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Proper UP-filters of UP-algebra

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Abstract

In this paper we introduce and discus a concept of proper UP-filters in UP-algebras. We present a connection between proper UP-filters and UP-ideals. We expect this concept of filters to play a significant role in research and understanding of algebras.

1. Introduction

The basic concepts of UP-algebra are taken from the text [3]. The author in his article has introduced and analyzed the concepts of UP-algebra, UP-subalgebra and UP-ideal. In the article [6] the authors introduced the concept of UP-filters in UP algebra. This latter concept has a non-standard attitude towards the concept of UP-ideals. That made us confused - we expected the filter to have a standard attitude towards the ideal. We were interested in why the authors of the concept of UP-filters opted for such definition of the UP-filter. Our first reaction to such a UP-filter determination was - the offered definition is not correct. Then we thought that the text of the UP-filter definition in the article [6] was incorrectly written. Viewing the available literature about the concept of filters in BCC-algebra (See [1, 2]) and KU-algebra ([4, 5]), algebras which close to UP-algebra, did not yield the expected results. It was possible to find the term 'deductive system' some authors called the filter. But this concept had a non-standard relationship to the concept of the ideal. That was the motive for our research of UP-algebra. In order to obtain a satisfactory definition of the proper UP-filter, we have permuted the positions of the logical atoms in the definition of the UP-ideals. In this text one obtained intriguing reflection of these variation is exposed.

Since in each of the previously known algebras the ideals play an important role, this is the case in this UP algebra, too. Filters in algebras, as substructures these algebras associated with ideals, could also play a significant role in our understanding of algebras.

2. Preliminaries

First, let us recall the definition of UP-algebra.

Definition 2.1 ([3], Definition 1.3)**.** *An algebra* $A = (A, \cdot, 0)$ *of type* $(2,0)$ *is called a* UP- algebra *if it satisfies the following axioms:*
(UP - 1)*:* $(\forall x,y,z \in A)((y\cdot z)\cdot((x\cdot y)\cdot(x\cdot z)) = 0)$,
(UP - 2)*:* $(\forall x \in A)(0\cdot x = x)$,
(UP - 3)*:* $(\forall x \in A)(x\cdot 0 = 0)$,
(UP - 4)*:* $(\forall x,y \in A)((x\cdot y = 0 \wedge y\cdot x = 0) \Longrightarrow x = y)$.

Second, in the following we give definition of the concept of UP-ideals of UP-algebra.

Definition 2.2 ([3], Definition 2.1)**.** *Let A be a UP-algebra. A subset J of A is called a* UP-ideal *of A if it satisfies the following properties:*
(1) $0 \in J$, *and*
(2) $(\forall x,y,z \in A)(x\cdot(y\cdot z) \in J \wedge y \in J \Longrightarrow x\cdot z \in J)$.

For this article, the recognizable feature of the UP-ideal is given in statement (1) of Proposition 2.7 in the article [3]:

Let A be a UP-algebra and B a UP-ideal of A. Then

$$(\forall x,y \in A)((x \in B \wedge x \leq y) \Longrightarrow y \in B).$$

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The concept of UP-filters is introduced by the following definition.

Definition 2.3 ([6], Definition 1.11)**.** *Let A be a UP-algebra. A subset F of A is called a* UP-filter *of A, if it satisfies the following properties:*
(i) $0 \in F$,
(ii) $(\forall x, y \in A)((x \in F \wedge x \cdot y \in F) \Longrightarrow y \in F)$.

3. The main results

Our intention in this short notice is to construct a substructure G in UP-algebras that will have the following property

$$(\forall x, y \in A)((y \in G \wedge x \leq y) \Longrightarrow x \in G)$$

and has a standard attitude toward the UP-ideal. We can transform this formula into the following formula

$$(\forall x, y \in A)((y \in G \wedge \neg(x \cdot y \in G)) \Longrightarrow x \in G).$$

The previous formula was the basis for us concept of UP-filters. The concept of proper UP-filter in a UP-algebra we introduce by the following definition.

Definition 3.1. *Let A be a UP-algebra. A subset G of A is called a* proper UP-filter *of A if it satisfies the following properties:*
(3) $\neg(0 \in G)$, *and*
(4) $(\forall x, y, z \in A)((\neg(x \cdot (y \cdot z) \in G) \wedge x \cdot z \in G) \Longrightarrow y \in G)$.

Subsets $\emptyset$ and $A_0 = \{x \in A : x \neq 0\}$ are trivial proper UP-filters and UP-algebras A. So, the family of all proper UP-filters of a UP-algebra is not empty.

Example 3.2. *Let A is as in Example 2.2 in* [3]*. Then the sets* $\{2, 4\}$ *and* $\{3, 4\}$ *are proper UP-filters of A.*

Example 3.3. *Let* $f : A \longrightarrow B$ *be a UP-homomorphism of UP-algebras. Then the set* $Coker(f) = \{x \in A : f(x) \neq 0\}$ *is a UP-filter of A. In addition, if H is a proper UP-filter of B, then* $f^{-1}(H)$ *is a proper UP-filter of A. Specifically, if* H/J *is a proper filter in* A/J*, where J is a UP-ideal in A, then* $\pi^{-1}(H/J)$ *is a proper UP-filter of A, where* $\pi : A \longrightarrow A/J$ *is the canonical UP-epimorphism.*

This determined substructure of a UP-algebra A has the following property.

