



## Elastic Beam Model and Bending Analysis of Silver Nanowires

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### Abstract

*In this study, bending analysis of silver (Ag) modeled nanowires has been carried out for six-various boundary conditions. Silver nanowires have great importance for Nano-electro-mechanical systems (NEMS) technology. The displacement, rotation of cross-section and bending moment values of elastic beam models of silver nanowires under uniform load have been calculated. Numerical results have been presented as graphics and tables. The influence of boundary conditions on deformation and bending moment has been discussed. As the boundary conditions become rigid, the values of displacement and cross-sectional rotation under uniform load reduce.*

**Keywords:** Silver nanowires, bending, NEMS, elastic beam.

### 1. Introduction

Nanotechnology is scientific disciplines that have applications that aim all the tools and equipment that we use have superior properties. Finding superior properties by processing the materials at the nano-scale is the basic concept of this discipline. The general investigation area of this discipline is structures with 1 – 100 nm in length. In the Nanotechnology, the physical, electrical, optical, elastic and thermal properties of one-dimensional structures such as carbon nanotubes, boron nitride nanotubes, silica carbide nanotubes, zinc oxide nanowire, gold nanorod as well as two-dimensional nanostructures such as graphene and silicene have been intensively investigated. The discovery of the superior properties of these materials enables the production and using of materials with new properties. Carbon nanotubes (CNT) material has a very popular research field in nanotechnology. Carbon (C) atoms are arranged to form a two-dimensional structure called “Graphene”, one or more of graphene are wrapped like cylinder in space. These tubes form the carbon nanotubes structure by intertwining. Japanese scientist Iijima discovered these materials in 1991 and from this date forward, carbon nanotubes have been an intense research topic [1]. Chemical sensors, medical and industrial applications, quality control, detection of war and security threats exemplify to its potential uses [2]. Another material that is intensively studied in nanotechnologies researches is boron nitride nanotubes (BNNT). Similar to structure of carbon nanotubes, they consist of equal numbers of boron (B) and nitrogen (N) atoms. They are synthetically produced because they are not found naturally. Their production can be grouped under two headings: Synthesis



at high temperature such as arc-discharge, and synthesis at medium or low temperatures such as carbo-metallic methods and chemical vapor synthesis [3]. Because of strong insulating properties, it is an important material for nanocables. These cables are used in complex electronic circuits [4]. Optical devices operating with UV-light are another using area [5].

The element of silver (Ag) is a transition metal in group-1B and period-5 of the periodic table. It reflects the light, very well. It shows ductile behavior and high resistance to oxidation. It is used intensively in electrical wires [6]. The nanowire structure can be described as a one-dimensional structure with a diameter of less than 1 nm and its ratio of length-width is roughly equal to 1000. Silver nanowires can be used in optical industry, conductive materials, anti-bacterial applications [7]. There are several methods about its synthesis: Hard template methods and soft template methods. Soft template methods are divided into two. One of the methods is typical soft templates, another of methods is the polyol method [8]. It is very important to know the behavior of nano-scale material based devices such as micro-processor, transistor, sensor, conductor wire structures under external influences. Researchers have worked intensively with the mechanical analyses of beam and rod models of one-dimensional nanostructures and plate models of two-dimensional nanostructures [25-36]. Because of the nanowires are one-dimensional structures, they can be modeled as beams and rods. In this study, the analysis of the beams modeled with silver nanowires under the uniform load has been carried out and obtained numerical results have been discussed.

