Optimal Allocation of Photo Voltaic Arrays in Radial Distribution System with Various Load Models

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Abstract

In this paper, an effective methodology has been proposed for optimal allocation of renewable generation sources in the distribution system. The objective of this work is to minimize real and reactive power losses in the distribution systems. The output power of solar DGs mainly depends upon solar irradiance level. So, before placement of these sources random nature of solar irradiance should be effectively modeled. A beta probability density function is used to model the solar irradiance and determine the exact output power of these sources. Further, the best location for placement of solar DGs is found out using loss sensitivity factor. The optimal sizing of photo voltaic arrays corresponding to these locations is determined using firefly algorithm (FA). Different time varying load models such as residential, commercial and industrial has been considered for the study along with probabilistic generation pattern. The developed method is tested on IEEE 69 bus test system. The obtained results show that optimal allocation of solar DGs in the distribution system gives a positive impact in terms of achieving better loss reduction and voltage profile enhancement. A comparative study is made for all the load models and the importance of considering different load models is projected.

1. INTRODUCTION

In the recent years, penetration of renewable distributed generation (RDG) sources (solar, wind and biomass) to the distribution network (DN) is increasing significantly throughout the globe due to its environmental and economic benefits. As of 2016 international energy agency report, the power generated with alternative energy sources will reach 37% by the year 2040 [1]. The installed capacity of solar arrays in the entire world will increase from 40 GW to 227GW during the years 2010-2016 [1]. From the utility point of view, integration of these sources (Solar based DGs) to the existing DN improves the performance of system in terms of better power loss reduction, voltage profile improvement, network upgrade deferral and peak demand shaving, etc. [2]. The above mentioned advantages can only be achieved by optimal allocation of the available resources.

DG placement in the DN with an objective of active power loss reduction is presented [3-7]. Further, optimal planning of these resources in the DS considering multiple objectives has been discussed [8-14]. Most of the above mentioned methods are applicable for placement of conventional DGs like gas turbines, diesel generators and micro turbines because the output power of these sources is controllable.

In recent years, different authors placed RDGs in the DN and studied the performance of the system. Optimal allocation of RDGs (solar, wind) in the DN using PSO technique is presented in [15]. The objective is to reduce losses and improve the voltage stability. Cuckoo search algorithm for placement of wind based DGs in the DN is presented in [16]. In [15] & [16] authors considered average wind speed and solar irradiance values for calculating output power from wind turbines (WTs) and photo voltaic
arrays (PVAs) but the drawback of the articles [15] & [16] is that the considered wind speed and solar irradiance does not give the exact output power. Optimal allocation of RDGs (solar, wind) in the DS using ant lion optimization is presented in [17]. Analytical method for placement of solar based DGs in the DS is presented in [18]. The objective of this approach is to minimize the power loss, improve voltage profile and economical analysis. A two-step algorithm for placement of WTs in the DS, with an aim of improving loss reduction is presented in [19]. In [17]-[19] authors placed PVAs and WTs in the DS, with an aim of improving the performance of DS, but the generation uncertainties associated with the renewable energy resources are not considered. Also, the demand of the system is considered to be constant. In practical cases, power generated from these sources is stochastic and the demand varies continuously. So, it is very important to consider the generation uncertainties associated with these renewable energy resources and also to consider the load as a variable parameter.

Few authors considered the generation uncertainties with RDGs (solar, wind) and placed them in the DS. In [20, 21] authors placed solar based DGs in the DS, with an aim of minimizing power loss of the system. Evolutionary programming technique is used for RDGs allocation (solar, wind) in the DS, with an aim of improving energy loss reduction [22]. A combination of different types of RDGs placed (solar, wind and biomass) in the DS using analytical approach is presented in [23]. The objective of this method is to reduce energy loss of the system. In the above methods, the authors considered only energy loss or power loss as the objective function. Further, in the articles [20]-[23] objectives like reduction of reactive power loss is not considered. Also the effect of different time varying load models is not addressed. Few authors [24-26] considered different load models and DG penetration levels effect on the DS. Although, these methods consider that DG units as dis-patchable sources and the output power of these sources is controllable. The above mentioned methods have not considered the generation uncertainties associated with the renewable generation sources. The impact of different load models on the system with placement of wind based DGs in the DS is considered in [27]. The objective of this approach is to minimize energy loss of the DS, however the best location and size of WTs is not calculated. In [28] authors described the effect and importance of time varying load models on the system along with DG location and sizing. But, the limitation of this approach is that the authors have not considered the generation uncertainties associated with the wind based DGs.

