http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 04.01.2018 *Year*: **2018**, *Number*: **22**, *Pages*: **39-50**
Published: 26.03.2018 *Original Article*

Partial Constant Hesitant Fuzzy Sets on UP-Algebras

Phakawat Mosrijai[1] <phakawat.mo@gmail.com>
Akarachai Satirad[1] <akarachai.sa@gmail.com>
Aiyared Iampan[1,*] <aiyared.ia@up.ac.th>

[1] *Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand*

Abstaract − In this paper, partial constant hesitant fuzzy sets on UP-algebras are introduced and proved some results. Further, we discuss the relation between partial constant hesitant fuzzy sets and UP-subalgebras (resp. UP-filters, UP-ideals and strongly UP-ideals).

Keywords − *UP-algebra, hesitant fuzzy set, partial constant hesitant fuzzy set.*

1 Introduction and Preliminaries

Among many algebraic structures, algebras of logic form important class of algebras. Examples of these are BCK-algebras [5], BCI-algebras [6], BCH-algebras [3], KU-algebras [15], SU-algebras [12], UP-algebras [4] and others. They are strongly connected with logic. For example, BCI-algebras introduced by Iséki [6] in 1966 have connections with BCI-logic being the BCI-system in combinatory logic which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Iséki [5, 6] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras.

A fuzzy subset f of a set S is a function from S to a closed interval $[0, 1]$. The concept of a fuzzy subset of a set was first considered by Zadeh [21] in 1965. The fuzzy set theories developed by Zadeh and others have found many applications in the domain of mathematics and elsewhere.

In 2009 - 2010, Torra and Narukawa [20, 19] introduced the notion of hesitant fuzzy sets, that is a function from a reference set to a power set of the unit interval. The notion of hesitant fuzzy sets is the other generalization of the notion fuzzy sets.

[*] *Corresponding Author.*

The hesitant fuzzy set theories developed by Torra and others have found many applications in the domain of mathematics and elsewhere.

After the introduction of the notion of hesitant fuzzy sets by Torra and Narukawa [20, 19], several researches were conducted on the generalizations of the notion of hesitant fuzzy sets and application to many logical algebras such as: In 2012, Rodríguez et al. [16] introduced the notion of hesitant fuzzy linguistic term sets and several basic properties and operations to carry out the processes of computing with words. Zhu et al. [22] introduced the notion of dual hesitant fuzzy sets, which is a new extension of fuzzy sets. In 2014, Jun et al. [8] introduced the notions of hesitant fuzzy soft subalgebras and (closed) hesitant fuzzy soft ideals in BCK/BCI-algebras, and investigated related properties. Jun and Song [10] introduced the notions of (Boolean, prime, ultra, good) hesitant fuzzy filters and hesitant fuzzy MV-filters of MTL-algebras, and investigated their relations. In 2015, Ali et al. [1] introduced the notions of hesitant fuzzy products, characteristic hesitant fuzzy sets, hesitant fuzzy $\mathcal{AG}$-groupoids, hesitant fuzzy left (resp. right, twosided) ideals, hesitant fuzzy bi-ideals, hesitant fuzzy interior ideals and hesitant fuzzy quasi-ideals on $\mathcal{AG}$-groupoids, and investigated several properties. They also characterized regular, completely regular, weakly regular and quasi-regular $\mathcal{AG}$-groupoids in term of hesitant fuzzy ideals. Jun and Song [11] introduced the notions of hesitant fuzzy prefilters (resp. filters) and positive implicative hesitant fuzzy prefilters (resp. filters) of EQ-algebras, and investigated several properties. Jun et al. [9] introduced the notions of hesitant fuzzy (generalized) bi-ideals, and investigated related properties. In 2016, Jun and Ahn [7] introduced the notions of hesitant fuzzy subalgebras and hesitant fuzzy ideals of BCK/BCI-algebras, and investigated their relations and properties. Muhiuddin [14] introduced the notion of hesitant fuzzy filters of residuated lattices. In 2017, Mosrijai et al. [13] introduced the notion of hesitant fuzzy sets which is a new extension of fuzzy sets on UP-algebras and the notions of hesitant fuzzy UP-subalgebras, hesitant fuzzy UP-filters, hesitant fuzzy UP-ideals and hesitant fuzzy strongly UP-ideals of UP-algebras and investigated some of its essential properties. Satirad et al. [17] characterized the relationships among (prime, weakly prime) hesitant fuzzy UP-subalgebras (resp. hesitant fuzzy UP-filters, hesitant fuzzy UP-ideals and hesitant fuzzy strongly UP-ideals) and some level subsets of a hesitant fuzzy set on UP-algebras.

The notions of hesitant fuzzy subalgebras, hesitant fuzzy filters and hesitant fuzzy ideals play an important role in studying the many logical algebras. In this paper, partial constant hesitant fuzzy sets are introduced and proved some results. Further, we discuss the relation between partial constant hesitant fuzzy sets and UP-subalgebras (resp. UP-filters, UP-ideals and strongly UP-ideals), and study the concept of prime and weakly prime of subsets and of hesitant fuzzy sets of a UP-algebra.

Before we begin our study, we will introduce the definition of a UP-algebra.

Definition 1.1. [4] An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra*, where A is a nonempty set, $\cdot$ is a binary operation on A, and 0 is a fixed element of A (i.e., a nullary operation) if it satisfies the following axioms: for any $x, y, z \in A$,

(UP-1) $(y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0$,

(UP-2) $0 \cdot x = x$,

(UP-3) $x \cdot 0 = 0$, and

(UP-4) $x \cdot y = y \cdot x = 0$ implies $x = y$.

From [4], we know that the notion of UP-algebras is a generalization of KU-algebras.

Example 1.2. [4] Let X be a universal set. Define a binary operation $\cdot$ on the power set of X by putting $A \cdot B = A' \cap B$ for all $A, B \in \mathcal{P}(X)$. Then $(\mathcal{P}(X), \cdot, \emptyset)$ is a UP-algebra and we shall call it the *power UP-algebra of type 1*.

Example 1.3. [4] Let X be a universal set. Define a binary operation $*$ on the power set of X by putting $A * B = A' \cup B$ for all $A, B \in \mathcal{P}(X)$. Then $(\mathcal{P}(X), *, X)$ is a UP-algebra and we shall call it the *power UP-algebra of type 2*.