## 2. Bending Analysis of Nanobeams

The deformations of one-dimensional bending elements under uniform load constitute a continuous function. In the differential geometry, this function is expressed as below

$$\frac{1}{\rho} = \pm \frac{w''}{[1 + (w')^2]^{3/2}} \quad (1)$$

Here,  $w$  is elastic curve of beams, namely geometric locus,  $\rho$  is radius of curvature. Eq. (1) which is being seen, can be reduced by two reasons. Firstly, extremely flat curves become  $w \cong 0$  because elastic curves are extremely flat in applications. On the other hand, second derivative of elastic curve become  $w'' < 0$  under positive bending moment. Hereby, Eq. (2) is written,

$$\frac{1}{\rho} = -w'' \quad (2)$$

The curvature expression of one – directional bending according to Euler – Bernoulli beam theory,

$$\frac{1}{\rho} = \frac{M}{EI} \quad (3)$$

Here,  $M$  is bending moment,  $E$  is elasticity modulus and  $I$  is moment of inertia. If Eq. (3) is replaced in the Eq. (2), Bending moment-displacement relation is obtained as below,

$$M = -EIw'' \quad (4)$$

According to differential equilibrium of elastic body under external forces,

$$V = \frac{dM}{dz}, \quad q = -\frac{dV}{dz} \quad (5)$$

If Eq. (5) is replaced in the Eq. (4), Eq. (6) is obtained and Eq. (6) is elastic curve equation of beams

$$q = EIw^{(4)}(z) \quad (6)$$

where,  $q$  is uniform load in this equation. If Eq. (6) is integrated four times one after another, Displacement equation is obtained. First-order derivative of displacement equation is rotation equation. These equations are as below,

$$w' = \frac{1}{EI} \left[ \frac{q}{6} z^3 + \frac{C_1}{2} z^2 + C_2 z + C_3 \right] \quad (7)$$

$$w = \frac{1}{EI} \left[ \frac{q}{24} z^4 + \frac{C_1}{6} z^3 + \frac{C_2}{2} z^2 + C_3 z + C_4 \right] \quad (8)$$

Eq. (7) and Eq. (8) are expresses cross-sectional rotation and displacement, respectively.  $C_i$  ( $i = 1,2,3,4$ ) is constant of indefinite integral and is found with the help of boundary conditions. The boundary conditions are as below

- Free end (F)                       $w'' = 0$                       and                       $w''' = 0$
- Simply end (S)                       $w = 0$                       and                       $w'' = 0$
- Cantilever end (C)                       $w = 0$                       and                       $w' = 0$
- Guided end (G)                       $w' = 0$                       and                       $w''' = 0$

### 3. Numerical Examples

The beam models to be analyzed are as shown in Fig.1. For the all numerical examples, elasticity modulus of Ag nanowires is  $E = 0.102$  TPa [9]. The diameter of circular cross-section has been chosen as  $d = 5$  nm. When we examine the Results in the Table-1, as the length of the beam increases, we understand the maximum displacement value increases. On the other hand, it is seen the most displacement is in the S-G beam, the least displacement is in the C-C beam. The elastic curves of the beams with different boundary conditions are given in Fig.2. The displacements of S-S, C-C and C-S beams are very low alongside other beam types. Rotation curves are plotted in Fig.3. S-G beam have made most cross-sectional rotation. The least rotation values have been shown in C-C beams. When we look to Results of Fig.4, The most bending moment have been shown guided end of S-G beam and cantilever end of C-F beams and theirs values are equal. The maximum bending moment of C-S and S-S are equal. In addition, C-C beams have the least bending moment.