From the literature, it is clear that optimal allocation of RDGs (solar) in the DS considering both generation uncertainties and different time varying load models effect on the DS has not been addressed. In this paper an effective methodology is proposed for solving solar based DG allocation problem in the DS, considering the importance of various time varying load models. An integrated approach for loss sensitivity factor (LSF) and a search based technique namely firefly algorithm (FA) has been used to solve the proposed objective. Initially best locations for the placement of PVAs have been identified using LSF. The number of photo voltaic arrays (PVAs) placed at these identified locations is determined using FA. The objective of the developed method is to minimize real, reactive power loss and enhance voltage profile of the system.

The remaining sections of the paper are organized as follows. In section 2, load and solar PV modeling is explained. Problem formulation with constraints is explained in section 3. The best location for placement of PVAs is identified using LSF technique is explained in section 4. Optimal sizing of PVAs is determined using FA technique and the same is explained in section 5. Results and discussion followed by the conclusion of the article is explained in sections 6 and 7.

2. LOAD AND SOLAR PV MODELING

2.1. Load modeling

In this paper, the load demand of the system follows three different load patterns (residential, commercial and industrial) with a peak demand of 1 p.u as shown in Figure 1. [29]. Voltage dependent loads for period ‘t’ can be modeled as [30]:

$$\text{Load}_t = \text{Load}_t^{\text{base}} \times (1 + \text{V Dependency})$$
Figure 1. Load demand curve for various customers

Where $P_m$ and $Q_m$ are the real and reactive power injection at bus m, $P_{om}$ and $Q_{om}$ are active and reactive power operating points at bus m. $\alpha$ and $\beta$ are the real and reactive power exponents that are given in Table 1 [24]. Finally $V_m$ is the voltage at the bus m.

$$P_m(t) = P_{om}(t) * V_m^\alpha(t); \; Q_m(t) = Q_{om}(t) * V_m^\beta(t)$$ (1)

Table 1. Load types and exponent values

<table>
<thead>
<tr>
<th>Different load types</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>0.92</td>
<td>4.04</td>
</tr>
<tr>
<td>Commercial</td>
<td>1.51</td>
<td>3.40</td>
</tr>
<tr>
<td>Industrial</td>
<td>0.18</td>
<td>6.00</td>
</tr>
</tbody>
</table>

2.2 Solar PV modeling

The output power of solar PV module mainly depends upon the solar irradiance level. So, in order to determine the exact power from the PV module there is a necessity to model the solar irradiance effectively. Different probability functions are available in the literature to model the solar irradiance, but beta PDF is found to be performing better [31, 32]. Historical data is taken from [33], corresponding mean and standard deviation of the hourly solar irradiance of the day is calculated from the considered historical data. Further each hour of the day is again divided into number of possible states. Each hour of the day is divided into 20 possible states for solar irradiance (s) with a step size of 0.05 kW/m$^2$ and then PDF of each state is calculated. Finally, the output of power of PV module is determined for that hour. The explanation for the model is as follows:

A Beta PDF of solar irradiance (s) can be calculated as follows

$$f_b(s) = \frac{\Gamma(\alpha_a + \beta_a)}{\Gamma(\alpha_a)\Gamma(\beta_a)}s^{\alpha_a-1}(1-s)^{\beta_a-1} \text{ for } \alpha_a > 0; \; \beta_a > 0$$ (2)