Example 1.4. [4] Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	0	0
2	0	1	0	3
3	0	1	2	0

Then $(A, \cdot, 0)$ is a UP-algebra.

In what follows, let A and B denote UP-algebras unless otherwise specified. The following proposition is very important for the study of UP-algebras.

Proposition 1.5. [4] In a UP-algebra A, the following properties hold: for any $x, y, z \in A$,

(1) $x \cdot x = 0$,

(2) $x \cdot y = 0$ and $y \cdot z = 0$ imply $x \cdot z = 0$,

(3) $x \cdot y = 0$ implies $(z \cdot x) \cdot (z \cdot y) = 0$,

(4) $x \cdot y = 0$ implies $(y \cdot z) \cdot (x \cdot z) = 0$,

(5) $x \cdot (y \cdot x) = 0$,

(6) $(y \cdot x) \cdot x = 0$ if and only if $x = y \cdot x$, and

(7) $x \cdot (y \cdot y) = 0$.

Definition 1.6. [4] A subset S of A is called a *UP-subalgebra* of A if the constant 0 of A is in S, and $(S, \cdot, 0)$ itself forms a UP-algebra.

Proposition 1.7. [4] A nonempty subset S of a UP-algebra $A = (A, \cdot, 0)$ is a UP-subalgebra of A if and only if S is closed under the $\cdot$ multiplication on A.

Definition 1.8. [4] A subset B of A is called a *UP-ideal* of A if it satisfies the following properties:

(1) the constant 0 of A is in B, and

(2) for any $x, y, z \in A, x \cdot (y \cdot z) \in B$ and $y \in B$ imply $x \cdot z \in B$.

Definition 1.9. [18] A subset F of A is called a *UP-filter* of A if it satisfies the following properties:

(1) the constant 0 of A is in F, and

(2) for any $x, y \in A, x \cdot y \in F$ and $x \in F$ imply $y \in F$.

Definition 1.10. [2] A subset C of A is called a *strongly UP-ideal* of A if it satisfies the following properties:

(1) the constant 0 of A is in C, and

(2) for any $x, y, z \in A, (z \cdot y) \cdot (z \cdot x) \in C$ and $y \in C$ imply $x \in C$.

From [2], we know that the notion of fuzzy UP-subalgebras is a generalization of fuzzy UP-filters, the notion of fuzzy UP-filters is a generalization of fuzzy UP-ideals, and the notion of fuzzy UP-ideals is a generalization of fuzzy strongly UP-ideals.

Definition 1.11. [18] A nonempty subset B of A is called a *prime subset* of A if it satisfies the following property: for any $x, y \in A$,

$$x \cdot y \in B \text{ implies } x \in B \text{ or } y \in B.$$

Definition 1.12. [18] A UP-subalgebra (resp. UP-filter, UP-ideal, strongly UP-ideal) B of A is called a *prime UP-subalgebra* (resp. *prime UP-filter*, *prime UP-ideal*, *prime strongly UP-ideal*) of A if B is a prime subset of A.

Theorem 1.13. [2] Let S be a subset of A. Then the following statements are equivalent:

(1) S is a prime UP-subalgebra (resp. prime UP-filter, prime UP-ideal, prime strongly UP-ideal) of A,

(2) $S = A$, and

(3) S is a strongly UP-ideal of A.

Definition 1.14. [2] A nonempty subset B of A is called a *weakly prime subset* of A if it satisfies the following property: for any $x, y \in A$ and $x \neq y$,

$$x \cdot y \in B \text{ implies } x \in B \text{ or } y \in B.$$

Definition 1.15. [2] A UP-subalgebra (resp. UP-filter, UP-ideal, strongly UP-ideal) B of A is called a *weakly prime UP-subalgebra* (resp. *weakly prime UP-filter*, *weakly prime UP-ideal*, *weakly prime strongly UP-ideal*) of A if B is a weakly prime subset of A.

Definition 1.16. [19] Let X be a reference set. A *hesitant fuzzy set* on X is defined in term of a function h that when applied to X return a subset of $[0,1]$, that is, $\mathrm{h}\colon X \to \mathcal{P}([0,1])$.

If $Y \subseteq X$, the *characteristic hesitant fuzzy set* h_Y on X is a function of X into $\mathcal{P}([0,1])$ defined as follows:

$$\mathrm{h}_Y(x) = \begin{cases} [0,1] & \text{if } x \in Y, \\ \emptyset & \text{if } x \notin Y. \end{cases}$$

By the definition of characteristic hesitant fuzzy sets, h_Y is a function of X into $\{\emptyset, [0,1]\} \subset \mathcal{P}([0,1])$. Hence, h_Y is a hesitant fuzzy set on X.

If $Y \subseteq X$ and $\varepsilon \in \mathcal{P}([0,1])$, the *partial constant hesitant fuzzy set* $\mathrm{P}_{Y,\varepsilon}$ on X is a function of X into $\mathcal{P}([0,1])$ defined as follows:

$$\mathrm{P}_{Y,\varepsilon}(x) = \begin{cases} [0,1] & \text{if } x \in Y, \\ \varepsilon & \text{if } x \notin Y. \end{cases}$$

By the definition of partial constant hesitant fuzzy sets, $\mathrm{P}_{Y,\varepsilon}$ is a function of X into $\{\varepsilon, [0,1]\} \subset \mathcal{P}([0,1])$. Hence, $\mathrm{P}_{Y,\varepsilon}$ is a hesitant fuzzy set on X. We note that $\mathrm{P}_{Y,\emptyset} = \mathrm{h}_Y$.

Definition 1.17. [13] Let h be a hesitant fuzzy set on A. The hesitant fuzzy set $\overline{\mathrm{h}}$ defined by $\overline{\mathrm{h}}(x) = [0,1] - \mathrm{h}(x)$ for all $x \in A$ is said to be the *complement* of h on A.

Remark 1.18. For all hesitant fuzzy set h on A, we have $\mathrm{h} = \overline{\overline{\mathrm{h}}}$.

Definition 1.19. [13] A hesitant fuzzy set h on A is called a *hesitant fuzzy UP-subalgebra* (HFUPS) of A if it satisfies the following property: for any $x, y \in A$,

$$\mathrm{h}(x \cdot y) \supseteq \mathrm{h}(x) \cap \mathrm{h}(y).$$

By Proposition 1.5 (1), we have $\mathrm{h}(0) = \mathrm{h}(x \cdot x) \supseteq \mathrm{h}(x) \cap \mathrm{h}(x) = \mathrm{h}(x)$ for all $x \in A$.