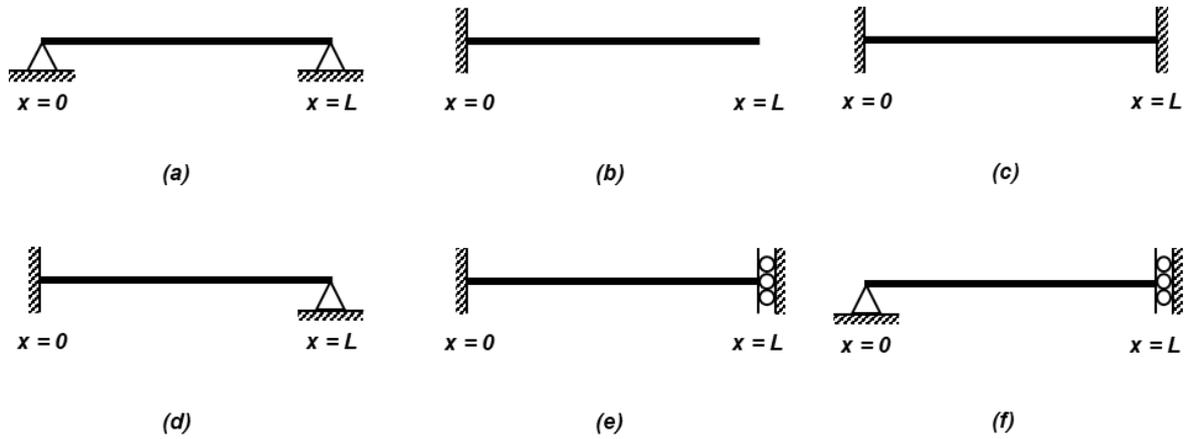


Fig.1. Nanobeams with various boundary conditions

(a) S – S (b) C – F (c) C – C (d) C – S (e) C – G (f) S – G

Table 1. The displacement values of nanobeams with various boundary conditions under  $q=0.05$  nN/nm uniform load along span (nm)

Length (nm)	S – S	C – F	C – C	C – S	C – G	S – G
10	0.00208	0.01997	0.00080	0.00042	0.03329	0.00666
20	0.03329	0.31956	0.01278	0.00666	0.53260	0.10652
30	0.16852	1.61776	0.06471	0.03370	2.69627	0.53925
40	0.53260	5.11293	0.20452	0.10652	8.52155	1.70431
50	1.30029	12.48274	0.49931	0.26006	20.80457	4.16091

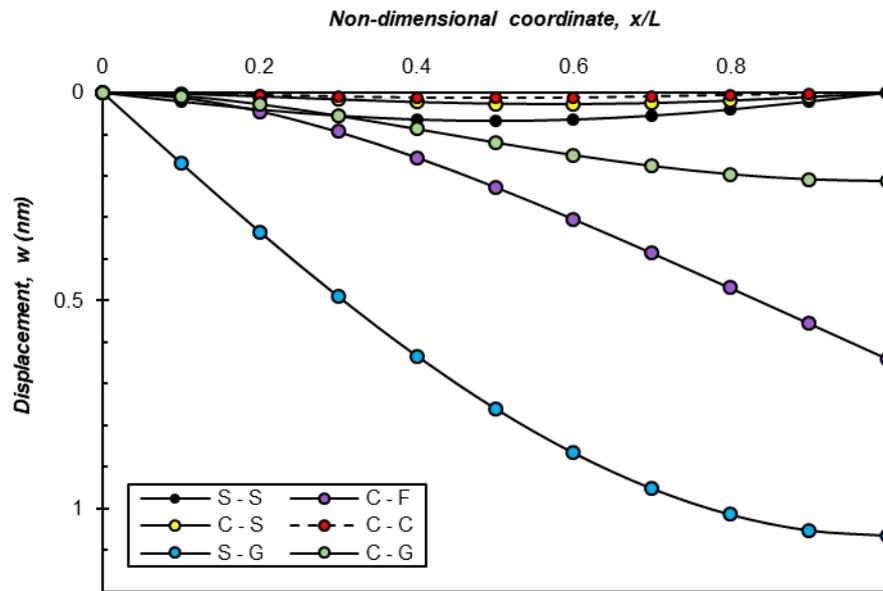


Fig. 2. Plotting the elastic curves of nanobeams with various boundary conditions under  $q=0.1$  nN/nm uniform load