Where $\alpha_a$ and $\beta_a$ are the shape parameters, $s$ is the solar irradiance (kW/m$^2$) and $\Gamma$ is the gamma function. Shape parameters of $f_b(s)$ can be calculated using mean (\( \mu \)) and standard deviation (\( \sigma \)) of $s$.

$$\alpha_a = \frac{\mu\beta_a}{1-\mu}$$ (3)

$$\beta_a = (1-\mu)\left(\frac{\mu(1+\mu)}{\sigma^2} - 1\right)$$ (4)
The maximum output power of PV module at corresponding solar irradiance \( s \) can be calculated as shown below [34].

\[
P_{PVM}(s) = N_{PVM} \cdot FF \cdot V_y \cdot I_y
\]

(5)

Where

\[
FF = \frac{V_{MPP} \cdot I_{MPP}}{V_{oc} \cdot I_{sc}}
\]

(6)

\[
V_y = V_{oc} - K_T T_{cy}
\]

(7)

\[
I_y = s[I_{sc} + K_i(T_cy - 25)]
\]

(8)

\[
T_{cy} = T_A + s\left(\frac{N_{OT} - 20}{0.8}\right)
\]

(9)

\[\text{Where } FF \text{ is the fill factor and } N_{PVM} \text{ is the number of PV modules used. } V_{oc}, I_{sc} \text{ and } N_{OT} \text{ are open circuit voltage, short circuit current and nominal operating temperature of PV module respectively. } T_{cy} \text{ and } T_A \text{ are cell and ambient temperature. Finally, } K_i \text{ and } K_T \text{ are voltage and current temperature coefficients respectively.}\]

The expected output power of PV module at corresponding solar irradiance \( s \) is determined using Eq. (10).

\[
Ep(s) = P_{PVM}(s) \cdot f_b(s)
\]

(10)

The total expected output power of PV module at a specific time segment \((t=1\text{h})\) can be determined as follows [34].

\[
TEP(t) = \int_0^1 P_{PVM}(s) \cdot f_b(s)ds
\]

(11)

In this paper solar based DGs which operates at unity power factor has been considered for optimal placement according to the standard of IEEE 1547 [35].

For example, from the collected historical data, mean and standard deviation is calculated and the beta PDF for 20 solar irradiance states and the total expected output powers at 9th, 11th and 13th hours are calculated and the same is illustrated in Figures 2 and 3.

![Figure 2. PDF for solar irradiance at 9th, 11th and 13th hours](image-url)
3. PROBLEM FORMULATION

3.1. Impact Indices

The active, reactive power loss and voltage profile of the DS is majorly dependent on proper location and sizing of PVAs and it can be defined as follows:

3.1.1 Active power loss index [36]

\[ APLI = \frac{P_{LPVA}}{P_L} \]  
(12)

3.1.2 Reactive power loss index

\[ RPLI = \frac{Q_{LPVA}}{Q_L} \]  
(13)

3.2 Formulation of multi objective function

In order to form a multi objective function, combine all indices adding proper weights and the same can be formulated as follows.

\[ MOF = \beta_1 APLI + \beta_2 RQLI \]  
(14)

Where

\[ \sum_{q=1}^{2} \beta_q = 1.0 \quad \text{and} \quad \beta_q \in [0,1], \quad q = 1, 2 \]  
(15)

The weights are assigned according to the importance of respective objective function [24, 25, 36]. Determining the best values of weights depends upon the experience of the system planners. PVA placement in the DS has a significant impact on the system losses and voltage profile. In case of DS, active power loss is a very significant issue compared to reactive power loss [24, 25, 36]. So, highest weight is assigned to active power loss (0.6) and a less weight is assigned to reactive power loss (0.4). It has to be noted that these weights can be changed according to the priority.

