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Operator Splitting Solution of Equal Width Wave Equation Based on the Lie-Trotter and Strang Splitting Methods

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Abstract

In the present work, we have applied four different algoritms based on the Lie-Trotter and Strang splitting methods to obtain numerical solution of Equal Width (EW) equation. For this purpose, EW equation is split up into two sub equation which one is linear and the other is nonlinear and then cubic B-spline collocation finite element method applied to each sub equation. The main advantage of this method is to obtain simpler and easier to solve sub-equations. The accuracy of the suggested method is displayed by calculating error norms L_2, L_∞ and conservation laws on the solution of a single wave motion. It was seen that cubic B-spline collocation schemes obtained via Lie-Trotter and Strang splitting methods led to lower error norms and quate easy to implement. The stability analysis of obtained schemes are investigated by von Neumann (Fourier Series) method in accordance with the structure of splitting methods. We considered single wave motion and Maxwellian initial pulse to examine the numerical solutions of the EW equation and to compare it with other studies.

Keywords: *Operator splitting method; Lie-Trotter splitting; Strang splitting; Equal Width equation; Collocation method.*
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1. Introduction

Many different kinds of wave phenomena such as surface waves on the ocean, sound waves in the air or other media, and electromagnetic waves, of which visible light is a special case can be described by partial differential equations (PDE) [1]. The regularized long wave (RLW) equation was first introduced by Peregrine [2] to describe the development of an undular bore problem. The RLW equation has solitary wave solutions propagating in shallow water channels and it is also suitable to describe Rossby waves in geophysics. These waves propagate in nonlinear media by preserving wave forms and velocity even after interaction [3]. Morrison et al. [3] introduced one dimensional PDE as a model for the non-linear dispersive waves. It represents an alternative to the well known the regularised long wave (RLW) equation. The one dimensional EW equation has form

$$U_t - \mu U_{xxt} + UU_x = 0 \tag{1.1}$$

with the physical boundary conditions $U \to 0$ as $x \to \pm\infty$, where t is time, x is the space coordinate, $U(x,t)$ is the wave amplitude and μ is a positive parameter. EW equation is used to model the waves generated in a shallow water channel the variables are normalized so that the distance x and water elevation U are scaled to the water depth h, and time t is scaled to $\sqrt{h/g}$, where g is the acceleration due to gravity [4]. It is known that there are several analytical solutions for some boundary and initial values of the EW equation, so numerically solving the equation is an essential activity. Some of these numerical studies in the literature, Dağ and Saka [5] have obtained numerical solution of the Equal Width equation based on a collocation method incorporated cubic B-splines. Banaja and Bakodah [6] have solved the EW equation using the method of lines (MOL) based on Runge-Kutta integration. Uddin [7] has presented a RBF-PS scheme to solve EW equation and Irk [8] has solved the EW equation using the Galerkin finite element method based on the Adams-Moulton method for time integration and quadratic/cubic B-splines for space integration. Saka et al. [9] have obtained solutions of the equation by Galerkin method with quartic B-spline finite elements, a differential quadrature method with cosine expansion basis and a meshless method with radial-basis functions. Yusufoğlu and Bekir [10] have solved the Equal Width wave equation numerically by means of the variational iteration method and compared with the Adomian decomposition method. Saka [11] has obtained numerical solution of the Equal Width (EW) equation using space-splitting technique and quadratic B-spline Galerkin finite element method. Esen and Kutluay [12] have found numerical solutions of the Equal Width wave equation using A linearized implicit finite difference method. Esen [13] has investigated numerical solution of the Equal Width wave (EW) equation based on a lumped Galerkin method using quadratic B-spline finite elements. Doğan [14] has solved the non-linear Equal Width equation by Galerkin method using linear finite elements. Evans and Raslan [15] have considered solitary wave

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solutions of the Generalized Equal Width (GEW) wave equation by collocation method using quadratic B-splines at midpoints as element shape functions. Raslan [4, 16] has solved the Equal Width equation numerically via collocation method using quintic and quartic B-splines at the knot points as element shape. Hamdi et al. [17] have derived exact solitary wave solutions for the general form of the EW equation and the generalized EW-Burgers equation with nonlinear terms of any order. Dereli and Schaback [18] have solved the EW equation using the Method of Lines using a somewhat unusual setup. Archilla [19] has used a spectral discretization of the Equal Width equation (EW) to obtain its solutions. Ali [20] has applied the collocation method using Chebyshev polynomials as a basis for the approximate solution of the EW equation. Gardner and Gardner [21] have implemented Galerkin method with cubic B-spline finite elements to obtain accurate and efficient numerical solution EW equation. Ghafoor and Haq [22] have proposed a new numerical scheme in which Haar wavelet method is coupled with finite difference scheme for the solution of the EW equation. Zaki [23] has solved the EW equation by a least-squares technique using linear space-time finite elements. In addition to these studies, recently splitting methods have been used in the solution of many partial differential equations. Lee and Lee [24] have proposed a simple and stable second order operator splitting method for the numerical solution of Allen–Cahn type equations. Seydaoğlu and Blanes [25] have considered the numerical integration of non-autonomous separable parabolic equations using high order splitting methods with complex coefficients. Arnold and Ringhofer [26] have analyzed an operator splitting method for the linear Wigner equation and the coupled Wigner-Poisson problem. Zhang et al. [27] have presented the nonlinear stability and convergence analyses for a second order operator splitting scheme applied to the “good” Boussinesq equation. Seydaoğlu et al. [28] have used high order splitting methods for calculating the numerical solutions of Burgers’ equation in one space dimension with periodic, Dirichlet, Neumann and Robin boundary conditions, etc.
In this study, the EW equation was split into two sub-equations, and then two numerical schemes were obtained using cubic B-spline bases for each sub-equation. Four numerical algorithms applied to these numerical schemes based on the Lie-Trotter and Strang splitting methods.

