseteq SS(\tilde{X})_A$ then (F, E) is called β -open soft set if $((F, E) \subseteq Cl(int(Cl(F, E)))$). The set of all β -open soft set is denoted by $S\beta O(X, \tau, E)$ or $S\beta O(\tilde{X})$ and the set of all β -closed soft set is denoted by $S\beta C(\tilde{X}, \tau, E)$ or $S\beta C(\tilde{X})$.

Proposition 1. Let (X, τ, E) be a soft topological space over X . If (X, τ, E) is soft βT_3 -space, then for all $x \in X, x_E = (x, E)$ is β -closed soft set.

Proposition 2. Let (Y, τ_Y, E) be a soft sub space of a soft topological space (X, τ, E) and $(F, E) \in SS(X)$ then

1. If (F, E) is soft β open set in Y and $Y \in \tau$, then $(F, E) \in \tau$
2. (F, E) is soft β open soft set in Y if and only if $(F, E) = Y \cap (G, E)$ for some $(G, E) \in \tau$.
3. (F, E) is soft β closed soft set in Y if and only if $(F, E) = Y \cap (H, E)$ for some (H, E) is τ soft β closed.

4. Soft β -Separation Axioms in Soft Quad Topological Spaces

In this section we inaugurated soft β separation axioms in soft quad topological space with respect to ordinary points and discussed some results with respect to these points in detail.

Definition 27. Let $(X, \tau_1, E), (X, \tau_2, E), (X, \tau_3, E)$ and (X, τ_4, E) be four different soft topologies on X . Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called a *soft quad topological space*. The soft four topologies $(X, \tau_1, E), (X, \tau_2, E), (X, \tau_3, E)$ and (X, τ_4, E) are independently satisfying the axioms of soft topology. The members of τ_1 are called τ_1 soft open set. And complement of τ_1 Soft open set is called τ_1 soft closed set. Similarly, the member of τ_2 are called τ_2 soft open sets and the complement of τ_2 soft open sets are called τ_2 soft closed set. The members of τ_3 are called τ_3 soft open set. And complement of τ_3 Soft open set is called τ_3 soft closed set and the members of τ_4 are called τ_4 soft open set. And complement of τ_4 Soft open set is called τ_4 soft closed set.

Definition 28. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X and Y be a non-empty subset of X . Then $\tau_{1Y} = \{(Y_F, E): (F, E) \in \tau_1\}, \tau_{2Y} = \{(Y_G, E): (G, E) \in \tau_2\}, \tau_{3Y} = \{(Y_H, E): (H, E) \in \tau_3\}$ and $\tau_{4Y} = \{(Y_I, E): (I, E) \in \tau_4\}$ are said to be the relative topological on Y . Then $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is called relative soft quad-topological space of $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$.

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X , where $(X, \tau_1, E), (X, \tau_2, E), (X, \tau_3, E)$ and (X, τ_4, E) be four different soft topologies on X . Then a sub set (F, E) is said to be quad-open (in short hand q-open) if $(F, E) \subseteq \tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4$ and its complement is said to be soft q-closed.

4.1 Soft β -Separation Axioms of Soft Quad Topological Spaces with Respect to Ordinary Points.

In this section inauguration of soft β separation axioms in soft quad topological space with respect to ordinary points is launched and discussed some eye-catching results with respect to these points in detail.

Definition 29. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X and $x, y \in X$ such that $x \neq y$. If we can find soft q-open sets (F, E) and (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft qT_0 space.

Definition 30. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X and $x, y \in X$ such that $x \neq y$. If we can find two soft q-open sets (F, E) and (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$ then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft qT_1 space.

Definition 31. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X and $x, y \in X$ such that $x \neq y$. If we can find two q- open soft sets such that $x \in (F, E)$ and $y \in (G, E)$ moreover $(F, E) \cap (G, E) = \phi$. Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called a soft qT_2 space.

Definition 32. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft topological space (G, E) be q-closed soft set in X and $x \in X_A$ such that $x \notin (G, E)$. If there occurs soft q-open sets (F_1, E) and (F_2, E) such that $x \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft q-regular spaces. A soft q-regular qT_1 Space is called soft qT_3 space.

Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called a soft q -regular spaces . A soft q -regular T_1 Space is called soft qT_3 space.

Definition 33. $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space $(F_1, E), (G, E)$ be closed soft sets in X such that $(F, E) \cap (G, E) = \varnothing$ If there exists q - open soft sets (F_1, E) and (F_2, E) such that $(F, E) \subseteq (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \varnothing$. Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called a q -soft normal space. A soft q -normal qT_1 Space is called soft qT_4 Space.

Definition 34. Let (X, τ, A) be a soft Topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if there can happen at least one soft semi open set (F_1, A) or (F_2, A) such that $e_G \in (F_1, A), e_H \notin (F_1, A)$ or $e_H \in (F_2, A), e_G \notin (F_2, A)$ then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called a soft qT_0 space.

Definition 35. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft Topological spaces over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if there can happen soft q -open sets (F_1, A) and (F_2, A) such that $e_G \in (F_1, A), e_H \notin (F_1, A)$ and $e_H \in (F_2, A), e_G \notin (F_2, A)$ then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft qT_1 space.