In this paper, PVAs is considered for placement in the DS. So, the output power of these sources depends upon solar irradiance level which is a random variable. So, the power generation from these sources changes from time to time. The hourly output power at time period t, and corresponding MOF (t) is obtained from Eqs (10) and (14) and this can be represented as follows i.e. Eq. (16).
MOF(t) = \int_{0}^{1} MOF(s)Ep(s)ds  \tag{16}

Hence, average MOF (AMOF) over the entire day (T=24) after placement of PVAs can be calculated as follows.

\begin{equation}
AMOF = \frac{1}{T} \int_{0}^{T} MOF(t)dt = \frac{1}{24} \sum_{t=1}^{24} MOF(t)\Delta t
\end{equation}  \tag{17}

Where \( \Delta t \) is the time segment of period t (1h). Optimal allocation of PVAs results in lesser value of AMOF.

The objective function represented in Eq.(17) should satisfy the voltage magnitude, power flow, thermal and DG penetration level constraints of the DS. The above constraints are represented as follows.

\begin{equation}
V_{\text{min}} \leq V_{m}(t) \leq V_{\text{max}}
\end{equation}  \tag{18}

\begin{equation}
P_{\text{sub}} + \sum_{PVA=1}^{k} P_{PVA} = P_{L} + P_{\text{loss}}
\end{equation}  \tag{19}

Here 'k' represents the total no of PVAs added to the system.

\begin{equation}
I_{\text{line}} \leq I_{\text{max}}
\end{equation}  \tag{20}

\begin{equation}
P_{PVA} \leq P_{D}
\end{equation}  \tag{21}

4. LOSS SENSITIVITY FACTOR (LSF) FOR PVAS INSTALLATION

In this paper, LSF technique is used to determine the best locations for placement of PVAs [37]. The LSF for buses m and m+1 can be determined using Eq. (22).

\begin{equation}
\frac{\partial P_{\text{line}\text{loss}}}{\partial P_{r+1,\text{eff}}} = \frac{2P_{r+1,\text{eff}} R_{r,r+1}}{|V_{r+1}|^2}
\end{equation}  \tag{22}

By using Eq. (21) it is essential to calculate the LSF values from distribution load flow and arrange them in decreasing order. The buses with least value of LSF are considered as optimal buses for placement of PVAs. This procedure reduces the problem search space during the optimization process.

5. FIREFLY ALGORITHM (FA)

Firefly algorithm is developed by Xin-She Yang based on the flashing behaviour of fireflies, The development of FA technique is based on three important steps [38].

1) An individual firefly attracts the other fireflies regardless of sex.
2) Attractiveness mainly depends upon brightness, if brighter firefly is available remaining fireflies move towards that firefly. The brightness decreases with respect to distance.
3) If there are no brighter fireflies available in the surroundings fireflies will move randomly.

Based on the above rules Pseudo code of FA is developed [38] that is as shown in Figure 4.

5.1. Important steps to solve PV allocation problem on the DS using FA

1) Read the system data and initialize the parameters of FA (n=20, iter_{max}=200, \alpha=0.25, \beta_{min}=0.2, \gamma=1 and dimension of search space d=3) [38].
2) Develop a generation load model with collected historical and load data for a given number of PVAs.
3) Run the base case load flow (backward-forward distribution load flow) [39].
4) Identify the best locations using LSF technique and these identified locations are given as input to the FA.
5) In FA, attractiveness mainly depends on light intensity that is calculated as follows.

\[ \beta = \beta_0 e^{-\gamma r^2} \]  

(23)

Where \( \beta_0 \) is the attractiveness at \( r=0 \). The distance between any two fireflies can be calculated as follows.

\[ r_{ij} = \| x_i - x_j \| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \]  

(24)

6) Less brighter firefly gets attracted to another brighter firefly for determining the optimal value of PV sizes that is calculated by using Eq. (25).

\[ x_i = x_i + \beta_0 e^{-\gamma r^2} (x_j - x_i) + \alpha (\text{rand} - 0.5) \]  

(25)

Where \( \alpha \) is the randomization parameter and “rand” is a random number generator over uniform distributed in \([0, 1]\).