2. Splitting methods

Operator splitting is a successful approach in numerical investigation of splitting complex and high-dimensional equations. The basic idea of the operator splitting methods to divide and conquer: split the complex problem into a sequence of simpler subproblems and solve these subproblems [29]. We will concentrate on the situation consisting of two linear operators. Now let us consider the following Cauchy problem

$$\frac{du(t)}{dt}=Cu(t),\ \ t\in[0,T],\quad u(0)=u_0 \tag{2.1}$$

and assume that it is split as $C=A+B$. Eq. (2.1) can be seen as a semi-discretization of a linear PDE with a homogeneous periodic boundary condition. Here, the initial function $u_0\in X$ is given and $C=A+B$, A and B are assumed to be bounded linear operators in X Banach space, with $A,B:X\to X$. There is also a norm associated with the X-space, and if A and B are matrices, then this norm is the Euclid norm [30]. The formal solution of Eq. (2.1) is in the form of $u(t_{n+1})=e^{tC}u(t_n)$. Then, $\Delta t=t_{n+1}-t_n$ is being the simplest splitting method, one can get

$$u(t_{n+1})\cong e^{\Delta tB}e^{\Delta tA}u(t_n). \tag{2.2}$$

If the operators A and B are commutative, then the method is exact. (2.2) is the simplest splitting technique and refers to the solution of two sub-problems as follows (2.2)

$$\begin{aligned}\frac{du^*(t)}{dt}&=Au^*(t),\ u^*(0)=u_0\text{ on }[0,\Delta t],\\ \frac{du^{**}(t)}{dt}&=Bu^{**}(t),\ u^{**}(0)=u^*(\Delta t)\text{ on }[0,\Delta t].\end{aligned} \tag{2.3}$$

Thus, the solutions at the desired time step are calculated by $u^{**}(\Delta t)$. This technique is called the $A-B$ splitting scheme. One can easily obtain the $B-A$ splitting scheme by replacing the locations of operators A and B [31].

2.1. Symmetric Strang splitting

For better accuracy, Strang [32] first handles the following scheme

$$u(\Delta t)=\frac{1}{2}\left[u_{AB}(\Delta t)+u_{BA}(\Delta t)\right] \tag{2.4}$$

where u_{AB} and u_{BA} are solutions calculated by AB and BA splitting schemes, respectively. Since each operator needs to be calculated twice in this scheme, the calculation cost is high. In place of Eq. (2.4), the symmetric scheme $u(t_{n+1})\simeq\left(e^{\frac{\Delta t}{2}A}e^{\frac{\Delta t}{2}B}\right)\left(e^{\frac{\Delta t}{2}B}e^{\frac{\Delta t}{2}A}\right)u(t_n)=e^{\frac{\Delta t}{2}A}e^{\Delta tB}e^{\frac{\Delta t}{2}A}u(t_n)$ is proposed due to its low computational cost [33]. This scheme can be explicitly stated as follows

$$\begin{aligned}\frac{du^*(t)}{dt}&=Au^*(t),\ u^*(0)=u_0\text{ on }[0,\Delta t/2],\\ \frac{du^{**}(t)}{dt}&=Bu^{**}(t),\ u^{**}(0)=u^*(\Delta t/2)\text{ on }[0,\Delta t],\\ \frac{du^{***}(t)}{dt}&=Au^{***}(t),\ u^{***}(0)=u^{**}(\Delta t)\text{ on }[0,\Delta t/2].\end{aligned} \tag{2.5}$$

Finally, the numerical schemes are solved with the term $u^{***}(\Delta t/2)$. As it is seen in the Eq. (2.5), the term $u^*(0)$ is calculated from the original initial condition of the problem, the other two initial conditions are taken from the previous calculated ones. If the scheme in Eq. (2.5) is called "$A-B-A$", one can obtain the scheme "$B-A-B$" in a similar way.

