Introduction

Induction motors are preferred in many applications because of their ease of maintenance, affordable price and robustness. Moreover squirrel cage induction motors are currently dominating the market since they can even be operated under faulty conditions [1, 2]. Motor faults are mainly caused by the windings, rotor, bearings, and air gap eccentricity and they have been analyzed and published by different researchers for many years [3, 4]. In this paper, the focus will be on faults caused by air gap eccentricity.

Various studies have been conducted on eccentricity faults and its detection since the first quarter of the 20th century. In the early years, researchers were focused on the mechanical reasons leading to faults and the magnetic and mechanical problems resulting from faults. Dating from the 1970s, different diagnostic methods have been developed to predict faults that might happen due to eccentricity. There are many diagnostic methods in the literature such as electromagnetic field monitoring, noise, vibrations, temperature measurements, and infrared recognition, etc. [4-6]. However, vibrations and motor current signals are predominant to detect eccentricity faults and have been preferred in applications [7]. The analysis of motor current to diagnose faults has been the subject of much research in the 1990s. The researchers addressed the modeling of the dynamic and/or static eccentricity faults with different eccentricity levels for different types of motors and investigated the motor parameters [8-10]. Finite element method (FEM) based programs have routinely been utilized to model such eccentricity faults. The work on eccentricity harmonics were the basis of the studies in the 2000s [11, 12].

Until today, many studies have been conducted to detect faults in induction motors. One of the simplest procedures applied to fault detection is Fast Fourier Transform (FFT) on the stator phase currents due to the fact that it includes many of the motor harmonics whose properties on faulty conditions (this is a fragment – finish the sentence please – Revise for clarity) [13]. A challenge can occur while using FFT analysis because the signals to be analyzed con-
tain many non-stationary and transitory components. That’s why, for fault detection or estimation, wavelet analysis based methods have recently been preferred. Three basic and significant motivations for using the new generation methodologies (including wavelet analysis) are listed as follows [14]:

- Requirement of non-stationary signal observation;
- Necessity of precise fault detection by acquired features even in light-load conditions;
- Challenge in fault detection by FFT feature extraction under harmonics caused by power electronic converters supplying induction motors.

Wavelet analysis is a promising technique that provides a powerful tool for fault analysis and has been used since the late 1980s. Wavelet transformation gives both time and time frequency analysis with multiresolution characteristics. It is possible to obtain the desired part of the signal in both time and frequency domains. Therefore, fault diagnosis based on wavelet analysis is becoming a trend for electrical machine designers [15].

The word “wavelet” was firstly used in the literature at the beginning of the 20th century and many related studies which contribute to wavelet theory have been conducted till the end of 1980s. The main algorithm of the theory is based on Mallat’s work from 1988 [16].

Wavelet analysis in fault detection of electrical machines first started to be used from the mid-1990s. The current and vibration signals in healthy and faulty situations have been investigated using different wavelet analysis methods, e.g. Discrete Wavelet Transform (DWT), Continuous Wavelet Transform (CWT), Wavelet Packet Transform (WPT) and MRWA to determine the faults caused by rotor bar failures, switching errors in inverter-fed machines, and eccentricity [17-21]. The research on fault diagnosis has gained a different perspective owing to the combined use of wavelet analysis and artificial neural network methods from the 2000s onwards [22]. These advancements are currently being performed in the work on PMSM and BLDC electric motors, which are becoming widespread, as well as for 3-phase asynchronous motors, which are widely used in the industry [23]. As a result, wavelet analysis is commonly used in fault diagnosis, online monitoring and protection areas.

In this study, an induction motor was simulated using a two dimensional (2D) finite-element analysis. The stator current was analyzed by means of the MRWA via MATLAB/Wavelet Toolbox. In the literature, there is no approach for the eccentricity failures with the wavelet analysis, they were investigated from either the tests or FEM based results [3, 9]. The results were compared in terms of machine performance characteristics such as current, flux density, torque etc. Simulations were also compared with experimental results, and were used to verify the validity of the approach.