Definition 1.20. [13] A hesitant fuzzy set h on A is called a *hesitant fuzzy UP-filter* (HFUPF) of A if it satisfies the following properties: for any $x, y \in A$,

(1) $\mathrm{h}(0) \supseteq \mathrm{h}(x)$, and

(2) $\mathrm{h}(y) \supseteq \mathrm{h}(x \cdot y) \cap \mathrm{h}(x)$.

Definition 1.21. [13] A hesitant fuzzy set h on A is called a *hesitant fuzzy UP-ideal* (HFUPI) of A if it satisfies the following properties: for any $x, y, z \in A$,

(1) $\mathrm{h}(0) \supseteq \mathrm{h}(x)$, and

(2) $\mathrm{h}(x \cdot z) \supseteq \mathrm{h}(x \cdot (y \cdot z)) \cap \mathrm{h}(y)$.

Definition 1.22. [13] A hesitant fuzzy set h on A is called a *hesitant fuzzy strongly UP-ideal* (HFSUPS) of A if it satisfies the following properties: for any $x, y, z \in A$,

(1) $\mathrm{h}(0) \supseteq \mathrm{h}(x)$, and

(2) $\mathrm{h}(x) \supseteq \mathrm{h}((z \cdot y) \cdot (z \cdot x)) \cap \mathrm{h}(y)$.

From [13], we know that the notion of hesitant fuzzy UP-ideals of UP-algebras is the generalization of the notion of hesitant fuzzy strongly UP-ideals, the notion of hesitant fuzzy UP-filters of UP-algebras is the generalization of the notion of hesitant fuzzy UP-ideals, and the notion of hesitant fuzzy UP-subalgebras of UP-algebras is the generalization of the notion of hesitant fuzzy UP-filters.

2 Main Results

In this section, we discuss the relation between partial constant hesitant fuzzy sets and UP-subalgebras (resp. UP-filters, UP-ideals and strongly UP-ideals), and study the concept of prime and weakly prime of subsets and of hesitant fuzzy sets of a UP-algebra.

Theorem 2.1. Let S be a nonempty subset of A. Then the following statements hold:

(1) if S is a UP-subalgebra of A, then the partial constant hesitant fuzzy set $\mathrm{P}_{S,\varepsilon}$ is a hesitant fuzzy UP-subalgebra of A for all $\varepsilon \in \mathcal{P}([0,1])$, and

(2) if there exists $\varepsilon \in \mathcal{P}([0,1])$ such that the partial constant hesitant fuzzy set $\mathrm{P}_{S,\varepsilon}$ is a hesitant fuzzy UP-subalgebra of A, then S is a UP-subalgebra of A.

Proof. (1) Assume that S is a UP-subalgebra of A. For any $\varepsilon \in \mathcal{P}([0,1])$ and let $x, y \in A$.

Case 1: $x \in S$ and $y \in S$. Then $\mathrm{P}_{S,\varepsilon}(x) = [0,1]$ and $\mathrm{P}_{S,\varepsilon}(y) = [0,1]$. Thus $\mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y) = [0,1]$. Since S is a UP-subalgebra of A, we have $x \cdot y \in S$ and so $\mathrm{P}_{S,\varepsilon}(x \cdot y) = [0,1]$. Therefore, $\mathrm{P}_{S,\varepsilon}(x \cdot y) = [0,1] \supseteq [0,1] = \mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y)$.

Case 2: $x \in S$ and $y \notin S$. Then $\mathrm{P}_{S,\varepsilon}(x) = [0,1]$ and $\mathrm{P}_{S,\varepsilon}(y) = \varepsilon$. Thus $\mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y) = \varepsilon$. If $x \cdot y \in S$, then $\mathrm{P}_{S,\varepsilon}(x \cdot y) = [0,1]$ and so $\mathrm{P}_{S,\varepsilon}(x \cdot y) = [0,1] \supseteq \varepsilon = \mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y)$. If $x \cdot y \notin S$, then $\mathrm{P}_{S,\varepsilon}(x \cdot y) = \varepsilon$ and so $\mathrm{P}_{S,\varepsilon}(x \cdot y) = \varepsilon \supseteq \varepsilon = \mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y)$. Therefore, $\mathrm{P}_{S,\varepsilon}(x \cdot y) \supseteq \mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y)$.

Case 3: $x \notin S$ and $y \in S$. Then $\mathrm{P}_{S,\varepsilon}(x) = \varepsilon$ and $\mathrm{P}_{S,\varepsilon}(y) = [0,1]$. Thus $\mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y) = \varepsilon$. If $x \cdot y \in S$, then $\mathrm{P}_{S,\varepsilon}(x \cdot y) = [0,1]$ and so $\mathrm{P}_{S,\varepsilon}(x \cdot y) = [0,1] \supseteq \varepsilon = \mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y)$. If $x \cdot y \notin S$, then $\mathrm{P}_{S,\varepsilon}(x \cdot y) = \varepsilon$ and so $\mathrm{P}_{S,\varepsilon}(x \cdot y) = \varepsilon \supseteq \varepsilon = \mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y)$. Therefore, $\mathrm{P}_{S,\varepsilon}(x \cdot y) \supseteq \mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y)$.

Case 4: $x \notin S$ and $y \notin S$. Then $\mathrm{P}_{S,\varepsilon}(x) = \varepsilon$ and $\mathrm{P}_{S,\varepsilon}(y) = \varepsilon$. Thus $\mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y) = \varepsilon$. If $x \cdot y \in S$, then $\mathrm{P}_{S,\varepsilon}(x \cdot y) = [0,1]$ and so $\mathrm{P}_{S,\varepsilon}(x \cdot y) = [0,1] \supseteq \varepsilon = \mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y)$. If $x \cdot y \notin S$, then $\mathrm{P}_{S,\varepsilon}(x \cdot y) = \varepsilon$ and so $\mathrm{P}_{S,\varepsilon}(x \cdot y) = \varepsilon \supseteq \varepsilon = \mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y)$. Therefore, $\mathrm{h}_S(x \cdot y) \supseteq \mathrm{h}_S(x) \cap \mathrm{h}_S(y)$.
Hence, $\mathrm{P}_{S,\varepsilon}$ is a hesitant fuzzy UP-subalgebra of A.

(2) Assume that $\mathrm{P}_{S,\varepsilon}$ is a hesitant fuzzy UP-subalgebra of A for some $\varepsilon \in \mathcal{P}([0,1])$. Let $x, y \in S$. Then $\mathrm{P}_{S,\varepsilon}(x) = [0,1]$ and $\mathrm{P}_{S,\varepsilon}(y) = [0,1]$. Thus $\mathrm{P}_{S,\varepsilon}(x \cdot y) \supseteq \mathrm{P}_{S,\varepsilon}(x) \cap \mathrm{P}_{S,\varepsilon}(y) = [0,1]$, so $\mathrm{P}_{S,\varepsilon}(x \cdot y) = [0,1]$. Since $\varepsilon \neq [0,1]$, we have $x \cdot y \in S$. Hence, S is a UP-subalgebra of A. □

Lemma 2.2. Let B be a nonempty subset of A. Then the following statements hold:

(1) if the constant 0 of A is in B, then $\mathrm{P}_{B,\varepsilon}(0) \supseteq \mathrm{P}_{B,\varepsilon}(x)$ for all $\varepsilon \in \mathcal{P}([0,1])$ and for all $x \in A$, and

(2) if there exists $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$ such that $\mathrm{P}_{B,\varepsilon}(0) \supseteq \mathrm{P}_{B,\varepsilon}(x)$ for all $x \in A$, then the constant 0 of A is in B.

Proof. (1) If $0 \in B$, then $\mathrm{P}_{B,\varepsilon}(0) = [0,1]$ for all $\varepsilon \in \mathcal{P}([0,1])$. Thus $\mathrm{P}_{B,\varepsilon}(0) = [0,1] \supseteq \mathrm{P}_{B,\varepsilon}(x)$ for all $\varepsilon \in \mathcal{P}([0,1])$ and for all $x \in A$.

(2) Assume that there exists $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$ such that $\mathrm{P}_{B,\varepsilon}(0) \supseteq \mathrm{P}_{B,\varepsilon}(x)$ for all $x \in A$. Since B is a nonempty subset of A, we have $a \in B$ for some $a \in A$. Then $\mathrm{P}_{B,\varepsilon}(0) \supseteq \mathrm{P}_{B,\varepsilon}(a) = [0,1]$, so $\mathrm{P}_{B,\varepsilon}(0) = [0,1]$. Since $\varepsilon \neq [0,1]$, we have $0 \in B$. □

Theorem 2.3. Let F be a nonempty subset of A. Then the following statements hold:

(1) if F is a UP-filter of A, then the partial constant hesitant fuzzy set $\mathrm{P}_{F,\varepsilon}$ is a hesitant fuzzy UP-filter of A for all $\varepsilon \in \mathcal{P}([0,1])$, and

(2) if there exists $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$ such that the partial constant hesitant fuzzy set $\mathrm{P}_{F,\varepsilon}$ is a hesitant fuzzy UP-filter of A, then F is a UP-filter of A.

Proof. (1) Assume that F is a UP-filter of A. Let $\varepsilon \in \mathcal{P}([0,1])$. Since $0 \in F$, it follows from Lemma 2.2 (1) that $\mathrm{P}_{F,\varepsilon}(0) \supseteq \mathrm{P}_{F,\varepsilon}(x)$ for all $x \in A$. Next, let $x, y \in A$.

Case 1: $x \in F$ and $y \in F$. Then $\mathrm{P}_{F,\varepsilon}(x) = [0,1]$ and $\mathrm{P}_{F,\varepsilon}(y) = [0,1]$. Therefore, $\mathrm{P}_{F,\varepsilon}(y) = [0,1] \supseteq \mathrm{P}_{F,\varepsilon}(x \cdot y) = \mathrm{P}_{F,\varepsilon}(x) \cap \mathrm{P}_{F,\varepsilon}(x \cdot y)$.

Case 2: $x \notin F$ and $y \in F$. Then $\mathrm{P}_{F,\varepsilon}(x) = \varepsilon$ and $\mathrm{P}_{F,\varepsilon}(y) = [0,1]$. Thus $\mathrm{P}_{F,\varepsilon}(y) = [0,1] \supseteq \varepsilon = \mathrm{P}_{F,\varepsilon}(x) \cap \mathrm{P}_{F,\varepsilon}(x \cdot y)$.

Case 3: $x \in F$ and $y \notin F$. Then $\mathrm{P}_{F,\varepsilon}(x) = [0,1]$ and $\mathrm{P}_{F,\varepsilon}(y) = \varepsilon$. Since F is a UP-filter of A, we have $x \cdot y \notin F$ or $x \notin F$. But $x \in F$, so $x \cdot y \notin F$. Then $\mathrm{P}_{F,\varepsilon}(x \cdot y) = \varepsilon$. Thus $\mathrm{P}_{F,\varepsilon}(y) = \varepsilon \supseteq \varepsilon = \mathrm{P}_{F,\varepsilon}(x) \cap \mathrm{P}_{F,\varepsilon}(x \cdot y)$.

Case 4: $x \notin F$ and $y \notin F$. Then $\mathrm{P}_{F,\varepsilon}(x) = \varepsilon$ and $\mathrm{P}_{F,\varepsilon}(y) = \varepsilon$. Thus $\mathrm{P}_{F,\varepsilon}(y) = \varepsilon \supseteq \varepsilon = \mathrm{P}_{F,\varepsilon}(x) \cap \mathrm{P}_{F,\varepsilon}(x \cdot y)$.

Hence, $\mathrm{P}_{F,\varepsilon}$ is a hesitant fuzzy UP-filter of A.