Theorem 3.4. *Let A be a UP-algebra and G a proper UP-filter of A. Then*
(5) $(\forall x, y \in A)((\neg(x \cdot y \in G) \wedge y \in G) \Longrightarrow x \in G)$.
(6) $(\forall x, y \in A)(x \cdot y \in G \Longrightarrow y \in G)$.

Proof. The first statement follows directly from definition when we put $x = 0, \quad y = x$ and $z = y$.
If we put y = z in formula (4), we get $\neg(x \cdot (y \cdot y) = x \cdot 0 = 0 \in G)$ and $x \cdot y \in G$. Thus $y \in G$. Therefore, (6) is proved. □

Corollary 3.5. *Let A be a UP-algebra and G a proper UP-filter of A. Then*
(7) $(\forall x, y \in A)((x \leqslant y \wedge y \in G) \Longrightarrow x \in G)$.

Proof. Let $x, y \in A$ be arbitrary elements such that $x \leqslant y$ and $y \in G$. Thus $\neg(x \cdot y = 0 \in G)$ and $y \in G$. Then $x \in G$ by (5). □

Remark 3.6. *The usual term used for property* (7) *of an algebra subset is a 'deductive system'. So, the concept of 'proper filters', introduced by definition 3.1, is a deductive system in the UP-algebra A. The important difference between the concept of 'deductive systems' and our concept of 'proper UP-filters' is in the requirement* (3).

Theorem 3.7. *A subset G of a UP-algebra A is a proper UP-filter of A if and only if the set* $A \backslash G$ *is a UP-ideal of A.*

Proof. Suppose that G is a proper UP-filter in UP-algebra A. It is clear $0 \in A \backslash G$. Let $x, y, z \in A$ be arbitrary elements such that $\neg(x \cdot (y \cdot z) \in G)$ and $\neg(y \in G)$. Thus $\neg(x \cdot z \in G)$. Indeed, if it were not, from $\neg(x \cdot (y \cdot z) \in G)$ and $x \cdot z \in G$ would follow $y \in G$ what is in a contradiction with $\neg(y \in G)$. So it have to be $\neg(x \cdot z \in G)$. Therefore, the set $A \backslash G$ is a UP-ideal of A.
In opposite, let J be a UP-ideal of UP-algebra A. It is obvious that $\neg(0 \in A \backslash J)$ is valid. Let $x, y, z \in A$ be arbitrary elements such that $\neg(x \cdot (y \cdot z) \in A \backslash J)$ and $x \cdot z \in A \backslash J$. Thus $y \in A \backslash J$. Indeed, if it were $y \in J$, then $x \cdot z \in J$ would follow from $x \cdot (y \cdot z) \in J$ and $y \in J$, contrary to the hypothesis $\neg(x \cdot z \in J)$. Therefore, the set $A \backslash J$ is a proper UP-filter of A. □

Theorem 3.8. *The family* $\mathfrak{G}_A$ *of all proper UP-filters in UP-algebra A forms a completely lattice.*

Proof. Let A be a UP-algebra and $\{G_{i \in I}\}$ a family of proper UP-filters of A.
(a) Let $x, y, z \in A$ be elements such that $\neg(x \cdot (y \cdot z) \in \bigcup_{i \in I} G_i)$ and $x \cdot z \in \bigcup_{i \in I} G_i$. Then there exists an index $i \in I$ such that $\neg(x \cdot (y \cdot z) \in G_i)$ and $x \cdot z \in G_i$. Thus $y \in G_i$ by (4). Therefore, $y \in \bigcup_{i \in I} G_i$.
(b) Let $\mathfrak{X}$ be a family of all proper UP-filter contained in $\bigcap_{i \in I} G_i$. Thus, by part (a) of this proof, the union $\bigcup \mathfrak{X}$ is a proper UP-filter of A.
(c) If we define $\sqcap_{i \in I} G_i = \bigcup \mathfrak{X}$ and $\sqcup_{i \in I} G_i = \bigcup_{i \in I} G_i$, then $(\mathfrak{G}, \sqcap, \sqcup)$ is a completely lattice. □

4. Final observation

In the present paper, we have introduced a new algebraic substructure in UP-algebra, called a proper UP-filter. We present some connections between proper UP-filters and UP-ideals. This concept of UP-filters has almost a standard connection with the UP-ideal. The author believes that this new structure enriches the family of substructures in UP-algebras. Of course, while the academic community of researchers algebras accept this concept of filters in algebras, it can be expected that further research involves relations of the concept of proper UP-filters and some other concepts, for example as concepts of orders, homomorphisms and congruences in algebras.

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