**Dynamic Modeling of Induction Motor**

**Air Gap Eccentricity in Induction Motor**

Eccentricity is defined as the condition of unequal air gap that exists between the stator and rotor. There are three types of eccentricity: static, dynamic and mixed of which the later is the combination of the previous two.

The stator center is located at the first axis and the rotor center is located at the second axis as shown in Figure 1. The eccentricity is named as dynamic or static eccentricity if the center of rotation is at the first or second axis respectively. The rotation axis can be placed anywhere between the first and second axis. Such a situation is called mixed eccentricity [3]. For correctly operating motors (healthy case), however, the center of the rotor and stator must be coincident.

One of the reasons for non-uniform air gap of an induction motor is due to manufacturing defects. The stator bore and/or rotor face may not be manufactured in a perfectly cylindrical shape. Another reason is bad mechanical bearing accommodation as the center of rotor and stator may not be concentric. Beside these reasons, there are other causes of eccentricity such as bearing wear, rotor shaft bending, or resonance at critical speeds, etc.

**Finite Element Method (FEM)**

The use of numerical models to analyze electrical machines for the purpose of calculating electric and magnetic fields is rapidly changing. When the difficulty of performing analytical solutions due to the complex structures of electrical machines and non-linear characteristics of used materials is considered, it is clearly comprehended why numerical solutions are frequently used.

![Figure 1. Cross-section of the eccentric motor](image-url)
Finite element method is one of the most suitable numerical methods for this purpose since it is preferred to solve problems that are time dependent or include nonhomogeneous, directional and nonlinear components where analytical methods are inadequate [24]. The unknown parameters in the finite number are expressed in terms of known parameters of the system in FEM based analysis. The procedure to solve a problem using FEM is as follows;

- Separate solution region to finite elements;
- Write fundamental equations for each element;
- Combine all elements in solution region;
- Solve achieved equation set.

Finite element method simulations are principally based on the solutions of Maxwell equations including the expressions of magnetic flux density, magnetic field intensity, electrical flux density and current density.

To simplify the solution of field problems, the vector potential $\mathbf{A}$ is used in the calculations instead of the field. The relation between magnetic vector potential and magnetic flux density can be given as shown below.

$$ \mathbf{B} = \nabla \times \mathbf{A} \quad (1) $$

Flux density components of the air gap in the Cartesian coordinates are expressed as follows;

$$ B_x = \frac{\partial A_y}{\partial y} - \frac{\partial A_z}{\partial z} \quad (2) $$

$$ B_y = \frac{\partial A_z}{\partial z} - \frac{\partial A_x}{\partial x} \quad (3) $$

$$ B_z = \frac{\partial A_x}{\partial x} - \frac{\partial A_y}{\partial y} \quad (4) $$

2-D finite element analysis was chosen for electrical machines since it decreases the computation memory requirement and reduces computational time. In this case, the components in z direction are assumed as zero and only x and y directions are used for magnetic vector potential $\mathbf{A}$. The magnetic flux density is stated in two dimensional space as shown below.

$$ B_x = \frac{\partial A_y}{\partial y}, \quad B_y = -\frac{\partial A_x}{\partial x}, \quad B_z = 0 \quad (5) $$

For the two dimensional model, the relationship between the magnetic vector potential and current density due to the Maxwell Equation is given as:

$$ \frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial \mathbf{A}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu} \frac{\partial \mathbf{A}}{\partial y} \right) = -\mathbf{J}_s \quad (6) $$

where $\mathbf{A}$, $\mu$ and $\mathbf{J}$ are magnetic vector potential, magnetic permeability and current density respectively.

A two-dimensional problem region is bounded point by point and the points are joined with line segments. More points and consequently smaller line segments should be preferred in order to represent the real boundary accurately. The density of network elements affects the accuracy of the solution and the element density is scattered into fields with respect to its structure [24]. Smaller and more frequent elements are used in the regions with small radius of curvature where the field values change rapidly, while larger and less frequent elements are preferred in the regions with larger radius of curvature as in Figure 2.