3. Operator Splitting B-Spline Collocation Method

Let us assume that the solution domain with respect to space is $[a,b]$ and a uniform discretization of the domain is constructed with nodal points at x_m, $m=0,1,...,N$, such that $a=x_0<x_1<...<x_N=b$. If we define the distance between two consecutive points as $h=x_{m+1}-x_m$, then by taking $\Phi_m(x)$, $m=-1(1)N+1$, cubic B-spline functions are presented on the domain $[a,b]$ having nodal points at x_m

$$\Phi_m(x)=\frac{1}{h^3}\begin{cases} (x-x_{m-2})^3, & x\in[x_{m-2},x_{m-1}] \\ h^3+3h^2(x-x_{m-1})+3h(x-x_{m-1})^2-3(x-x_{m-1})^3, & x\in[x_{m-1},x_m] \\ h^3+3h^2(x_{m+1}-x)+3h(x_{m+1}-x)^2-3(x_{m+1}-x)^3, & x\in[x_m,x_{m+1}] \\ (x_{m+2}-x)^3, & x\in[x_{m+1},x_{m+2}] \\ 0, & \text{otherwise} \end{cases} \tag{3.1}$$

as stated by Prenter [34]. It is obvious that the set $\{\Phi_{-1}(x),\Phi_0(x),...,\Phi_{N+1}(x)\}$ constitutes a base on the domain $[a,b]$. If we assume that the function $U(x,t)$ we are looking for is defined on the domain $[a,b]$, then the function $U(x,t)$ can be approximated by the following formula in terms of cubic B-spline functions and time dependent parameters $\delta_m(t)$

$$U(x,t)\cong\sum_{m=-1}^{N+1}\delta_m(t)\Phi_m(x). \tag{3.2}$$

Here the time dependent parameters $\delta_m(t)$ are going to be found using Eq. (1.1) and auxiliary conditions. Since Eq. (1.1) contains the first and second order derivatives of U, we need the nodal values of u and its first and second order derivatived in terms of cubic B-spline functions, using Eqs. (3.1) and (3.2) nodal values are obtained as follows

$$\begin{aligned} U_m &= U(x_m)=\delta_{m-1}+4\delta_m+\delta_{m+1} \\ U_m' &= U'(x_m)=\frac{3}{h}(\delta_{m+1}-\delta_{m-1}) \\ U_m'' &= U''(x_m)=\frac{6}{h^2}(\delta_{m-1}-2\delta_m+\delta_{m+1}) \end{aligned} \tag{3.3}$$

where $'$ and $''$ denote the first and second derivatives with respect to spatial variable x.
The time-split form of EW equation is as follows

$$u_t-\mu u_{xxt}+uu_x=0, \tag{3.4}$$

$$u_t-\mu u_{xxt}=0. \tag{3.5}$$

If the values of U, U' and U''in Eq. (3.3) are replaced in Eqs. (3.4) and (3.5), the following system of ordinary differential equations is obtained

$$\dot{\delta}_{m-1}+4\dot{\delta}_m+\dot{\delta}_{m+1}-\frac{6\mu}{h^2}\left(\dot{\delta}_{m-1}-2\dot{\delta}_m+\dot{\delta}_{m+1}\right)+\frac{3z_m}{h}(\delta_{m+1}-\delta_{m-1})=0, \tag{3.6}$$

$$\dot{\delta}_{m-1}+4\dot{\delta}_m+\dot{\delta}_{m+1}-\frac{6\mu}{h^2}\left(\dot{\delta}_{m-1}-2\dot{\delta}_m+\dot{\delta}_{m+1}\right)=0. \tag{3.7}$$

Here the symbol $\bullet$ denotes the first order derivative with respect to t and the linearization is made as follows

$$z_m=\delta_{m-1}+4\delta_m+\delta_{m+1}.$$

In Eqs. (3.6) and (3.7), if Crank-Nicolson approximation $\frac{\delta_m^{n+1}+\delta_m^n}{2}$ is written in place of parameters δ_m and forward difference scheme $\frac{\delta_m^{n+1}-\delta_m^n}{\nabla t}$ is written in place of parameters $\dot{\delta}_m$, the following system of equations are obtained, respectively

$$B:\quad \alpha_1\delta_{m-1}^{n+1}+\alpha_2\delta_m^{n+1}+\alpha_3\delta_{m+1}^{n+1}=\alpha_3\delta_{m-1}^n+\alpha_2\delta_m^n+\alpha_1\delta_{m+1}^n \tag{3.8}$$

$$A:\quad \beta_1\delta_{m-1}^{n+1}+\alpha_2\delta_m^{n+1}+\beta_1\delta_{m+1}^{n+1}=\beta_1\delta_{m-1}^n+\alpha_2\delta_m^n+\beta_1\delta_{m+1}^n \tag{3.9}$$

$$\begin{aligned} \alpha_1 &= 1-\frac{6\mu}{h^2}-\frac{3z_m\Delta t}{2h},\ \alpha_2=4+\frac{12\mu}{h^2},\ \alpha_3=1-\frac{6\mu}{h^2}+\frac{3z_m\Delta t}{2h} \\ \beta_1 &= 1-\frac{6\mu}{h^2}. \end{aligned}$$

The system (3.8) and (3.9) consist of $(N+1)$ equations and $(N+3)$ unknowns, namely δ_m, $m=0(1)N$, time dependent parameters. Using the boundary conditions $u(a,0)=u(b,0)=0$, we obtain a solvable $(N+1)\times(N+1)$ band matrix for solving the parameters δ_m. Then the systems (3.8) and (3.9) are arranged according to the Lie-Trotter and Strang algorithms given by (2.3) and (2.5) respectively, then the following algorithms are obtained,

$$\begin{aligned} \alpha_1\delta_{m-1}^*(t_{n+1})+\alpha_2\delta_m^*(t_{n+1})+\alpha_3\delta_{m+1}^*(t_{n+1}) &= \alpha_3\delta_{m-1}^*(t_n)+\alpha_2\delta_m^*(t_n)+\alpha_1\delta_{m+1}^*(t_n), \\ \delta_m^*(t_n) &= \delta_m^0,\ t\in[t_n,t_{n+1}], \\ \beta_1\delta_{m-1}^{**}(t_{n+1})+\alpha_2\delta_m^{**}(t_{n+1})+\beta_1\delta_{m+1}^{**}(t_{n+1}) &= \beta_1\delta_{m-1}^{**}(t_n)+\alpha_2\delta_m^{**}(t_n)+\beta_1\delta_{m+1}^{**}(t_n), \\ \delta_m^{**}(t_n) &= \delta_m^*(t_{n+1}),\ t\in[t_n,t_{n+1}] \end{aligned}$$

and

$$\begin{aligned}
\alpha_1\delta^{*}_{m-1}(t_{n+\frac{1}{2}})+\alpha_2\delta^{*}_{m}(t_{n+\frac{1}{2}})+\alpha_3\delta^{*}_{m+1}(t_{n+\frac{1}{2}}) &= \alpha_3\delta^{*}_{m-1}(t_n)+\alpha_2\delta^{*}_{m}(t_n)+\alpha_1\delta^{*}_{m+1}(t_n),\\
\delta^{*}_{m}(t_n) &= \delta^{0}_{m},\ t\in\left[t_n,t_{n+\frac{1}{2}}\right],\\
\beta_1\delta^{**}_{m-1}(t_{n+1})+\alpha_2\delta^{**}_{m}(t_{n+1})+\beta_1\delta^{**}_{m+1}(t_{n+1}) &= \beta_1\delta^{**}_{m-1}(t_n)+\alpha_2\delta^{**}_{m}(t_n)+\beta_1\delta^{**}_{m+1}(t_n),\\
\delta^{**}_{m}(t_n) &= \delta^{*}_{m}(t_{n+\frac{1}{2}}),\ t\in[t_n,t_{n+1}],\\
\alpha_1\delta^{***}_{m-1}(t_{n+1})+\alpha_2\delta^{***}_{m}(t_{n+1})+\alpha_3\delta^{***}_{m+1}(t_{n+1}) &= \alpha_3\delta^{***}_{m-1}(t_{n+\frac{1}{2}})+\alpha_2\delta^{***}_{m}(t_{n+\frac{1}{2}})+\alpha_1\delta^{***}_{m+1}(t_{n+\frac{1}{2}}),\\
\delta^{***}_{m}(t_{n+\frac{1}{2}}) &= \delta^{**}_{m}(t_{n+1}),\ t\in\left[t_{n+\frac{1}{2}},t_{n+1}\right]
\end{aligned}$$

respectively.

Now, we need the initial vector δ^0_m to be able to solve the system of Eqs (3.8) and (3.9). The initial vector is obtained as follows using the initial condition given together with the problem $u(x,0)=f(x)$ and approximation in Eq. (3.2)

$$\begin{aligned}
u(x_m,0) &= f(x_m)=U(x_m,0),\ m=0(1)N\\
u_m &= \delta^0_{m-1}+4\delta^0_m+\delta^0_{m+1}\\
u_0 &= \delta^0_{-1}+4\delta^0_0+\delta^0_1\\
u_1 &= \delta^0_0+4\delta^0_1+\delta^0_2\\
&\ \vdots\\
u_N &= \delta^0_{N-1}+4\delta^0_N+\delta^0_{N+1}.
\end{aligned}$$

Using the boundary conditions $U^{'}(a,0)=U^{'}(b,0)=0$ for this system; the initial vector is obtained as follows

$$\begin{bmatrix} 4 & 2 & 0 & & \\ 1 & 4 & 1 & & \\ & & \ddots & & \\ & & 1 & 4 & 1\\ & & 0 & 2 & 4 \end{bmatrix}\begin{bmatrix}\delta^0_0\\ \delta^0_1\\ \vdots\\ \delta^0_{N-1}\\ \delta^0_N\end{bmatrix}=\begin{bmatrix}u_0\\ u_1\\ \vdots\\ u_{N-1}\\ u_N\end{bmatrix}.$$

3.1. Stability analysis

(3.8) and (3.9) numerical schemes have been considered by the Fourier von Neumann [35] method. In this method $\delta^n_m=\xi^n e^{i\beta mh}$ is taken, where $i=\sqrt{-1}$, β is mode number, ξ is amplification factor and h is the space step in the method. In Eq. (2.3) in the term uu_x, since we take $u=z_m$ for linearization purpose, it will behave as a local constant. Let us assume that the amplification factors related to the schemes in (3.8) and (3.9) are ρ_A and ρ_B, respectively. If we write $\delta^n_m=\xi^n e^{i\beta mh}$ in Eq.(3.8), we obtain

$$\rho_A\left(\frac{\xi^{n+1/2}}{\xi^n}\right) = \frac{X-iY}{X+iY},$$

$$X = \alpha_2+(\alpha_1+\alpha_3)\cos\beta h,\ \ Y=(\alpha_3-\alpha_1)\sin\beta h.$$

Thus, since $\left|\rho_A\left(\frac{\xi^{n+1/2}}{\xi^n}\right)\right|\le 1$ is valid, the linearized schme is unconditionally stable. In a similar way, if we take $\delta^n_m=\xi^n_B e^{i\beta mh}$ in Eq. (3.9), we obtain $\left|\rho_B\left(\frac{\xi^{n+1}}{\xi^n}\right)\right|\le 1$. Thus,

$$\rho_L(\xi)=\rho_A^{n+1}\rho_B^{n+1}\qquad \text{or}\qquad \rho_L(\xi)=\rho_B^{n+1}\rho_A^{n+1}$$

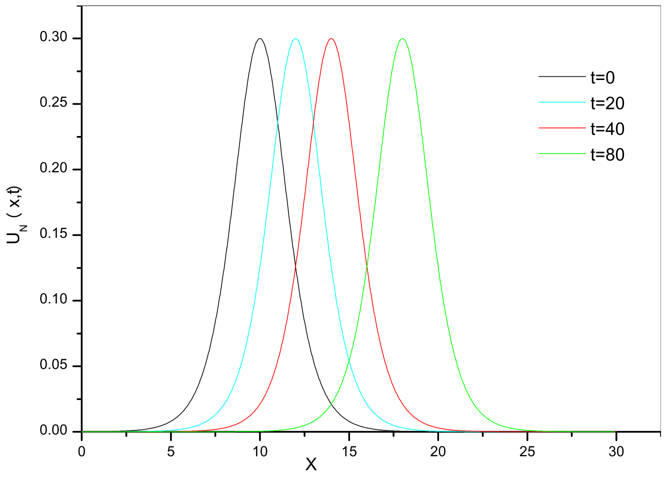
$$|\rho_L(\xi)|\le\left|\rho_A\left(\frac{\xi^{n+1}}{\xi^n}\right)\right|\left|\rho_B\left(\frac{\xi^{n+1}}{\xi^n}\right)\right|\le 1$$

the Lie-Trotter splitting schemes are unconditionally stable. In a similar way,

$$\rho_S(\xi)=\rho_A^{n+1/2}\rho_B^{n+1}\rho_A^{n+1/2}\qquad or\qquad \rho_S(\xi)=\rho_B^{n+1/2}\rho_A^{n+1}\rho_B^{n+1/2}$$

$$|\rho_S(\xi)|\le\left|\rho_A\left(\frac{\xi^{n+1/2}}{\xi^n}\right)\right|\left|\rho_B\left(\frac{\xi^{n+1}}{\xi^n}\right)\right|\left|\rho_A\left(\frac{\xi^{n+1/2}}{\xi^n}\right)\right|\le 1$$

so that the Strang splitting schemes are unconditionally stable.



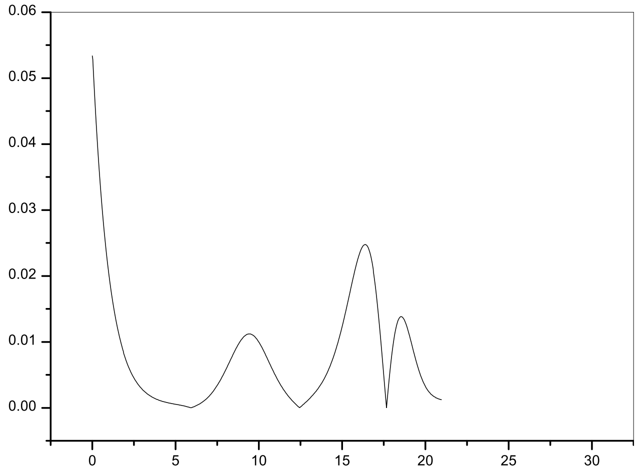


Figure 4.1: The movement of single solitary wave as time progress *a*) $\Delta t = 0.05$, $h = 0.03$, $c = 0.1$, $x_0 = 10$, $a = 0$ and $b = 30$. *b*) Error graph at time $t = 80$ for the same values.

4. Numerical Studies and Results

In this section numerical solutions of EW equation obtained by operator splitting cubic B-spline collocation method are presented. For this purpose, the method is investigated for the motion of single solitary wave and Maxwellian initial pulse problems. The correctness and effectiveness of the method are measured by the following error norms

$$\begin{aligned} L_2 &= \left\|U^{exact} - U^N\right\|_2 = \sqrt{h\sum_{i=1}^{N}\left|U_i^{exact} - U_i^N\right|}, \\ L_\infty &= \left\|U^{exact} - U^N\right\|_\infty = \max_i \left|U_i^{exact} - U_i^N\right| \end{aligned}$$

where U^N is the numerical solution and $h = (b-a)/N$. The effectiveness of a numerical method other than the error norms it is also measured by conservation constants provided by the equation. The three conservation laws namely mass, momentum, energy have been given by Olver [36] as follows

$$\begin{aligned} I_1 &= \int_a^b U\,dx \cong h\sum_{i=0}^{N} U_i^N, \\ I_2 &= \int_a^b \left(U^2 + \mu\left(U_x\right)^2\right)dx \cong h\sum_{i=0}^{N}\left[\left(U_i^N\right)^2 + \mu\left(\left(U_x\right)_i^N\right)^2\right], \\ I_3 &= \int_a^b U^3 dx \cong h\sum_{i=0}^{N}\left(U_i^N\right)^3 \end{aligned}$$

respectively. The analytic values of the invariants I_1, I_2, I_3 for $c = 0.1$ given by [18] as follows

$$I_1 = \frac{6c}{k} = 1.2, \quad I_2 = \frac{12c^2}{k} + \frac{48c^2k}{5} = 0.288, \quad I_3 = \frac{144c^3}{5k} = 0.0576$$

and $I_1 = 0.36000$, $I_2 = 0.02592$, $I_3 = 0.001555$ for $c = 0.3$.

4.1. The Progress of a Single Wave

With the boundary conditions $U \to 0$ as $x \to \pm\infty$ the solitary wave solution of the EW equation given by [3] is

$$U(x,t) = 3c\sec h^2\left[k\left(x - vt - x_0\right)\right]$$

where $v = c$ the wave velocity and $k = \sqrt{1/(4\mu)}$ width of the wave. This solution is a solution that preserves the shape and speed of with a $3c$-amplitude wave located at center x_0 as it moves to the right. To compare with other studies in the literature, $c = 0.1$, $x_0 = 10$, $\mu = 1$ and $U(a,t) = U(b,t) = 0$ have taken on the region $0 \le x \le 30$. For the four methods we have applied to the EW equation, the conservation laws I_1 ,I_2 ,I_3 with the L_2 and L_∞ error norms were calculated over time $t = 80$. In the Table 1 for values of $\Delta t = 0.05$, $h = 0.03$, $c = 0.1$, $x_0 = 10$, $a = 0$ and $b = 30$ using the schemes *A-B*, *B-A*, *A-B-A* and *B-A-B* the conservative properties I_1, I_2, I_3 together with the error norms L_2 and L_∞ are presented. In the Table 1, it is seen that among the results obtained using the four schemes the invariant I_1 changes a little with time, but the invariants I_2 and I_3 remain almost constant and using the schemes *B-A-B* reluts in lower error norms L_2 and L_∞. In Table 2, the results obtained using the scheme *B-A-B* are compared with those in Refs [5, 6]. From this table it is seen that the invariant I_1 is in good agreement with those in Refs [5, 6] but the invariants I_2 and I_3 are more preserved than those in these studies. Moreover the study in Ref. [5]

Table 1: A comparison of numerical results and error norms of single solitary wave for values of $\Delta t = 0.05$, $h = 0.03$, $c = 0.1$, $x_0 = 10$, $a = 0$ and $b = 30$.

method	t	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$
	0	1.199945	0.288000	0.057600	0.000000	0.000000
	5	1.199989	0.288000	0.057600	0.015912	0.020800
	10	1.200014	0.288000	0.057600	0.026310	0.033416
A-B	20	1.200040	0.288000	0.057600	0.038555	0.045711
	40	1.200052	0.288000	0.057600	0.049568	0.051898
	80	1.200041	0.288000	0.057600	0.057386	0.052850
	0	1.199945	0.288000	0.057600	0.000000	0.000000
	5	1.199989	0.288000	0.057600	0.015912	0.020800
	10	1.200014	0.288000	0.057600	0.026310	0.033416
B-A	20	1.200040	0.288000	0.057600	0.038555	0.045711
	40	1.200052	0.288000	0.057600	0.049568	0.051898
	80	1.200041	0.288000	0.057600	0.057386	0.052850
	0	1.199945	0.288000	0.057600	0.000000	0.000000
	5	1.199989	0.288000	0.057600	0.015912	0.020800
	10	1.200014	0.288000	0.057600	0.026310	0.033416
A-B-A	20	1.200040	0.288000	0.057600	0.038555	0.045711
	40	1.200052	0.288000	0.057600	0.049568	0.051898
	80	1.200041	0.288000	0.057600	0.057386	0.052850
	0	1.199945	0.288000	0.057600	0.000000	0.000000
	5	1.199989	0.288000	0.057600	0.015841	0.020800
	10	1.200014	0.288000	0.057600	0.026141	0.033416
B-A-B	20	1.200040	0.288000	0.057600	0.038115	0.045711
	40	1.200052	0.288000	0.057600	0.048432	0.051898
	80	1.200041	0.288000	0.057600	0.054761	0.052850

Table 2: A comparison of numerical results and error norms of single solitary wave with [5] and [6] for values of $\Delta t = 0.05$, $h = 0.03$, $c = 0.1$, $x_0 = 10$, $a = 0$ and $b = 30$.

method	t	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$
	0	1.199945	0.288000	0.057600	0.000000	0.000000
	5	1.199989	0.288000	0.057600	0.015841	0.020800
	10	1.200014	0.288000	0.057600	0.026141	0.033416
B-A-B	20	1.200040	0.288000	0.057600	0.038115	0.045711
	40	1.200052	0.288000	0.057600	0.048432	0.051898
	80	1.200041	0.288000	0.057600	0.054761	0.052850
	0	1.19995	0.2880	0.05760	0.000	0.000
	5	1.19999	0.2880	0.05760	0.021	0.033
	10	1.20010	0.28804	0.05761	0.048	0.033
[5]	20	1.20015	0.28805	0.05761	0.064	0.046
	40	1.20005	0.28800	0.05760	0.049	0.052
	80	1.19998	0.28798	0.05759	0.056	0.053
	10	1.20001	0.287997	0.0576	0.033297	0.033415
	20	1.20004	0.287997	0.0576	0.056561	0.045709
[6]	40	1.20005	0.287997	0.0576	0.098784	0.051895
	80	1.20004	0.287997	0.0576	0.183785	0.095878

Table 3: A comparison of numerical results and error norms of single solitary wave with Refs. [6, 11, 12, 13, 14] for values of $\Delta t = h = 0.05$, $c = 0.03$, $x_0 = 10$, $a = 0$ and $b = 30$.

method	t	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$
	0	0.359984	0.025920	0.001555	0.000000	0.000000
	10	0.359992	0.025920	0.001555	0.003959	0.004029
B-A-B	20	0.359999	0.025920	0.001555	0.007352	0.007014
	40	0.360007	0.025920	0.001555	0.013007	0.010863
	80	0.360014	0.025920	0.001555	0.021446	0.014528
	10	0.359991	0.0259194	0.0015552	0.006478	0.0040286
	20	0.359998	0.0259194	0.0015552	0.012628	0.007013
[6]	40	0.360006	0.0259194	0.0015552	0.024398	0.01254
	80	0.360013	0.0259194	0.0015552	0.047152	0.024708
	10	0.35998	0.02592	0.00156	0.008724	0.012107
	20	0.35998	0.02592	0.00156	0.006448	0.008969
[11]	40	0.35999	0.02592	0.00156	0.003515	0.004923
	80	0.36000	0.02592	0.00156	0.001025	0.001483
	10	0.35999	0.02592	0.00156	0.0063	0.0040
	20	0.36000	0.02592	0.00156	0.0123	0.0081
[12]	40	0.36001	0.02592	0.00156	0.0236	0.0165
	80	0.36001	0.02592	0.00156	0.0436	0.0309
[13]	80	0.36000	0.02592	0.00156	0.013	0.007
[14]	80	0.36665	0.02658	0.00162	2.683	1.836

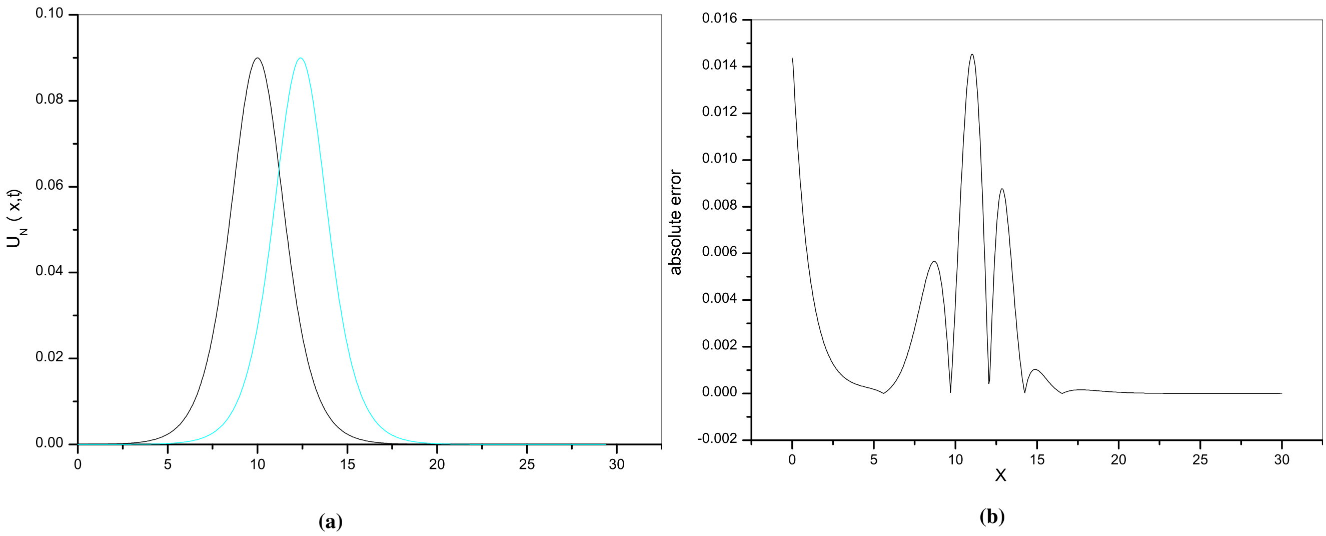


Figure 4.2: The movement of single solitary wave as time progress *a*) $\Delta t = 0.05$, $h = 0.03$, $c = 0.03$, $x_0 = 10$, $a = 0$ and $b = 30$. *b*) Error graph at time $t = 80$ for the same values.

Table 4: A comparison of numerical results and chance factor (c. f.) of Maxwellian initial case with Refs. [16, 20] for $\mu = 0.2$, 0.04 and 0.001.

μ	t	I_1	I_2	I_3	I_1	I_2	I_3
			B-A-B			[16]	
	0.5	1.7724540	1.5039736	1.0233267	1.772454	1.503339	1.023327
	1	1.7724542	1.5039646	1.0233268	1.772457	1.503303	1.023327
0.2	2	1.7724544	1.5039358	1.0233271	1.772531	1.503190	1.023328
	3	1.7724545	1.5039076	1.0233274	1.774443	1.505232	1.023329
	4	1.7724546	1.5038948	1.0233275	1.796324	1.583276	1.023345
c. f.		6×10^{-7}	7.88×10^{-5}	8×10^{-7}	2.387×10^{-2}	8×10^{-2}	1.8×10^{-5}
c. f. in [20]		2.2×10^{-3}	6.3×10^{-2}	3×10^{-6}			
	0.5	1.7724539	1.3034387	1.0233269	1.772454	1.303289	1.023328
	1	1.7724541	1.3034109	1.0233278	1.772453	1.303178	1.023331
0.04	2	1.7724550	1.3032596	1.0233337	1.772454	1.302571	1.023355
	3	1.7724565	1.3030066	1.0233446	1.772447	1.301551	1.023400
	4	1.7724576	1.3028363	1.0233515	1.765447	1.301041	1.023424
c. f.		3.7×10^{-6}	6.024×10^{-4}	2.46×10^{-5}	7×10^{-3}	2.88×10^{-3}	9.6×10^{-5}
c. f. in [20]		2.2×10^{-3}	7.5×10^{-2}	3×10^{-6}			
	0.5	1.7724539	1.2545674	1.0233267	1.772453	1.254567	1.023326
	1	1.7724539	1.2545672	1.0233267	1.772454	1.254566	1.023327
0.001	2	1.7724544	1.2545020	1.0233279	1.772453	1.254304	1.023330
	3	1.7724549	1.2544085	1.0233289	1.772434	1.253925	1.023327
	4	1.7724548	1.2543581	1.0233285	1.760800	1.255103	1.023311
c. f.		9×10^{-7}	2.093×10^{-4}	1.8×10^{-6}	1.16×10^{-2}	6.42×10^{-4}	1.9×10^{-5}
c. f. in [20]		1.2×10^{-2}	7.8×10^{-2}	3×10^{-6}			

has applied cubic B-spline collocation method to the EW equation without splitting, but we have applied the same method by splitting the equation and obtained lower error norms L_2 and L_∞. In the Table 3, for values of $\Delta t = h = 0.05$, $c = 0.03$, $x_0 = 10$, $a = 0$ and $b = 30$ using the scheme *B-A-B* the conservative properties I_1, I_2, I_3 together with the error norms L_2 and L_∞ are presented. Moreover, the obtained results are compared with those given in Refs. [6, 11, 12, 13, 14]. It is seen from tha table that the results calculated using the scheme *B-A-B* are in agreement with those given in Refs. [11, 13] and the present method yields lower error norms L_2 and L_∞ than those obtained in Refs. [6, 12, 14]. The graphics of single solitary wave for $c = 0.1$ at times $t = 0, 20, 40, 80$ and for $c = 0.03$ at times $t = 0, 80$ are illustared in the Figures 4.1 and 4.2. Is is seen from the figures that tha wave moves towards right preserving its shape, speed and amplitude. While the amplitude of the wave is 0.3 for $c = 0.1$ at $x_0 = 10$ and $t = 0$, it is 0.299992 at $x = 18.00$ and $t = 80$. Again While the amplitude of the wave is 0. for $c = 0.03$ at $x_0 = 10$ and $t = 0$, it is 0.089994 at $x = 12.40$ and $t = 80$.

5. Initial Maxwellian Pulse

The last problem, we have worked, for different values of the μ, the evolution of an initial Maxwellian pulse into solitary waves, taken as an initial condition given by Refs. [6, 16, 20, 21]

$$U(x,0) = \exp\left[-(x-7)^2\right].$$

This initial pulse developed into a series of solitons for KdV equation when μ bigger than a critical value $\mu = 0.0625$, if not, generates a rapidly oscillating wave packet [21]. The results of this problem has been obtained by scheme *B-A-B* for values of $\mu = 0.2, 0.04, 0.001$, $a = 0$ and $b = 30$. In the Table 4, the calculated invariants I_1, I_2, I_3 are given and compared with the invariants I_1, I_2, I_3 given in Ref. [16]. Together with this, the change factor of the ivariants is calculated as the difference between the first and last time values and also compared with those obtained in Refs. [16, 20]. As it is seen from the table, the results obtained using the Strang scheme *B-A-B* are much more better than those obtained in those studies.

6. Conclusion

In this study, operator splitting cubic B-spline collocation method has been applied to EW equation based on the Lie-Trotter and Strang splitting. For this purpose, the equation has been split up into two sub-equations and then numerical schemes were obtained by applying the cubic B-spline collocation method to these sub-equations. These numerical schemas are solved using *A-B*, *B-A*, *A-B-A* and *B-A-B* algorithms. The results obtained by these four algorithms have been compared in the single solitary wave problem for $c = 0.1$ and the results calculated by *B-A-B* algorithm have been compared with other studies in the literature for the two problems we are dealing with. As seen the result we give, the method we use is a convenient and efficient method for many nonlinear mathematical and physical problems.

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