7) From the obtained PV sizes, calculate the active and reactive power loss indices for the DS and if these values are best and all the constraints of DS are satisfied then stop the procedure.

8) Otherwise, repeat steps 5 to 7 until reaching the stopping criteria or until attaining the maximum number of iterations.

6. RESULTS AND DISCUSSION

The proposed method is implemented on an IEEE 69 bus RDS which has a peak demand of 3.8 MW and 2.69 MVAr respectively, and test system data is taken from [40]. The single line diagram of the 69 bus RDS is shown in Figure 5. Three different types of load models (Residential, commercial and industrial)
are considered for the analysis. The hourly load demand pattern of these load models is as shown in Figure 1. In this method, solar based DGs injecting only active power into the system are considered and these are placed in the DS [35]. Before DG placement, generation uncertainties associated with solar based DGs are modeled using beta PDF from the considered historical data taken from [33]. From this historical data, mean and standard deviation is calculated. Further specifications of PV module are taken from [34]. The solar irradiance ‘s’ is considered at an interval of 0.05 kW/m² and using Eqs (2)-(11) the hourly expected output power of PV module is estimated and is as shown in Figure 6. From Figure 6 it is clear that the output power of PV module varies with respect to time. Also, the output power mainly depends on solar irradiance level and specification of PV module. Further in this paper, the number of PV modules considered to form a PV array is 1600 [41]. The maximum number of PV arrays placed at any bus is 8, considering the space and land availability.

![Figure 5. Single line diagram of 69 bus RDS](image)

![Figure 6. Hourly expected PV module output](image)
The best locations for placement of PVAs are identified by using LSF technique and the identified buses for the placement of DGs are 27, 61 and 64 respectively. The optimal sizes of PVAs determined at these locations during each hour of the day considering different load models are found out by using FA technique and is as shown in Figure 7. From Figure 7 it is clear that the power outputs patterns of PVAs exactly follow the hourly expected output power of a PV module. Further the hourly power loss indices considering various load models after placement of PVAs are as shown in Figure 8 and also it is clear that both the active and reactive power loss indices are reduced significantly with the optimal allocation of PVAs. This clearly represents that optimal placement of PVAs in the DS shows positive impact on the system. Also from Figs it is observed that the maximum PV size is attained at the commercial load model which is higher compared to residential and industrial load models. The reason for this is the commercial load profile exactly follows the PV module output power pattern. In case of commercial load model, the load demand is maximum during day times and the output power of PV module gives maximum output during these time periods only.
Figure 7. Hourly PV outputs considering various load models: (a) Residential (b) Commercial (c) Industrial

In case of residential and industrial load models, the load demand pattern does not exactly match the output power availability of PV modules. From Figure 9 it is observed that overall multi objective function (MOI=1p.u.) is reduced significantly with placement of PVAs. From different types of load models it can be observed that the impact of PVA placement for the commercial load model is better compared to residential and industrial models.

From Figure 7 it is clear that maximum output of each PVA unit is attained at hour 12, because the output power of PVAs mainly depends upon solar irradiance level. The optimal sizes of PVAs at optimal buses and MOI considering various load models at 12th hour is tabulated in Table 2. From Table 2, it is clear that the overall objective function is less and PV size obtained at optimal buses is higher with commercial load model. It is clear that, PV allocation in the DS significantly matches the commercial load model pattern compared to other load models.
Figure 9. Hourly expected MOI for 69 bus RDS with placement of PVAs considering different types of load models

Table 2. Optimal allocation of PVAs in the 69 bus RDS considering various load models