Boundary conditions need to be assigned carefully in order to solve electromagnetic field problems that affect accuracy and precision. There are three type of the boundary conditions for the solution of electromagnetic problems: Neumann, Dirichlet and mixed boundary conditions. Neumann boundary conditions are defined by setting all point values of the potential of the boundary to the specified value, and only interior nodes are unknowns. Dirichlet boundary condition can be applied by using additional equations at the boundary which requires boundary point values to be equal to the nearest interior neighbor. In this study, the boundary condition of the motor was set to zero by applying Dirichlet boundary condition.

**Finite Element Method Simulation Results**

The squirrel cage induction motor used in this study had the following ratings: 3 phase, 2.2 kW, 4 poles, 36 slots, 28 rotor bars, 50 Hz, 400 V, 5 A, 1440 min$^{-1}$. Stator windings were connected using delta connection. The different degrees of static eccentric cases of the induction motor were simulated and compared with the results of a healthy motor by using FEM based commercial software FLUX 2D.

The induction motor simulation results were obtained under rated operating conditions with no asymmetry. Figure 3, 4 show the equi-flux lines for a two-pole pairs induction motor and radial/tangential components of the flux density in the middle of the air gap, respectively.

In Figure 5, the color spectrum of flux density was obtained as well as equi-flux lines.
The achieved experimental results using the test motor verified the simulation studies for healthy condition since the difference between the simulation and experimental results in terms of rated current, rated torque, and output power was below 1%. The obtained motor characteristics for the healthy condition were utilized in the analysis of asymmetrical operating conditions where the models with different degrees of static eccentricity were produced.

Eccentric motor characteristics were investigated by defining the rotor and its rotation axes separately. Asymmetrical conditions were obtained by offsetting the rotor and its rotation axes by 10%, 30%, 50% and 80%, respectively. The geometry of the highest degree of the eccentric condition (80%) is given in Figure 6.

**Overview of Wavelet Analysis**

In this section, wavelet transformation for discrete signals is explained briefly. The easy localization of the signal in the time and frequency plane in the wavelet analysis makes the method suitable for the analysis of non-stationary signals and a good alternative to conventional Short Time Fourier Transform (STFT).

In classical approaches, the frequency components of the signal are determined and it is assumed that the components are identical in each part of the signal. However, this approach may not always be valid. The idea of analyzing the signal in short time windows was revealed to eliminate this deficiency and was successfully applied by Gabor in 1946. However, the window size affects the accuracy of the application since it remains constant. The frequency resolution is better when the window size is large. So the size of the window is very important in order to get a more accurate result in terms of frequency or time components from the analyzed signal parts.

Wavelet analysis is based on a windowing technique for variable-sized regions. Moreover, it allows to get results in both high and low frequencies at short and long time intervals respectively.

Fourier analysis is achieved by means of Fourier Transformation which is the sum of the multiplications of the signal and complex exponential multipliers. The purpose of Fourier transformation is to calculate Fourier coefficients. Thus, the signal is divided into components each having a separate amplitude and frequency. Similarly, the wavelet function is shifted and summed over time after being multiplied by a scaling factor in wavelet analysis. The mathematical expression is,

\[ \psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi \left( \frac{t-b}{a} \right) \]  

(7)

where ‘a’ and ‘b’ are scaling and shifting parameters respectively. The translation parameter ‘b’ refers to the position of the wave in time and the scale parameter ‘a’ refers to the change in different frequencies. In this context, the following conditions must be satisfied.
Energy conservation and completeness are guaranteed by a real wavelet transformation on the weak admissibility condition is satisfied.

A wavelet is like an oscillation with an amplitude that starts at zero, increases to its maximum, and then goes back to zero. Wavelet analysis is generally focused in time unlike Fourier transformation for the decomposition of a signal. While the Fourier transformation using sine functions is only shown in frequency domain, wavelet provides an analysis of the signal in both time and frequency domains.

In practice, DWT transforms discrete signals to discrete coefficients in the wavelet domain. This transformation is basically a sampled version of CWT [25]. The length of the signal satisfies N = 2^J for a positive J in the case of DWT. The transformation can be performed efficiently using Mallat’s algorithm [16]. The decomposition of the signal into different scales can be considered as different frequency bands.

The high-pass (HP) and low-pass (LP) filters utilized in the algorithm are chosen due to the mother wavelet in use. The outputs of the LP filters are assigned as the approximation coefficients, and the outputs of the HP filters are assigned to the detail coefficients.

Each decomposition level of the produced signal by the filters covers the half of the frequency band. This helps to increase the frequency resolution so that the frequency uncertainty can be reduced. Hence, each filter output can be reduced by a factor of two by using Nyquist’s theorem. Frequency resolution gives good outputs at low frequencies. However, time resolution becomes better at high frequencies. The number of decomposition steps should be decided by considering the properties of the signal.

One of the discrete wavelet transformations is MRWA. Wavelet filters, called H and G, are utilized in MRWA for decomposition and reconstruction of the signal to be analyzed. Figure 7 shows a one-level fundamental procedure of the decomposition and reconstruction of the signal. It should be noted that it may not be possible to obtain the desired frequency region by wavelet decomposition at once, so it can be applied repeatedly.

**Methodology**

The decomposed signal which was achieved by wavelet analysis needed to be reprocessed according to the desired frequency band since the effects of the fundamental and high frequency harmonic components on the flux, current and torque characteristics were being investigated in this research. Although, as explained previously, using Mallat’s algorithm gave the fundamental component of the signal and the algorithm was not able to completely show the effects of high frequency components. The characteristic including fundamental component of the signal was expressed in the approximation a₃, whereas the details d₁, d₂, and d₃ did not represent the highest frequency harmonic components as in Figure 8. Therefore, the algorithm needed to be iterated again to decompose d₁ and d₂. This process helped to observe the effects of high frequency harmonic components separately.

In this direction, the decomposition process seen in Figure 9 was performed and the signals in the frequency range 7f₁-8f₁ which belong to the harmonic components was compared for both healthy and faulty cases.

In this work, Daubechies-25 (Db-25) wavelet filter is used in all decompositions. An example of Db scaled and wavelet functions are taken from MATLAB (Figure 10).

**Simulation Results**

A set of tests are applied to achieve a dynamic model of the induction motor. MRWA is utilized in the analysis of some performance characteristics such as air gap flux density, current and torque.
The current and torque characteristics are obtained for the faulty (with static eccentricity) models and healthy model by using the finite elements method (FEM). In this study, wavelet analysis is used to determine how the fundamental component and high frequency components of some parameters such as flux density, torque and current were going to change in the case of eccentricity.

**Flux Results**

The harmonics components of the reference motor are obtained by using Fourier decomposition [3, 9]. The amplitudes of the basic harmonic components of the reference motor are given in Table 1. The 7th and 11th belt harmonics and 17th and 19th space harmonics in the spectrum are increased with the increased eccentricity. Fourier analysis gave the amplitude of the harmonic components in the frequency band as seen in Figure 11 (the blue bars show healthy conditions and the red bars show the eccentric conditions).

As seen in Figure 12, the positions where the flux density took its largest and lowest values varied with respect to fault degrees. The small windows have been identified as ‘a’ and ‘b’ in Figure 12 to show this inconsistency.

For instance, the largest flux value in the window ‘a’ occurred for the model with 80% static eccentricity whereas the largest flux value in the window ‘b’ belonged to the healthy model as shown in Figure 13 respectively.

Besides the above mentioned results, the simulation results of the flux density for the healthy and eccentric conditions were
decomposed by using wavelet analysis and the flux characteristics of the fundamental components were plotted comparatively as shown in Figure 14 and given in Table 2.

Table 2. Flux density values for the healthy and eccentric motors

<table>
<thead>
<tr>
<th>Fundamental Component</th>
<th>Healthy</th>
<th>10% SE</th>
<th>30% SE</th>
<th>50% SE</th>
<th>80% SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux Density [T]</td>
<td>0.529</td>
<td>0.528</td>
<td>0.525</td>
<td>0.524</td>
<td>0.523</td>
</tr>
<tr>
<td>Error [%]</td>
<td>-0.23</td>
<td>-0.85</td>
<td>-1.04</td>
<td>-1.15</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Harmonic Components</th>
<th>Healthy</th>
<th>10% SE</th>
<th>30% SE</th>
<th>50% SE</th>
<th>80% SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux Density [T]</td>
<td>0.066</td>
<td>0.068</td>
<td>0.072</td>
<td>0.079</td>
<td>0.089</td>
</tr>
<tr>
<td>Error [%]</td>
<td>3.52</td>
<td>8.37</td>
<td>16.62</td>
<td>26.01</td>
<td></td>
</tr>
</tbody>
</table>

The consistency between FFT analysis and MRWA results showed the validity of MRWA results. The effects of high frequency components was also clearly seen with an increase of 26% in MRWA results.

Current Results

The current characteristics for the healthy motor obtained via simulation study are given in Figure 15.

The steady-state current signal is taken between 0.1 to 0.175 s of the current characteristic given in Fig 15 to analyze (Figure 16).

Based on the strategy described above, the current values of the fundamental component and high frequency components are obtained. The current characteristics after decomposition is shown broadly in Figure 17.

0-f, fundamental component and 7f₁,8f₁, high frequency component intervals are separately examined and the interval including the fundamental component of the current characteristic is shown in Figure 18.
The expected increase in the fundamental component is observed in the stator current with the increasing eccentricity as given in Table 3. Additionally, the components that distort the sinusoidal form of the current signal caused by high frequency components increased up to 5%.

**Torque Results**

A similar approach is applied to the torque signal given in Figure 19 for the healthy condition. Wavelet analysis is employed and the effects of the decomposed high frequency components under different failure levels are obtained.

**Test Results**

The current profile is obtained for the healthy condition in order to compare it to the simulation results.
The healthy motor airgap is distorted with a hammer to get a non-uniform rotor surface for the asymmetric conditions. The distorted rotor is given in Figure 21.

The current profiles of the healthy and faulty motor are given in Figure 22.

The high frequency components that could be seen in Figure 23 were decomposed with MRWA. The fundamental components were obtained as shown in Figure 24.

The fundamental and high frequency components of the current show an increasing trend in both the test and simulation results with the faulty condition. The results are given in Table 5.

The vibration values for the healthy and faulty conditions were obtained by experimental setup with an increase of 8.4% as $0.49 \text{ m/s}^2$ and $0.535 \text{ m/s}^2$, respectively.

High frequency components in the range of 1200-1600 Hz show a significant difference compared to those of the lower frequency components (Figure 25). The increase trends with the increasing eccentricity in terms of the flux density, current and torque are given in Figure 26.

| Table 5. Current test values for healthy and eccentric conditions |
|----------------------|----------------------|
| **Healthy** | **Faulty** |
| **Fundamental Component** | 4.437 | 4.671 |
| Error [%] | - | 5.27 |
| **Harmonic Components** | 0.023 | 0.032 |
| Error [%] | - | 41.1 |
Conclusion

In this study, a 2.2 kW squirrel cage induction motor having different levels of static eccentricity was analyzed by using FEM and MRWA. The healthy motor was also analyzed with FEM and tested to verify the FEM modeling consistency. The air gap flux density, current, torque and vibration parameters were the key parameters used to detect the eccentricity in the motor and these parameters show an increasing trend with increasing air gap eccentricity. The results were decomposed via MRWA and the high frequency components were obtained. The parameters, decomposed by using MRWA, gave more significant information about the increasing fault signal because of the inclusion of the higher frequency components.

The slot harmonics in the flux density, current characteristics and the torque pulsation caused by these harmonics were observed separately from the fundamental component with MRWA. The decomposition of the signal could be seen with wavelet analysis while only the magnitude of harmonic components and harmonic order could be seen with FFT. The results showed that wavelet analysis can potentially be more powerful than FFT in the diagnosis and monitoring of electrical machines.

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