(2) Assume that $\mathrm{P}_{F,\varepsilon}$ is a hesitant fuzzy UP-filter of A for some $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$. Since $\mathrm{P}_{F,\varepsilon}(0) \supseteq \mathrm{P}_{F,\varepsilon}(x)$ for all $x \in A$, it follows from Lemma 2.2 (2) that $0 \in F$. Next, let $x, y \in A$ be such that $x \cdot y \in F$ and $x \in F$. Then $\mathrm{P}_{F,\varepsilon}(x \cdot y) = [0,1]$ and $\mathrm{P}_{F,\varepsilon}(x) = [0,1]$. Thus $\mathrm{P}_{F,\varepsilon}(y) \supseteq \mathrm{P}_{F,\varepsilon}(x) \cap \mathrm{P}_{F,\varepsilon}(x \cdot y) = [0,1]$, so $\mathrm{P}_{F,\varepsilon}(y) = [0,1]$. Since $\varepsilon \neq [0,1]$, we have $y \in F$ and so F is a UP-filter of A. □

Theorem 2.4. Let B be a nonempty subset of A. Then the following statements hold:

(1) if B is a UP-ideal of A, then the partial constant hesitant fuzzy set $\mathrm{P}_{B,\varepsilon}$ is a hesitant fuzzy UP-ideal of A for all $\varepsilon \in \mathcal{P}([0,1])$, and

(2) if there exists $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$ such that the partial constant hesitant fuzzy set $\mathrm{P}_{B,\varepsilon}$ is a hesitant fuzzy UP-ideal of A, then B is a UP-ideal of A.

Proof. (1) Assume that B is a UP-ideal of A. Let $\varepsilon \in \mathcal{P}([0,1])$. Since $0 \in B$, it follows from Lemma 2.2 (1) that $\mathrm{P}_{B,\varepsilon}(0) \supseteq \mathrm{P}_{B,\varepsilon}(x)$ for all $x \in A$. Next, let $x, y, z \in A$.

Case 1: $x\cdot(y\cdot z) \in B$ and $y \in B$. Then $\mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) = [0,1]$ and $\mathrm{P}_{B,\varepsilon}(y) = [0,1]$. Thus $\mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) \cap \mathrm{P}_{B,\varepsilon}(y) = [0,1]$. Since B is a UP-ideal of A, we have $x\cdot z \in B$ and so $\mathrm{P}_{B,\varepsilon}(x\cdot z) = [0,1]$. Therefore, $\mathrm{P}_{B,\varepsilon}(x\cdot z) = [0,1] \supseteq [0,1] = \mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) \cap \mathrm{P}_{B,\varepsilon}(y)$.

Case 2: $x\cdot(y\cdot z) \in B$ and $y \notin B$. Then $\mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) = [0,1]$ and $\mathrm{P}_{B,\varepsilon}(y) = \varepsilon$. Thus $\mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) \cap \mathrm{P}_{B,\varepsilon}(y) = \varepsilon$. Therefore, $\mathrm{P}_{B,\varepsilon}(x\cdot z) \supseteq \varepsilon = \mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) \cap \mathrm{P}_{B,\varepsilon}(y)$.

Case 3: $x\cdot(y\cdot z) \notin B$ and $y \in B$. Then $\mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) = \varepsilon$ and $\mathrm{P}_{B,\varepsilon}(y) = [0,1]$. Thus $\mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) \cap \mathrm{P}_{B,\varepsilon}(y) = \varepsilon$. Therefore, $\mathrm{P}_{B,\varepsilon}(x\cdot z) \supseteq \varepsilon = \mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) \cap \mathrm{P}_{B,\varepsilon}(y)$.

Case 4: $x\cdot(y\cdot z) \notin B$ and $y \notin B$. Then $\mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) = \varepsilon$ and $\mathrm{P}_{B,\varepsilon}(y) = \varepsilon$. Thus $\mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) \cap \mathrm{P}_{B,\varepsilon}(y) = \varepsilon$. Therefore, $\mathrm{P}_{B,\varepsilon}(x\cdot z) \supseteq \varepsilon = \mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) \cap \mathrm{P}_{B,\varepsilon}(y)$.
Hence, $\mathrm{P}_{B,\varepsilon}$ is a hesitant fuzzy UP-ideal of A.

(2) Assume that $\mathrm{P}_{B,\varepsilon}$ is a hesitant fuzzy UP-ideal of A for some $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$. Since $\mathrm{P}_{B,\varepsilon}(0) \supseteq \mathrm{P}_{B,\varepsilon}(x)$ for all $x \in A$, it follows from Lemma 2.2 (2) that $0 \in B$. Next, let $x, y, z \in A$ be such that $x\cdot(y\cdot z) \in B$ and $y \in B$. Then $\mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) = [0,1]$ and $\mathrm{P}_{B,\varepsilon}(y) = [0,1]$. Thus $\mathrm{P}_{B,\varepsilon}(x\cdot z) \supseteq \mathrm{P}_{B,\varepsilon}(x\cdot(y\cdot z)) \cap \mathrm{P}_{B,\varepsilon}(y) = [0,1]$, so $\mathrm{P}_{B,\varepsilon}(x\cdot z) = [0,1]$. Since $\varepsilon \neq [0,1]$, we have $x\cdot z \in B$ and so B is a UP-ideal of A. □

Theorem 2.5. Let C be a nonempty subset of A. Then the following statements hold:

(1) if C is a strongly UP-ideal of A, then the partial constant hesitant fuzzy set $\mathrm{P}_{C,\varepsilon}$ is a hesitant fuzzy strongly UP-ideal of A for all $\varepsilon \in \mathcal{P}([0,1])$, and

(2) if there exists $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$ such that the partial constant hesitant fuzzy set $\mathrm{P}_{C,\varepsilon}$ is a hesitant fuzzy strongly UP-ideal of A, then C is a strongly UP-ideal of A.

Proof. (1) Assume that C is a strongly UP-ideal of A. Let $\varepsilon \in \mathcal{P}([0,1])$. Since $0 \in C$, it follows form Lemma 2.2 (1) that $\mathrm{P}_{C,\varepsilon}(0) \supseteq \mathrm{P}_{C,\varepsilon}(x)$ for all $x \in A$. Next, let $x, y, z \in A$.

Case 1: $(z\cdot y)\cdot(z\cdot x) \in C$ and $y \in C$. Then $\mathrm{P}_{C,\varepsilon}((z\cdot y)\cdot(z\cdot x)) = [0,1]$ and $\mathrm{P}_{C,\varepsilon}(y) = [0,1]$. Thus $\mathrm{P}_{C,\varepsilon}((z\cdot y)\cdot(z\cdot x)) \cap \mathrm{P}_{C,\varepsilon}(y) = [0,1]$. Since C is a strongly UP-ideal of A, we have $x \in C$ and so $\mathrm{P}_{C,\varepsilon}(x) = [0,1]$. Therefore, $\mathrm{P}_{C,\varepsilon}(x) = [0,1] \supseteq [0,1] = \mathrm{P}_{C,\varepsilon}((z\cdot y)\cdot(z\cdot x)) \cap \mathrm{P}_{C,\varepsilon}(y)$.

Case 2: $(z\cdot y)\cdot(z\cdot x) \in C$ and $y \notin C$. Then $\mathrm{P}_{C,\varepsilon}((z\cdot y)\cdot(z\cdot x)) = [0,1]$ and $\mathrm{P}_{C,\varepsilon}(y) = \varepsilon$. Thus $\mathrm{P}_{C,\varepsilon}((z\cdot y)\cdot(z\cdot x)) \cap \mathrm{P}_{C,\varepsilon}(y) = \varepsilon$. Therefore, $\mathrm{P}_{C,\varepsilon}(x) \supseteq \varepsilon = \mathrm{P}_{C,\varepsilon}((z\cdot y)\cdot(z\cdot x)) \cap \mathrm{P}_{C,\varepsilon}(y)$.

Case 3: $(z\cdot y)\cdot(z\cdot x) \notin C$ and $y \in C$. Then $\mathrm{P}_{C,\varepsilon}((z\cdot y)\cdot(z\cdot x)) = \varepsilon$ and $\mathrm{P}_{C,\varepsilon}(y) = [0,1]$. Thus $\mathrm{P}_{C,\varepsilon}((z\cdot y)\cdot(z\cdot x)) \cap \mathrm{P}_{C,\varepsilon}(y) = \varepsilon$. Therefore, $\mathrm{P}_{C,\varepsilon}(x) \supseteq \varepsilon = \mathrm{P}_{C,\varepsilon}((z\cdot y)\cdot(z\cdot x)) \cap \mathrm{P}_{C,\varepsilon}(y)$.

Case 4: $(z\cdot y)\cdot(z\cdot x) \notin C$ and $y \notin C$. Then $\mathrm{P}_{C,\varepsilon}((z\cdot y)\cdot(z\cdot x)) = \varepsilon$ and $\mathrm{P}_{C,\varepsilon}(y) = \varepsilon$. Thus $\mathrm{P}_{C,\varepsilon}((z\cdot y)\cdot(z\cdot x)) \cap \mathrm{P}_{C,\varepsilon}(y) = \varepsilon$. Therefore, $\mathrm{P}_{C,\varepsilon}(x) \supseteq \varepsilon = \mathrm{P}_{C,\varepsilon}((z\cdot y)\cdot(z\cdot x)) \cap \mathrm{P}_{C,\varepsilon}(y)$.
Hence, $\mathrm{P}_{C,\varepsilon}$ is a hesitant fuzzy strongly UP-ideal of A.

(2) Assume that $\mathrm{P}_{C,\varepsilon}$ is a hesitant fuzzy strongly UP-ideal of A for some $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$. Since $\mathrm{P}_{C,\varepsilon}(0) \supseteq \mathrm{P}_{C,\varepsilon}(x)$ for all $x \in A$, it follows from Lemma

2.2 (2) that $0 \in C$. Next, let $x, y, z \in A$ be such that $(z \cdot y) \cdot (z \cdot x) \in C$ and $y \in C$. Then $\mathrm{P}_{C,\varepsilon}((z \cdot y) \cdot (z \cdot x)) = [0, 1]$ and $\mathrm{P}_{C,\varepsilon}(y) = [0, 1]$. Thus $\mathrm{P}_{C,\varepsilon}(x) \supseteq \mathrm{P}_{C,\varepsilon}((z \cdot y) \cdot (z \cdot x)) \cap \mathrm{P}_{C,\varepsilon}(y) = [0, 1]$, so $\mathrm{P}_{C,\varepsilon}(x) = [0, 1]$. Since $\varepsilon \neq [0, 1]$, we have $x \in C$ and so C is a strongly UP-ideal of A. □

Definition 2.6. [13] A hesitant fuzzy set h on A is called a *prime hesitant fuzzy set* on A if it satisfies the following property: for any $x, y \in A$,

$$\mathrm{h}(x \cdot y) \subseteq \mathrm{h}(x) \cup \mathrm{h}(y).$$

Theorem 2.7. Let B be a nonempty subset of A. Then the following statements hold:

(1) if B is a prime subset of A, then the partial constant hesitant fuzzy set $\mathrm{P}_{B,\varepsilon}$ is a prime hesitant fuzzy set on A for all $\varepsilon \in \mathcal{P}([0, 1])$, and

(2) if there exists $\varepsilon \in \mathcal{P}([0, 1])$ and $\varepsilon \neq [0, 1]$ such that the partial constant hesitant fuzzy set $\mathrm{P}_{B,\varepsilon}$ is a prime hesitant fuzzy set on A, then B is a prime subset of A.

Proof. (1) Assume that B is a prime subset of A. For any $\varepsilon \in \mathcal{P}([0, 1])$ and let $x, y \in A$.

Case 1: $x \cdot y \in B$. Then $\mathrm{P}_{B,\varepsilon}(x \cdot y) = [0, 1]$. Since B is a prime subset of A, we have $x \in B$ or $y \in B$. Then $\mathrm{P}_{B,\varepsilon}(x) = [0, 1]$ or $\mathrm{P}_{B,\varepsilon}(y) = [0, 1]$, so $\mathrm{P}_{B,\varepsilon}(x) \cup \mathrm{P}_{B,\varepsilon}(y) = [0, 1]$. Therefore, $\mathrm{P}_{B,\varepsilon}(x \cdot y) = [0, 1] \subseteq [0, 1] = \mathrm{P}_{B,\varepsilon}(x) \cup \mathrm{P}_{B,\varepsilon}(y)$.

Case 2: $x \cdot y \notin B$. Then $\mathrm{P}_{B,\varepsilon}(x \cdot y) = \varepsilon \subseteq \mathrm{P}_{B,\varepsilon}(x) \cup \mathrm{P}_{B,\varepsilon}(y)$.
Therefore, $\mathrm{P}_{B,\varepsilon}$ is a prime hesitant fuzzy set on A.

(2) Assume that $\mathrm{P}_{B,\varepsilon}$ is a prime hesitant fuzzy set on A for some $\varepsilon \in \mathcal{P}([0, 1])$ and $\varepsilon \neq [0, 1]$. Let $x, y \in A$ be such that $x \cdot y \in B$. Then $\mathrm{P}_{B,\varepsilon}(x \cdot y) = [0, 1]$, so $[0, 1] = \mathrm{P}_{B,\varepsilon}(x \cdot y) \subseteq \mathrm{P}_{B,\varepsilon}(x) \cup \mathrm{P}_{B,\varepsilon}(y)$. Thus $\mathrm{P}_{B,\varepsilon}(x) \cup \mathrm{P}_{B,\varepsilon}(y) = [0, 1]$, so $\mathrm{P}_{B,\varepsilon}(x) = [0, 1]$ or $\mathrm{P}_{B,\varepsilon}(y) = [0, 1]$. Since $\varepsilon \neq [0, 1]$, we have $x \in B$ or $y \in B$ and so B is a prime subset of A. □

Definition 2.8. [13] A hesitant fuzzy UP-subalgebra (resp. hesitant fuzzy UP-filter, hesitant fuzzy UP-ideal, hesitant fuzzy strongly UP-ideal) h of A is called a *prime hesitant fuzzy UP-subalgebra* (resp. *prime hesitant fuzzy UP-filter, prime hesitant fuzzy UP-ideal, prime hesitant fuzzy strongly UP-ideal*) if h is a prime hesitant fuzzy set on A.

Definition 2.9. [13] A hesitant fuzzy set h on A is called a *weakly prime hesitant fuzzy set* on A if it satisfies the following property: for any $x, y \in A$ and $x \neq y$,

$$\mathrm{h}(x \cdot y) \subseteq \mathrm{h}(x) \cup \mathrm{h}(y).$$

Definition 2.10. [13] A hesitant fuzzy UP-subalgebra (resp. hesitant fuzzy UP-filter, hesitant fuzzy UP-ideal, hesitant fuzzy strongly UP-ideal) h of A is called a *weakly prime hesitant fuzzy UP-subalgebra* (resp. *weakly prime hesitant fuzzy UP-filter, weakly prime hesitant fuzzy UP-ideal, weakly prime hesitant fuzzy strongly UP-ideal*) if h is a weakly prime hesitant fuzzy set on A.

From [13], we know that the notion of weakly prime hesitant fuzzy UP-subalgebras (resp. weakly prime hesitant fuzzy UP-filters, weakly hesitant fuzzy UP-ideals) is a generalization of prime hesitant fuzzy UP-subalgebras (resp. prime hesitant fuzzy UP-filters, prime hesitant fuzzy UP-ideals), and the notions of weakly prime hesitant fuzzy strongly UP-ideals and prime hesitant fuzzy strongly UP-ideals coincide.

Theorem 2.11. Let B be a nonempty subset of A. Then the following statements hold:

(1) if B is a weakly prime subset of A, then the partial constant hesitant fuzzy set $\mathrm{P}_{B,\varepsilon}$ is a weakly prime hesitant fuzzy set on A for all $\varepsilon \in \mathcal{P}([0,1])$, and

(2) if there exists $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$ such that the partial constant hesitant fuzzy set $\mathrm{P}_{B,\varepsilon}$ is a weakly prime hesitant fuzzy set on A, then B is a weakly prime subset of A.

Proof. (1) Assume that B is a weakly prime subset of A and let $x, y \in A$ be such that $x \neq y$ and $\varepsilon \in \mathcal{P}([0,1])$.

Case 1: $x \cdot y \in B$. Then $\mathrm{P}_{B,\varepsilon}(x \cdot y) = [0,1]$. Since B is a weakly prime subset of A, we have $x \in B$ or $y \in B$. Then $\mathrm{P}_{B,\varepsilon}(x) = [0,1]$ or $\mathrm{P}_{B,\varepsilon}(y) = [0,1]$, so $\mathrm{P}_{B,\varepsilon}(x) \cup \mathrm{P}_{B,\varepsilon}(y) = [0,1]$. Therefore, $\mathrm{P}_{B,\varepsilon}(x \cdot y) = [0,1] \subseteq [0,1] = \mathrm{P}_{B,\varepsilon}(x) \cup \mathrm{P}_{B,\varepsilon}(y)$.

Case 2: $x \cdot y \notin B$. Therefore, $\mathrm{P}_{B,\varepsilon}(x \cdot y) = \varepsilon \subseteq \mathrm{P}_{B,\varepsilon}(x) \cup \mathrm{P}_{B,\varepsilon}(y)$.

Hence, $\mathrm{P}_{B,\varepsilon}$ is a weakly prime hesitant fuzzy set on A.

(2) Assume that $\mathrm{P}_{B,\varepsilon}$ is a weakly prime hesitant fuzzy set on A for some $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$. Let $x, y \in A$ be such that $x \neq y$ and $x \cdot y \in B$. Then $\mathrm{P}_{B,\varepsilon}(x \cdot y) = [0,1]$, so $[0,1] = \mathrm{P}_{B,\varepsilon}(x \cdot y) \subseteq \mathrm{P}_{B,\varepsilon}(x) \cup \mathrm{P}_{B,\varepsilon}(y)$. Thus $\mathrm{P}_{B,\varepsilon}(x) \cup \mathrm{P}_{B,\varepsilon}(y) = [0,1]$, so $\mathrm{P}_{B,\varepsilon}(x) = [0,1]$ or $\mathrm{P}_{B,\varepsilon}(y) = [0,1]$. Since $\varepsilon \neq [0,1]$, we have $x \in B$ or $y \in B$ and so B is a weakly prime subset of A. □

Theorem 2.12. Let S be a nonempty subset of A. Then the following statements hold:

(1) if S is a weakly prime UP-subalgebra of A, then the partial constant hesitant fuzzy set $\mathrm{P}_{S,\varepsilon}$ is a weakly prime hesitant fuzzy UP-subalgebra of A for all $\varepsilon \in \mathcal{P}([0,1])$, and

(2) if there exists $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$ such that the partial constant hesitant fuzzy set $\mathrm{P}_{S,\varepsilon}$ is a weakly prime hesitant fuzzy UP-subalgebra of A, then S is a weakly prime UP-subalgebra of A.

Proof. (1) It is straightforward by Theorem 2.1 (1) and 2.11 (1).