<table>
<thead>
<tr>
<th>Load models</th>
<th>Optimal bus</th>
<th>Optimal PV size (kW)</th>
<th>MOI in p.u</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>61 64 27</td>
<td>1359.33 194.19 388.38</td>
<td>0.3449</td>
</tr>
<tr>
<td>Commercial</td>
<td>61 64 27</td>
<td>1553.52 194.19 388.38</td>
<td>0.3433</td>
</tr>
<tr>
<td>Industrial</td>
<td>61 64 27</td>
<td>1165.19 194.19 388.38</td>
<td>0.5721</td>
</tr>
</tbody>
</table>

Moreover, active power loss during each hour of the day without and with placement of PVAs, considering different load models is illustrated in Figure 10. From Figure 10 it is clear that power loss reduces significantly from 7th to 19th hour because PVAs produce power at these time periods only. In view of different load models, the power loss at each hour of the day after placement of PVAs reduces significantly in case of the commercial load model. Figure 11 represents the voltage profile under various load models at peak period with and without placement of PVAs. It is observed that voltage profile is improved effectively at all buses after placement of PVAs.
Figure 10. (a), (b) and (c) Power loss without and with placement of PVAs considering Residential, Commercial and Industrial load models
The results of average power loss indices and AMOI with different load models are represented in Table 3. From Table 3, it is clear that the lesser value of active, reactive power indices and average multi objective index are attained with the commercial load model. This means that power loss reduction is more in case of the commercial load model compared to other load models. This clearly shows that choice of PV allocation in the distribution system is apt for the commercial load model.

Table 3. Average power loss indices for 69 bus test system with placement of PVAs in DS considering various load models

<table>
<thead>
<tr>
<th>Load models</th>
<th>APLI</th>
<th>AQLI</th>
<th>AMOI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>0.7169</td>
<td>0.7318</td>
<td>0.7229</td>
</tr>
<tr>
<td>Commercial</td>
<td>0.7031</td>
<td>0.7190</td>
<td>0.7094</td>
</tr>
<tr>
<td>Industrial</td>
<td>0.7378</td>
<td>0.7487</td>
<td>0.7422</td>
</tr>
</tbody>
</table>

Finally, the annual energy loss ($E_{A_{loss}}$) of the system with and without placement of PVAs under various load models is tabulated in Table 4. The annual energy loss is calculated as sum of power losses during 24 hours and multiplied by 365. The $E_{A_{loss}}$ after placement of PVAs is reduced effectively. Highest loss reduction is noticed with the commercial load model and lowest with industrial load model. This clearly states that, PV generation is exactly matches with commercial load model compared to residential and industrial load models.

Table 4. Annual Energy loss without and with placement of PVAs considering various load models

<table>
<thead>
<tr>
<th>Load models</th>
<th>$E_{A_{loss}}$ (MWh)</th>
<th>$E_{A_{loss}}$ PVA (MWh)</th>
<th>% Red $E_{A_{loss}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>2.039</td>
<td>1.351</td>
<td>33.74</td>
</tr>
<tr>
<td>Commercial</td>
<td>2.006</td>
<td>0.976</td>
<td>51.34</td>
</tr>
<tr>
<td>Industrial</td>
<td>1.899</td>
<td>1.486</td>
<td>21.74</td>
</tr>
</tbody>
</table>

7. CONCLUSION

This paper presented an effective methodology for optimal allocation of RDGs (Solar) in the RDS with an aim of minimizing active, reactive power losses and enhancing the voltage profile of the system. Different load models are considered for the analysis with probabilistic generation and load demand variation. The variable nature of solar irradiance is effectively modeled by Beta PDF. The best locations for placement of PVAs are identified by LSF technique and sizes corresponding to these locations are determined using FA technique. From the results, it is observed that the maximum PV size is obtained at commercial load model compared to industrial and residential load models. Also the active, reactive power loss index and overall multi objective function of the commercial load model is less compared to other load models. The annual energy loss of the system after placement of PVAs is reduced more significantly with the
commercial load model. Because, commercial load profile exactly matches the PV module generation pattern. So finally it can be concluded that PV generation pattern varies depending on the solar irradiance but it appropriately suits the commercial load model which makes it more viable for commercial installations.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES