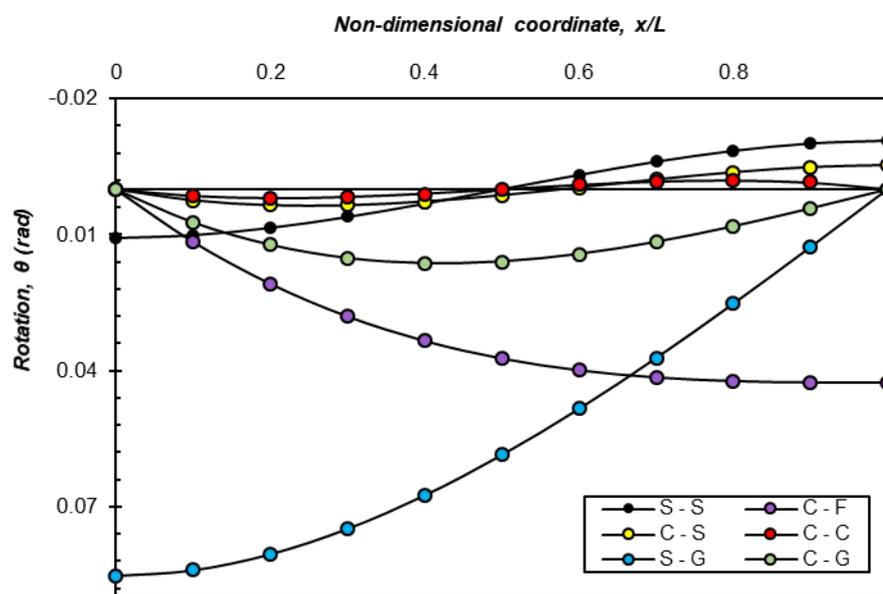


Fig. 3. Plotting the cross-sectional rotations of nanobeams with various boundary conditions under  $q=0.1$  nN/nm uniform load

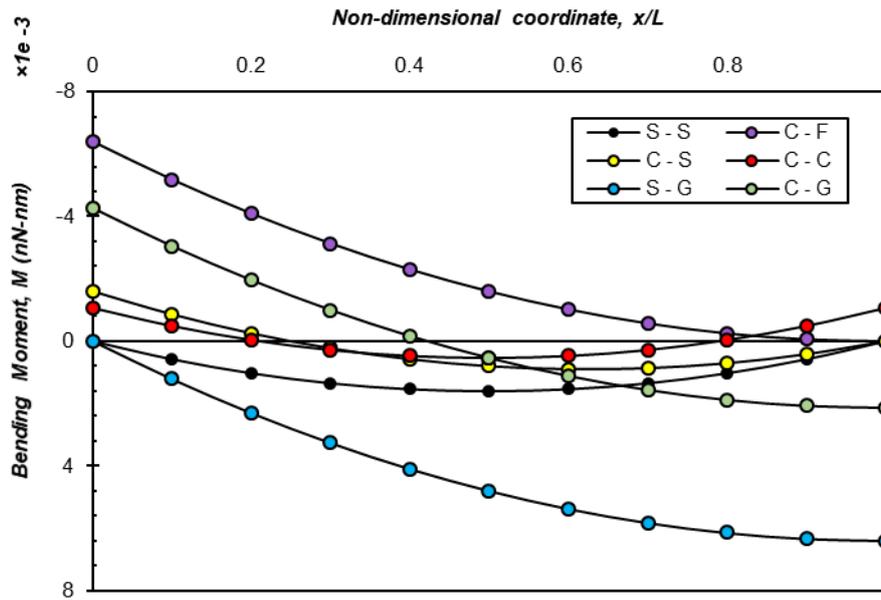


Fig. 4. Plotting the bending moment diagram of nanobeams with various boundary conditions under  $q=0.1$  nN/nm uniform load

#### 4. Concluding Remarks

As the ends of the beam become rigid, the displacement rotation and bending moment values have decreased. On the other hand, as the length increases in all beam types, the ratio of displacement to length of nanobeam increases. It is hoped that these results will contribute to the research on the design problem of nano-scaled elements under bending effect.

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