Definition 36. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft Topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if there can happen soft q -open sets (F_1, A) and (F_2, A) such that $e_G \in (F_1, A)$, and $e_H \in (F_2, A), (F_1, A) \cap (F_2, A) = \varnothing$. Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft qT_2 space

Definition 37. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft topological space (G, E) be q -closed soft set in X and $e_G \in X_A$ such that $e_G \notin (G, E)$. If there occurs soft q -open sets (F_1, E) and (F_2, E) such that $e_G \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \varnothing$. Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft q -regular spaces . A soft q -regular qT_1 Space is called soft qT_3 space.

Definition 38. In a soft quad topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$

1) $\tau_1 \cup \tau_2$ is said to be soft βT_0 space with respect to $\tau_3 \cup \tau_4$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists $\tau_1 \cup \tau_2$ soft β -open set (F, E) and a $\tau_3 \cup \tau_4$ soft β -open set (G, E) such that $x \in (F, E)$ and $y \notin (G, E)$ or $y \in (G, E)$ and $x \notin (G, E)$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft βT_0 space with respect to $\tau_1 \cup \tau_2$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists $\tau_3 \cup \tau_4$ soft β open set (F, E) and $\tau_1 \cup \tau_2$ soft β open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$. Soft quad topological spaces $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be pair wise soft βT_0 space if $\tau_1 \cup \tau_2$ is soft βT_0 space with respect to $\tau_3 \cup \tau_4$ and to $\tau_3 \cup \tau_4$ and is soft βT_0 space with respect to $\tau_1 \cup \tau_2$.

2) $\tau_1 \cup \tau_2$ is said to be soft βT_1 space with respect to $\tau_3 \cup \tau_4$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists $\tau_1 \cup \tau_2$ soft β open set (F, E) and to $\tau_3 \cup \tau_4$ soft β open set (G, E) such that $x \in (F, E)$ and $y \notin (G, E)$ and $y \in (G, E)$ and $x \notin (G, E)$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft βT_1 space with respect to $\tau_1 \cup \tau_2$ if for each pair of distinct points $x, y \in X$ such that $x \neq y$ there exists $\tau_3 \cup \tau_4$ soft β open set (F, E) and a to $\tau_1 \cup \tau_2$ soft β open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$. Soft quad topological spaces $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be pair wise soft

βT_1 space if $\tau_1 \cup \tau_2$ is soft βT_1 space with respect to $\tau_3 \cup \tau_4$ and to $\tau_3 \cup \tau_4$ is soft βT_1 space with respect to $\tau_1 \cup \tau_2$.

3) $\tau_1 \cup \tau_2$ is said to be soft βT_2 space with respect to $\tau_1 \cup \tau_2$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists a $\tau_1 \cup \tau_2$ soft β open set (F, E) and a $\tau_3 \cup \tau_4$ soft β open set (G, E) such that $x \in (F, E)$ and $y \in (G, E)$, $(F, E) \cap (G, E) = \phi$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft βT_2 space with respect to $\tau_1 \cup \tau_2$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists a $\tau_3 \cup \tau_4$ soft β open set (F, E) and a $\tau_1 \cup \tau_2$ soft β open set (G, E) such that $x \in (F, E)$, $y \in (G, E)$ and $(F, E) \cap (G, E) = \phi$. The soft quad topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be pair wise soft βT_2 space if $\tau_1 \cup \tau_2$ is soft βT_2 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft βT_2 space with respect to $\tau_1 \cup \tau_2$.

Definition 39. In a soft quad topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$

1) $\tau_1 \cup \tau_2$ is said to be soft βT_3 space with respect to a $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft βT_1 space with respect to $\tau_3 \cup \tau_4$ and for each pair of points $x, y \in X$ such that $x \neq y$ there exists $\tau_1 \cup \tau_2$ Soft β closed set (G, E) such that $x \notin (G, E)$, $\tau_1 \cup \tau_2$ soft β open set (F_1, E) and $\tau_3 \cup \tau_4$ soft β open set (F_2, E) such that $x \in (F_1, E)$, $(G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft βT_3 space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft βT_1 space with respect to $\tau_1 \cup \tau_2$ and for each pair of points $x, y \in X$ such that $x \neq y$ there exists a $\tau_3 \cup \tau_4$ soft β closed set (G, E) such that $x \notin (G, E)$, $\tau_3 \cup \tau_4$ soft β open set (F_1, E) and $\tau_1 \cup \tau_2$ soft β open set (F_2, E) such that $x \in (F_1, E)$, $(G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be pair wise soft βT_3 space if $\tau_1 \cup \tau_2$ is soft βT_3 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft βT_3 space with respect to $\tau_1 \cup \tau_2$.

2) $\tau_1 \cup \tau_2$ is said to be soft βT_4 space with respect to $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft βT_1 space with respect to $\tau_3 \cup \tau_4$, there exists a $\tau_1 \cup \tau_2$ soft β closed set (F_1, E) and $\tau_3 \cup \tau_4$ soft β closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$. Also there exists (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_1 \cup \tau_2$ β open set, (G_1, E) is soft $\tau_3 \cup \tau_4$ β open set such that $(F_1, E) \subseteq (F_3, E)$, $(F_2, E) \subseteq (G_1, E)$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft βT_4 space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft βT_1 space with respect to $\tau_1 \cup \tau_2$, there exists $\tau_3 \cup \tau_4$ soft β closed set (F_1, E) and $\tau_1 \cup \tau_2$ soft β closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$. Also there exist (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_3 \cup \tau_4$ β open set, (G_1, E) is soft $\tau_1 \cup \tau_2$ β open set such that $(F_1, E) \subseteq (F_3, E)$, $(F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$. Thus, (X, τ_1, τ_2, E) is said to be pair wise soft βT_4 space if $\tau_1 \cup \tau_2$ is soft βT_4 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft βT_4 space with respect to $\tau_1 \cup \tau_2$.

Proposition 3. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X . Then, if (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft βT_3 space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft βT_2 space.

Proof: Suppose (X, τ_1, τ_2, E) is a soft βT_3 space with respect to (X, τ_3, τ_4, E) then according to definition for $x, y \in X$, which distinct, by using Proposition 1, (Y, E) is soft β closed set in $\tau_3 \cup \tau_4$ and $x \notin (Y, E)$ there exists a $\tau_1 \cup \tau_2$ soft β open set (F, E) and a $\tau_3 \cup \tau_4$ soft β open set (G, E) such that $x \in (F, E)$, $y \in (Y, E) \subseteq (G, E)$ and $(F, E) \cap (G, E) = \phi$. Hence $\tau_1 \cup \tau_2$ is soft βT_2 space with respect to $\tau_3 \cup \tau_4$. Similarly, if (X, τ_3, τ_4, E) is a soft βT_3 space with respect to (X, τ_1, τ_2, E) then according to definition for $x, y \in X, x \neq y$, by using Theorem 2, (x, E) is β closed soft set in $\tau_1 \cup \tau_2$ and $y \notin (x, E)$ there exists a $\tau_3 \cup \tau_4$ soft β open set (F, E) and a $\tau_1 \cup \tau_2$ soft β open set (G, E) such that $y \in (F, E)$, $x \in (x, E) \subseteq$

(G, E) and $(F_1, E) \cap (F_2, E) = \phi$. Hence $\tau_3 \cup \tau_4$ is soft βT_2 space. This implies that $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft βT_2 space.

Proposition 4. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X. if (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft βT_3 space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft βT_3 space.

Proof: Suppose (X, τ_1, τ_2, E) is a soft βT_3 space with respect to (X, τ_3, τ_4, E) then according to definition for $x, y \in X, x \neq y$ there exists a (X, τ_1, τ_2, E) soft β open set (F, E) and a (X, τ_3, τ_4, E) soft β open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each point $x \in X$ and each (X, τ_1, τ_2, E) β closed soft set (G_1, E) such that $x \notin (G_1, E)$ there exists a (X, τ_1, τ_2, E) soft β open set (F_1, E) and (X, τ_3, τ_4, E) soft β open set (F_2, E) such that $x \in (F_1, E), (G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Similarly, to (X, τ_3, τ_4, E) is a soft βT_3 space with respect to (X, τ_1, τ_2, E) . So according to definition for $x, y \in X, x \neq y$ there exists a (X, τ_3, τ_4, E) soft β open set (F, E) and a (X, τ_1, τ_2, E) soft β open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each point $x \in X$ and each (X, τ_3, τ_4, E) β closed soft set (G_1, E) such that $x \notin (G_1, E)$ there exists (X, τ_3, τ_4, E) soft β open set (F_1, E) and (X, τ_1, τ_2, E) soft β open set (F_2, E) such that $x \in (F_1, E), (G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft βT_3 space.

Proposition 5. If $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X. if (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft βT_4 space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft βT_4 space.

Proof: Suppose (X, τ_1, τ_2, E) is soft βT_4 space with respect to (X, τ_3, τ_4, E) . So according to definition for $x, y \in X, x \neq y$ there exist a (X, τ_1, τ_2, E) soft β open set (F, E) and a (X, τ_3, τ_4, E) soft β open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ each (X, τ_1, τ_2, E) soft β closed set (F_1, E) and a (X, τ_3, τ_4, E) soft β closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$. There exist (F_3, E) and (G_1, E) such that (F_3, E) is soft (X, τ_3, τ_4, E) and soft β open set (G_1, E) is soft (X, τ_1, τ_2, E) β open set $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$. Similarly, (X, τ_3, τ_4, E) is soft βT_4 space with respect to (X, τ_1, τ_2, E) is so according to definition for $x, y \in X, x \neq y$ there exists a (X, τ_3, τ_4, E) soft semi open set (F, E) and a (X, τ_1, τ_2, E) soft β open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each (X, τ_3, τ_4, E) soft β closed set (F_1, E) and (X, τ_1, τ_2, E) soft β closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$. there exists soft β open sets (F_3, E) and (G_1, E) such that (F_3, E) is soft (X, τ_3, τ_4, E) β open set (G_1, E) is soft (X, τ_1, τ_2, E) β open set such that $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$. Hence $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft βT_4 space.

Proposition 6. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X and Y be a non-empty subset of X. if $(X, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft βT_3 space. Then $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft βT_3 space.

Proof: First we prove that $(X, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft βT_1 space. Let $x, y \in X, x \neq y$ if $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise space then this implies that $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft space. So there exists (X, τ_1, τ_2, E) soft β open (F, E) and (X, τ_3, τ_4, E) soft β open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ now

$x \in Y$ and $x \notin (G, E)$. Hence $x \in Y \cap (F, E) = (Y_F, E)$ then $y \notin Y \cap (\alpha)$ for some $\alpha \in E$. this means that $\alpha \in E$ then $y \notin Y \cap F(\alpha)$ for some $\alpha \in E$.

Therefore, $y \notin Y \cap (F, E) = (Y_F, E)$. Now $y \in Y$ and $y \in (G, E)$. Hence $y \in Y \cap (G, E) = (G_Y, E)$ where $(G, E) \in (X, \tau_3, \tau_4, E)$. Consider $x \notin (G, E)$ this means that $\alpha \in E$ then $x \notin Y \cap G(\alpha)$ for some $\alpha \in E$. There fore $x \notin Y \cap (G, E) = (G_Y, E)$ thus $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft βT_1 space.

Now we prove that $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft βT_3 space then $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft β regular space.

Let $y \in Y$ and (G, E) be a soft β closed set in Y such that $y \notin (G, E)$ where $(G, E) \in (X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ then $(G, E) = (Y, E) \cap (F, E)$ for some soft β closed set in $(X, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$. Hence $y \notin (Y, E) \cap (F, E)$ but $y \in (Y, E)$, so $y \notin (F, E)$ since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft βT_3 space $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft β regular space so there exists (X, τ_1, τ_2, E) soft β open set (F_1, E) and (X, τ_3, τ_4, E) soft β open set (F_2, E) such that $y \in (F_1, E), (G, E) \subseteq (F_2, E), (F_1, E) \cap (F_2, E) = \phi$

Take $(G_1, E) = (Y, E) \cap (F_2, E)$ then $(G_1, E), (G_2, E)$ are soft β open set in Y such that $y \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$
 $(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$
 $(G_1, E) \cap (G_2, E) = \phi$

There fore $\tau_{1Y} \cup \tau_{2Y}$ is soft β regular space with respect to $\tau_{3Y} \cup \tau_{4Y}$. Similarly, Let $y \in Y$ and (G, E) be a soft β closed sub set in Y such that $y \notin (G, E)$, where $(G, E) \in (X, \tau_3, \tau_4, E)$ then $(G, E) = (Y, E) \cap (F, E)$ where (F, E) is some soft β closed set in (X, τ_3, τ_4, E) . $y \notin (Y, E) \cap (F, E)$ But $y \in (Y, E)$ so $y \notin (F, E)$ since (X, τ_1, τ_2, E) is soft β regular space so there exists (X, τ_3, τ_4, E) soft β open set (F_1, E) and (X, τ_1, τ_2, E) soft β open set (F_2, E) . Such that

$y \in (F_1, E), (G, E) \subseteq (F_2, E)$
 $(F_1, E) \cap (F_2, E) = \phi$

Take $(G_1, E) = (Y, E) \cap (F_1, E)$
 $(G_2, E) = (Y, E) \cap (F_2, E)$

Then (G_1, E) and (G_2, E) are soft β open set in Y such that

$y \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$
 $(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$

There for $\tau_{3Y} \cup \tau_{4Y}$ is soft β regular space with respect $\tau_{1Y} \cup \tau_{2Y} \Rightarrow (Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft βT_3 space.

Proposition 7. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X and Y be a soft β closed sub space of X . if $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft βT_4 space then $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft βT_4 space.

Proof: Since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft βT_4 space so this implies that $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft βT_1 space as proved above.

We prove $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft β normal space.

Let $(G_1, E), (G_2, E)$ be soft β closed sets in Y such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Then $(G_1, E) = (Y, E) \cap (F_1, E)$

And $(G_2, E) = (Y, E) \cap (F_2, E)$

For some soft β closed sets such that (F_1, E) is soft β closed set in $\tau_1 \cup \tau_2$ soft β closed set (F_2, E) in $\tau_3 \cup \tau_4$.

And $(F_1, E) \cap (F_2, E) = \phi$

From Proposition 2. Since, Y is soft β closed sub set of X then $(G_1, E), (G_2, E)$ are soft β closed sets in X such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft β normal space. So there exists soft β open sets (H_1, E) and (H_2, E) such that

(H_1, E) is soft β open set in $\tau_1 \cup \tau_2$ and (H_2, E) is soft β open set in $\tau_3 \cup \tau_4$ such that

$$(G_1, E) \subseteq (H_1, E)$$

$$(G_2, E) \subseteq (H_2, E)$$

$$(H_1, E) \cap (H_2, E) = \phi$$

Since $(G_1, E), (G_2, E) \subseteq (Y, E)$

Then $(G_1, E) \subseteq (Y, E) \cap (H_1, E)$

$$(G_2, E) \subseteq (Y, E) \cap (H_2, E)$$

And $[(Y, E) \cap (H_1, E)] \cap [(Y, E) \cap (H_2, E)] = \phi$

Where $(Y, E) \cap (H_1, E)$ and $(Y, E) \cap (H_2, E)$ are soft β open sets in Y there fore $\tau_{1Y} \cup \tau_{2Y}$ is soft β normal space with respect to $\tau_{3Y} \cup \tau_{4Y}$. Similarly, let $(G_1, E), (G_2, E)$ be soft β closed sub set in Y such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Then $(G_1, E) = (Y, E) \cap (F_1, E)$

And $(G_2, E) = (Y, E) \cap (F_2, E)$

For some soft β closed sets such that (F_1, E) is soft β closed set in $\tau_3 \cup \tau_4$ and (F_2, E) soft β closed set in $\tau_1 \cup \tau_2$ and

$$(F_1, E) \cap (F_2, E) = \phi$$

From Proposition 2. Since, Y is soft β closed sub set in X then $(G_1, E), (G_2, E)$ are soft β closed sets in X such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft β normal space so there exists soft β open sets (H_1, E) and (H_2, E)

Such that (H_1, E) is soft β open set in $\tau_3 \cup \tau_4$ and (H_2, E) is soft β open set in $\tau_1 \cup \tau_2$ such that

$$(G_1, E) \subseteq (H_1, E)$$

$$(G_2, E) \subseteq (H_2, E)$$

$$(H_1, E) \cap (H_2, E) = \phi$$

Since $(G_1, E), (G_2, E) \subseteq (Y, E)$

Then $(G_1, E) \subseteq (Y, E) \cap (H_1, E)$

$$(G_2, E) \subseteq (Y, E) \cap (H_2, E)$$

And $[(Y, E) \cap (H_1, E)] \cap [(Y, E) \cap (H_2, E)] = \phi$

Where $(Y, E) \cap (H_1, E)$ and $(Y, E) \cap (H_2, E)$ are soft β open sets in Y there fore $\tau_{3Y} \cup \tau_{4Y}$ is soft β normal space with respect to $\tau_{1Y} \cup \tau_{2Y}$

$\Rightarrow (Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft βT_4 space.

4.2 Soft β -Separation Axioms in Soft Quad Topological Spaces with Respect to Soft Points.

In this section, we brought out soft topological structures known as β separation axioms in soft quad topology with respect to soft points. With the applications of this soft β separation axioms different result are brought under examination.

Definition 40. In a soft quad topological space $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$

1) $\tau_1 \cup \tau_2$ said to be soft βT_0 space with respect to $\tau_3 \cup \tau_4$ if for each pair of distinct points $e_G, e_H \in X_A$ there happens $\tau_1 \cup \tau_2$ soft β open set (F, E) and a $\tau_3 \cup \tau_4$ soft β open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (G, E)$, Similarly, $\tau_3 \cup \tau_4$ is said to be soft βT_0 space with respect to $\tau_1 \cup \tau_2$ if for each pair of distinct points $e_G, e_H \in X_A$ there happens $\tau_3 \cup \tau_4$ soft β open set (F, E) and a $\tau_1 \cup \tau_2$ soft β open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$. Soft quad topological spaces $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$ is said to be pair wise soft βT_0 space if $\tau_1 \cup \tau_2$ is soft βT_0 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft βT_0 spaces with respect to $\tau_1 \cup \tau_2$.

2) $\tau_1 \cup \tau_2$ is said to be soft βT_1 space with respect to $\tau_3 \cup \tau_4$ if for each pair of distinct points $e_G, e_H \in X_A$ there happens a $\tau_1 \cup \tau_2$ soft β open set (F, E) and $\tau_3 \cup \tau_4$ soft β open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (G, E)$ and $e_H \in (G, E)$ and $e_G \notin (G, E)$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft βT_1 space with respect to $\tau_1 \cup \tau_2$ if for each pair of distinct points $e_G, e_H \in X_A$ there exist a $\tau_3 \cup \tau_4$ soft β open set (F, E) and a $\tau_1 \cup \tau_2$ soft β open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (G, E)$ and $e_H \in (G, E)$ and $e_G \notin (G, E)$. Soft quad topological space $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$ is said to be pair wise soft βT_1 space if $\tau_1 \cup \tau_2$ is soft βT_1 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft βT_1 spaces with respect to $\tau_1 \cup \tau_2$.

3) $\tau_1 \cup \tau_2$ is said to be soft βT_2 space with respect to $\tau_3 \cup \tau_4$, if for each pair of distinct points $e_G, e_H \in X_A$ there happens a $\tau_1 \cup \tau_2$ soft β open set (F, E) and a $\tau_3 \cup \tau_4$ soft β open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (G, E)$ and $e_H \in (G, E)$ and $e_G \notin (G, E)$ and $(F, E) \cap (G, E) = \phi$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft βT_2 space with respect to $\tau_1 \cup \tau_2$ if for each pair of distinct points $e_G, e_H \in X_A$ there happens a $\tau_3 \cup \tau_4$ soft β open set (F, E) and a $\tau_1 \cup \tau_2$ soft β open set (G, E) such that $e_G \in (F, E)$ and $e_G \in (G, E)$ and $(F, E) \cap (G, E) = \phi$. The soft quad topological space $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$ is said to be pair wise soft βT_2 space if $\tau_1 \cup \tau_2$ is soft βT_2 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft βT_2 space with respect to $\tau_1 \cup \tau_2$.

Definition 41. In a soft quad topological space $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$

1) $\tau_1 \cup \tau_2$ is said to be soft βT_3 space with respect to $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft βT_1 space with respect to $\tau_3 \cup \tau_4$ and for each pair of distinct points $e_G, e_H \in X_A$, there exists a $\tau_1 \cup \tau_2$ β closed soft set (G, E) such that $e_G \notin (G, E)$, $\tau_1 \cup \tau_2$ soft β open set (F_1, E) and $\tau_3 \cup \tau_4$ soft β open set (F_2, E) such that $e_G \in (F_1, E)$, $(G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft βT_3 space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft βT_1 space with respect to $\tau_1 \cup \tau_2$ and for each pair of distinct points $e_G, e_H \in X_A$ there exists a $\tau_3 \cup \tau_4$ soft β closed set (G, E) such that $e_G \notin (G, E)$, $\tau_3 \cup \tau_4$ soft β open set (F_1, E) and $\tau_1 \cup \tau_2$ soft β open set (F_2, E) such that $e_G \in (F_1, E)$, $(G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$ is said to be pair wise soft βT_3 space if $\tau_1 \cup \tau_2$ is soft βT_3 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft βT_3 space with respect to $\tau_1 \cup \tau_2$.

2) $\tau_1 \cup \tau_2$ is said to be soft βT_4 space with respect to $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft βT_1 space with respect to $\tau_3 \cup \tau_4$, there exists a $\tau_1 \cup \tau_2$ soft β closed set (F_1, E) and $\tau_3 \cup \tau_4$ soft β closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \emptyset$, also, there exists (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_1 \cup \tau_2$ β open set, (G_1, E) is soft $\tau_3 \cup \tau_4$ β open set such that $(F_1, E) \subseteq (F_3, E)$, $(F_2, E) \subseteq (G_1, E)$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft βT_4 space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft βT_1 space with respect to $\tau_1 \cup \tau_2$ there exists $\tau_3 \cup \tau_4$ soft β closed set (F_1, E) and $\tau_1 \cup \tau_2$ soft β closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \emptyset$. Also there exists (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_3 \cup \tau_4$ β open set, (G_1, E) is soft $\tau_1 \cup \tau_2$ β soft set such that $(F_1, E) \subseteq (F_3, E)$, $(F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \emptyset$. Thus, $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be pair wise soft βT_4 space if $\tau_1 \cup \tau_2$ is soft βT_4 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft βT_4 space with respect to $\tau_1 \cup \tau_2$.

Proposition 8. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft topological space over X . $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft βT_3 space, then for all $e_G \in X_E$ $e_G = (e_G, E)$ is soft β -closed set.

Proof: We want to prove that e_G is β closed soft set, which is sufficient to prove that e_G^c is β open soft set for all $e_H \in \{e_G\}^c$. Since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft βT_3 space, then there exists soft β sets $(F, E)_{e_H}$ and (G, E) such that $e_{H,E} \subseteq (F, E)_{e_H}$ and $e_{G,E} \cap (F, E)_{e_H} = \emptyset$ and $e_{G,E} \subseteq (G, E)$ and $e_{H,E} \cap (G, E) = \emptyset$. It follows that, $\cup_{e_H \in (e_G)^c} (F, E)_{e_H} \subseteq e_{G,E}^c$. Now, we want to prove that $e_{G,E}^c \subseteq \cup_{e_H \in (e_G)^c} (F, E)_{e_H}$. Let $\cup_{e_H \in (e_G)^c} (F, E)_{e_H} = (H, E)$. Where $H(e) = \cup_{e_H \in (e_G)^c} (F, E)_{e_H}$ for all $e \in E$. Since $e_{G,E}^c(e) = (e_G)^c$ for all $e \in E$ from Definition 9, so, for all $e_H \in \{e_G\}^c$ and $e \in E$ $e_{G,E}^c(e) = \{e_G\}^c = \cup_{e_H \in (e_G)^c} \{e_H\} = \cup_{e_H \in (e_G)^c} (F, E)_{e_H} = H(e)$. Thus, $e_{G,E}^c \subseteq \cup_{e_H \in (e_G)^c} (F, E)_{e_H}$ from Definition 2, and so, $e_{G,E}^c = \cup_{e_H \in (e_G)^c} (F, E)_{e_H}$.

This means that, $e_{G,E}^c$ is soft β open set for all $e_H \in \{e_G\}^c$. Therefore, $e_{G,E}$ is β closed soft set.

Proposition 9. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X . Then, if (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft βT_3 space, then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft βT_2 space.

Proof: Suppose if (X, τ_1, τ_2, E) is a soft βT_3 space with respect to (X, τ_3, τ_4, E) , then according to definition for, $e_G \neq e_H, e_G, e_H \in X_A$, by using Theorem 8, (e_H, E) is soft β closed set in (X, τ_3, τ_4, E) and $e_G \notin (e_H, E)$ there exist a (X, τ_1, τ_2, E) soft β open set (F, E) and a (X, τ_3, τ_4, E) soft β open set (G, E) such that $e_G \in (F, E)$, $e_H \in (y, E) \subseteq (G, E)$ and $(F_1, E) \cap (F_2, E) = \emptyset$. Hence, (X, τ_1, τ_2, E) is soft βT_2 space with respect to (X, τ_3, τ_4, E) . Similarly, if (X, τ_3, τ_4, E) is a soft βT_3 space with respect to (X, τ_1, τ_2, E) , then according to definition for, $e_G \neq e_H, e_G, e_H \in X_A$, by using Theorem 8, (e_G, E) is β closed soft set in (X, τ_1, τ_2, E) and $y \notin (x, E)$ there exists a (X, τ_3, τ_4, E) soft β open set (F, E) and a (X, τ_1, τ_2, E) soft β open set (G, E) such that $e_H \in (F, E)$, $e_G \in (x, E) \subseteq (G, E)$ and $(F_1, E) \cap (F_2, E) = \emptyset$. Hence, (X, τ_3, τ_4, E) is a soft βT_2 space. Thus $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft βT_2 space.

Proposition 10. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X . If (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft βT_3 space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft βT_3 space.

Proof: Suppose (X, τ_1, τ_2, E) is a soft βT_3 space with respect to (X, τ_3, τ_4, E) then according to definition for $e_G, e_H \in X_A, e_G \neq e_H$ there happens $\tau_1 \cup \tau_2$ soft β open set (F, E) and a $\tau_3 \cup \tau_4$ soft β open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$ and for each point $e_G \in X_A$ and each $\tau_1 \cup \tau_2$ β closed soft set (G_1, E) such that $e_G \notin (G_1, E)$ there happens a $\tau_1 \cup \tau_2$ soft β open set (F_1, E) and $\tau_3 \cup \tau_4$ soft β open set (F_2, E) such that $e_G \in (F_1, E), (G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Similarly (X, τ_3, τ_4, E) is a soft βT_3 space with respect to (X, τ_1, E) . So according to definition for $e_G, e_H \in X_A, e_G \neq e_H$ there exists a $\tau_3 \cup \tau_4$ soft β open set (F, E) and $\tau_1 \cup \tau_2$ soft β open set (G, E) such that $e_H \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$ and for each point $e_G \in X_A$ and each $\tau_3 \cup \tau_4$ β closed soft set (G_1, E) such that $e_G \notin (G_1, E)$ there exists $\tau_3 \cup \tau_4$ soft β open set (F_1, E) and $\tau_1 \cup \tau_2$ soft β open set (F_2, E) such that $e_G \in (F_1, E), (G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft βT_3 space.

Proposition 11. If $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X . (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft βT_4 space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft βT_4 space.

Proof: Suppose $((X, \tau_1, \tau_2, E))$ is soft βT_4 space with respect to (X, τ_3, τ_4, E) . So according to definition for $e_G, e_H \in X_A, e_G \neq e_H$ there happens a $\tau_1 \cup \tau_2$ soft β open set (F, E) and a $\tau_3 \cup \tau_4$ soft β open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$ each $\tau_1 \cup \tau_2$ soft β closed set (F_1, E) and a $\tau_3 \cup \tau_4$ soft β closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$. There occurs (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_3 \cup \tau_4$ β open set (G_1, E) is soft a $\tau_1 \cup \tau_2$ β open set $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$. Similarly, $\tau_3 \cup \tau_4$ is soft βT_4 space with respect to $\tau_1 \cup \tau_2$ so according to definition for $e_G, e_H \in X_A, e_G \neq e_H$ there happens a $\tau_3 \cup \tau_4$ soft β open set (F, E) and a $\tau_1 \cup \tau_2$ soft β open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$ and for each $\tau_3 \cup \tau_4$ soft β closed set (F_1, E) and $\tau_1 \cup \tau_2$ soft β closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$. there occurs (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_3 \cup \tau_4$ β open set (G_1, E) is soft $\tau_1 \cup \tau_2$ β open set such that $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$ hence $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft βT_4 space.

Proposition 12. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X and Y be a non-empty subset of X . if $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft βT_3 space. Then $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft βT_3 space.

Proof: First we prove that $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft βT_1 space.

Let $e_G, e_H \in X_A, e_G \neq e_H$ if $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft space then this implies that $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft $\tau_1 \cup \tau_2$ space. So there exists $\tau_1 \cup \tau_2$ soft β open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$ now $e_G \in Y$ and $e_G \notin (G, E)$. Hence $e_G \in Y \cap (F, E) = (Y_F, E)$ then $e_H \notin Y \cap F(\alpha)$ for some $\alpha \in E$. this means that $\alpha \in E$ then $e_H \notin Y \cap F(\alpha)$ for some $\alpha \in E$.

There fore, $e_H \notin Y \cap (F, E) = (Y_F, E)$. Now $e_H \in Y$ and $e_H \in (G, E)$. Hence, $e_H \in Y \cap (G, E) = (G_Y, E)$ where $(G, E) \in \tau_3 \cup \tau_4$. Consider $x \notin (G, E)$. this means that $\alpha \in E$ then $x \notin Y \cap G(\alpha)$ for some $\alpha \in E$. There fore $e_G \notin Y \cap (G, E) = (G_Y, E)$ thus $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft βT_1 space.

Now, we prove that $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft βT_3 space.

Let $e_H \in Y$ and (G, E) be soft β closed set in Y such that $e_H \notin (G, E)$ where $(G, E) \in \tau_1 \cup \tau_2$ then $(G, E) = (Y, E) \cap (F, E)$ for some soft β closed set in $\tau_1 \cup \tau_2$ hence $e_H \notin (Y, E) \cap (F, E)$ but $e_H \in (Y, E)$, so $e_H \notin (F, E)$ since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft βT_3 space $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft β regular space so there happens $\tau_1 \cup \tau_2$ soft β open set (F_1, E) and $\tau_3 \cup \tau_4$ soft β open set (F_2, E) such that $e_H \in (F_1, E), (G, E) \subseteq (F_2, E), (F_1, E)(F_2, E) = \phi$

Take $(G_1, E) = (Y, E) \cap (F_2, E)$ then $(G_1, E), (G_2, E)$ are soft β open sets in Y such that
 $e_H \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$
 $(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$
 $(G_1, E) \cap (G_2, E) = \phi$

Therefore, (τ_{1Y}, τ_{2Y}) is soft β regular space with respect to (τ_{3Y}, τ_{4Y}) . Similarly, Let $e_H \in Y$ and (G, E) be a soft β closed sub set in Y such that $e_H \notin (G, E)$, Where $(G, E) \in \tau_3 \cup \tau_4$ then $(G, E) = (Y, E) \cap (F, E)$ where (F, E) is some soft β closed set in $\tau_3 \cup \tau_4$. $e_H \notin (Y, E) \cap (F, E)$ but $e_H \in (Y, E)$ so $e_H \notin (F, E)$ since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft β regular space so there happens $\tau_3 \cup \tau_4$ soft β open set (F_1, E) and $\tau_1 \cup \tau_2$ soft β open set (F_2, E) . Such that

$e_H \in (F_1, E), (G, E) \subseteq (F_2, E)$
 $(F_1, E) \cap (F_2, E) = \phi$

Take $(G_1, E) = (Y, E) \cap (F_1, E)$
 $(G_2, E) = (Y, E) \cap (F_2, E)$

Then (G_1, E) and (G_2, E) are soft β open set in Y such that

$e_H \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$
 $(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi.$

Therefore (τ_{3Y}, τ_{4Y}) is soft β regular space.

4. Conclusion

A soft set with single specific topological structure is unable to shoulder up the responsibility to construct the whole theory. So to make the theory healthy, some additional structures on soft set has to be introduced. It makes, it more springy to develop the soft topological spaces with its infinite applications. In this regard we introduce strong topological structure known as soft quad topological structure in this paper.

Topology is the supreme branch of pure mathematics which deals with mathematical structures. Freshly, many scholars have studied the soft set theory which is coined by Molodtsov [4] and carefully applied to many difficulties which contain uncertainties in our social life. Shabir and Naz [7] familiarized and deeply studied the origin of soft topological spaces. They also studied topological structures and exhibited their several properties with respect to ordinary points.

In the present work, we constantly study the behavior of soft semi separation axioms in soft quad topological spaces with respect to soft points as well as ordinary points of a soft topological space. We introduce soft βqT_0 structure, soft βqT_1 structure, soft βqT_2 structure, Soft βqT_3 and soft βqT_4 structure with respect to soft and ordinary points. In future we will plant these structures in different results. More over defined soft βT_0 structure

w.r.t. soft βT_1 structure and vice versa, soft βT_1 structure w.r.t soft βT_2 structure and vice versa and soft βT_3 space w.r.t soft βT_4 and vice versa with respect to ordinary and soft points in soft quad topological spaces and studied their activities in different results with respect to ordinary and soft points . We also planted these axioms to different results. These soft semi separation axioms in quad structure would be valuable for the development of the theory of soft topology to solve complicated problems, comprising doubts in economics, engineering, medical etc. We also attractively discussed some soft transmissible properties with respect to ordinary as well as soft points. We expect that these results in this article will do help the researchers for strengthening the toolbox of soft topological structures. In the forthcoming, we spread the idea of soft α - open, and soft b^{**} open sets in soft quad topological structure with respect to ordinary and soft points.

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