(2) It is straightforward by Theorem 2.1 (2) and 2.11 (2). □

Theorem 2.13. Let F be a nonempty subset of A. Then the following statements hold:

(1) if F is a weakly prime UP-filter of A, then the partial constant hesitant fuzzy set $\mathrm{P}_{F,\varepsilon}$ is a weakly prime hesitant fuzzy UP-filter of A for all $\varepsilon \in \mathcal{P}([0,1])$, and

(2) if there exists $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$ such that the partial constant hesitant fuzzy set $\mathrm{P}_{F,\varepsilon}$ is a weakly prime hesitant fuzzy UP-filter of A, then F is a weakly prime UP-filter of A.

Proof. (1) It is straightforward by Theorem 2.3 (1) and 2.11 (1).
(2) It is straightforward by Theorem 2.3 (2) and 2.11 (2). □

Theorem 2.14. Let B be a nonempty subset of A. Then the following statements hold:

(1) if B is a weakly prime UP-ideal of A, then the partial constant hesitant fuzzy set $\mathrm{P}_{B,\varepsilon}$ is a weakly prime hesitant fuzzy UP-ideal of A for all $\varepsilon \in \mathcal{P}([0,1])$, and

(2) if there exists $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$ such that the partial constant hesitant fuzzy set $\mathrm{P}_{B,\varepsilon}$ is a weakly prime hesitant fuzzy UP-ideal of A, then B is a weakly prime UP-ideal of A.

Proof. (1) It is straightforward by Theorem 2.4 (1) and 2.11 (1).
(2) It is straightforward by Theorem 2.4 (2) and 2.11 (2). □

Theorem 2.15. Let C be a nonempty subset of A. Then the following statements hold:

(1) if C is a weakly prime strongly UP-ideal of A, then the partial constant hesitant fuzzy set $\mathrm{P}_{C,\varepsilon}$ is a weakly prime hesitant fuzzy strongly UP-ideal of A for all $\varepsilon \in \mathcal{P}([0,1])$, and

(2) if there exists $\varepsilon \in \mathcal{P}([0,1])$ and $\varepsilon \neq [0,1]$ such that the partial constant hesitant fuzzy set $\mathrm{P}_{C,\varepsilon}$ is a weakly prime hesitant fuzzy strongly UP-ideal of A, then C is a weakly prime strongly UP-ideal of A.

Proof. (1) It is straightforward by Theorem 2.5 (1) and 2.11 (1).
(2) It is straightforward by Theorem 2.5 (2) and 2.11 (2). □

Acknowledgement

This work was financially supported by the University of Phayao.

References

[1] A. Ali, M. Khan, F. G. Shi, *Hesitant fuzzy ideals in Abel-Grassmann's groupoid*, Ital. J. Pure Appl. Math. 35 (2015) 537-556.

[2] T. Guntasow, S. Sajak, A. Jomkham, A. Iampan, *Fuzzy translations of a fuzzy set in UP-algebras*, J. Indones. Math. Soc. 23(2) (2017) 1-19.

[3] Q. P. Hu, X. Li, *On BCH-algebras*, Math. Semin. Notes, Kobe Univ. 11 (1983) 313-320.

[4] A. Iampan, *A new branch of the logical algebra: UP-algebras*, J. Algebra Relat. Top. 5(1) (2017) 35-54.

[5] Y. Imai, K. Iséki, *On axiom system of propositional calculi, XIV*, Proc. Japan Acad. 42(1) (1966) 19-22.

[6] K. Iséki, *An algebra related with a propositional calculus*, Proc. Japan Acad. 42(1) (1966) 26-29.

[7] Y. B. Jun, S. S. Ahn, *Hesitant fuzzy set theory applied to BCI/BCK-algebras*, J. Comput. Anal. Appl. 20(4) (2016) 635-646.

[8] Y. B. Jun, S. S. Ahn, G. Muhiuddin, *Hesitant fuzzy soft subalgebra and ideal in BCI/BCK-algebras*, Sci. World J. 2014 (2014) Article ID 763929, 7 pages.

[9] Y. B. Jun, K. J. Lee, S. Z. Song, *Hesitant fuzzy bi-ideals in semigroups*, Commun. Korean Math. Soc. 30(3) (2015) 143-154.

[10] Y. B. Jun, S. Z. Song, *Hesitant fuzzy set theory applied to filters in MTL-algebras*, Honam Math. J. 36(4) (2014) 813-830.

[11] Y. B. Jun, S. Z. Song, *Hesitant fuzzy prefilters and filters of EQ-algebras*, Appl. Math. Sci. 9(11) (2015) 515-532.

[12] S. Keawrahun, U. Leerawat, *On isomorphisms of SU-algebras*, Sci. Magna 7(2) (2011) 39-44.

[13] P. Mosrijai, W. Kamti, A. Satirad, A. Iampan, *Hesitant fuzzy sets on UP-algebras*, Konuralp J. Math. 5(2) (2017) 268-280.

[14] G. Muhiuddin, *Hesitant fuzzy filters and hesitant fuzzy G-filter in residuated lattices*, J. Comput. Anal. Appl. 21(2) (2016) 394-404.

[15] C. Prabpayak, U. Leerawat, *On ideals and congruences in KU-algebras*, Sci. Magna 5(1) (2009) 54-57.

[16] R. M. Rodríguez, L. Martínez, F. Herrera, *Hesitant fuzzy linguistic term sets for decision making*, IEEE Trans. Fuzzy Syst. 20(1) (2012) 109-119.

[17] A. Satirad, P. Mosrijai, W. Kamti, A. Iampan, *Level subsets of a hesitant fuzzy set on UP-algebras*, Ann. Fuzzy Math. Inform. 14(3) (2017) 279-302.

[18] J. Somjanta, N. Thuekaew, P. Kumpeangkeaw, A. Iampan, *Fuzzy sets in UP-algebras*, Ann. Fuzzy Math. Inform. 12(6) (2016) 739-756.

[19] V. Torra, *Hesitant fuzzy sets*, Int. J. Intell. Syst. 25 (2010) 529-539.

[20] V. Torra, Y. Narukawa, *On hesitant fuzzy sets and decision*, 18th IEEE Int. Conf. Fuzzy Syst. (2009) 1378–1382.

[21] L. A. Zadeh, *Fuzzy sets*, Inf. Cont. 8 (1965) 338-353.

[22] B. Zhu, Z. Xu, M. Xia, *Dual hesitant fuzzy sets*, J. Appl. Math. 2012 (2012) Article ID 879629, 13 pages.