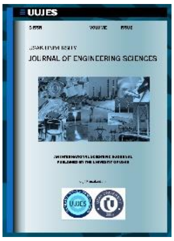

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Research article

A TUTORIAL ON BANG-BANG ALGORITHM FOR ATTITUDE CONTROL SYSTEM

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Abstract

The physical environment of a space vehicle shows that bang-bang method for attitude control system of a space vehicle is a viable and natural choice. This tutorial describes the algorithm and the specific aspects of its implementation. It derives the equations and the switching surface used by the method. The immediate benefits of this design is short response time which results in larger fuel margin, less fuel consumption and less weight.

Keywords: *Bang-bang; relay design; on-off control; attitude control design.*

1. Introduction

Typically, a space vehicle (SV) uses warm gas generators for achieving attitude control. These generators spew off gas through two pairs of opposing thrusters to control the pitch and the yaw channels respectively. Once activated these generators deliver gas to the thrusters continually through pulse width modulated valves until exhausted.

Attitude control systems (ACS) may apply a classical feedback control to pulse width modulate these valves. Computer simulations show that the design may meets the control requirements, albeit the spontaneous net thrust is chaotic. This behavior causes sluggish response, higher gas consumption than needed and limiting the operating dynamic pressure to a level below what could be achieved. As such, an alternative is needed to avert the chaotic thrust so we can fully utilize the available energy by delivering it to the right thruster. The alternative is naturally offered by means of bang-bang control.

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Bang-bang algorithm is a powerful control design and finds wide and farfetched applications as for example in references [1-3]. A vivid example is its use in attitude control systems. This control algorithm has been introduced under other names as relay, linear switching, on-off and contactor control [4-8]. Using it in our SV application is highly intuitive. Here a SV in the outer space needs to be reoriented along a desired direction. To do that a thruster is fired to generate a moment to slew the vehicle towards that direction. Problem, at least theoretically, is that once the thruster is fired, no matter for how long, the vehicle will continue to turn in the same direction indefinitely. Thus a second moment must be generated to counter the first one and halt the vehicle turn, hopefully along the desired vector. This shows that a pair of opposing moments, or a doublet, is needed to accomplish the maneuver. One might ask again, how strong this doublet should be and for how long it should last. Although this is the core of the mathematical derivation in this note, one might intuitively feel that each of the opposing moments would be enacted for one-half of the entire duration. The first moment will accelerate the vehicle until it is midway towards the final destination and the second moment will decelerate the vehicle until it reaches a steady state at the desired destination.

In the following, we formally present the problem and provide the mathematical derivation towards a complete control algorithm and confirm it with the simulation results.

2. Problem formulation and mathematical derivation

Herein, we consider a SV in 2-dimensional plane. To control its pitch attitude, in the absence of atmospheric pressure, the SV is equipped with two collinear thrusters that are placed in the perpendicular plane at its base. The plane is l feet from the body center of mass. The thrusters are capable of generating thrust forces F or $-F$ respectively (see Fig. 1).

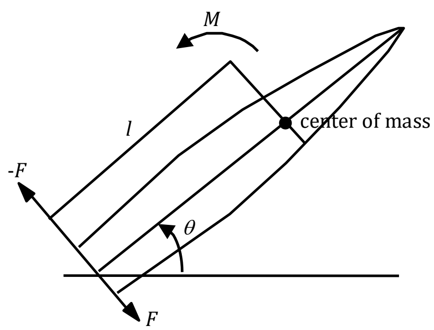

Fig. 1 Body dynamics.

Thus, the magnitude of the moment M generated by the thrust force in either direction is:

$$M = Fl \tag{1}$$

Suppose pitch attitude initial conditions are:

$$\theta(0) = \theta_0, \qquad \dot{\theta}(0) = \dot{\theta}_0 \tag{2}$$

It is desired to generate a doublet, as in Fig. 2, that restores the SV to steady state conditions,

$$\theta(t) = 0, \quad \dot{\theta}(t) = 0, \quad t \geq T \tag{3}$$

where T is the doublet duration.

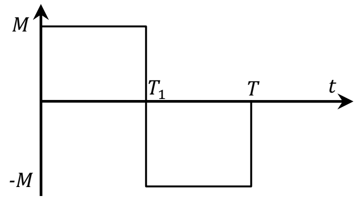

Fig. 2 Moment doublet.

Now, suppose that the durations of the opposing moments are T_1 and T_2 seconds respectively, then the Newton moment equation gives:

$$\begin{aligned} I\ddot{\theta} &= M, \qquad & 0 < t \le T_1 \\ I\ddot{\theta} &= -M, \qquad & T_1 < t < T_1 + T_2 \end{aligned} \tag{4}$$

where I is second moment of inertia about the pitch axis. Integrating Eq. (4) twice with respect to time and using the initial conditions in Eq. (2) gives:

$$\begin{aligned} \dot{\theta}(t) &= Ht + \dot{\theta}_0, \qquad & 0 \le t \le T_1 \\ \theta(t) &= \frac{H}{2}t^2 + \dot{\theta}_0 t + \theta_0, \quad & 0 \le t \le T_1 \end{aligned} \tag{5}$$

where

$$H = \frac{M}{I} \tag{6}$$

Thus, at $t = T_1$, Eq. (5) becomes:

$$\begin{aligned} \dot{\theta}_1 &= \dot{\theta}(T_1) = HT_1 + \dot{\theta}_0 \\ \theta_1 &= \theta(T_1) = \frac{H}{2}T_1^2 + \dot{\theta}_0 T_1 + \theta_0 \end{aligned} \tag{7}$$

The variables θ_1 and $\dot{\theta}_1$ become the initial conditions for the second part of Eq. (4). Carrying out the integration steps for the second part of Eq. (4) and using the above initial conditions gives:

$$\begin{aligned} \dot{\theta}(t') &= -Ht' + \dot{\theta}_1, \qquad & 0 \le t' \le T_2 \\ \theta(t') &= -\frac{H}{2}t'^2 + \dot{\theta}_1 t' + \theta_1, \quad & 0 \le t' \le T_2 \end{aligned} \tag{8}$$

where t' is the time referenced to the end of the first moment, *i.e.*

$$t' = t - T_1 \tag{9}$$

At the end of the second moment, $t' = T_2$, Eq. (8), using Eq. (7), become as follow:

$$\begin{aligned} \dot{\theta}_2 &= \dot{\theta}(T_2) = -HT_2 + HT_1 + \dot{\theta}_0 \\ \theta_2 &= \theta(T_2) = -\frac{H}{2}T_2^2 + \left(HT_1 + \dot{\theta}_0\right)T_2 + \frac{H}{2}T_1^2 + \dot{\theta}_0 T_1 + \theta_0 \end{aligned} \tag{10}$$

By collecting terms, these equations simplify to:

$$\begin{aligned} \dot{\theta}_2 &= H\left(T_1 - T_2\right) + \dot{\theta}_0 \\ \theta_2 &= \frac{H}{2}\left(T_1^2 - T_2^2\right) + HT_1T_2 + \dot{\theta}_0\left(T_1 + T_2\right) + \theta_0 \end{aligned} \tag{11}$$

With no loss of generality, we will require the steady state of the system be:

$$\dot{\theta}_2 = 0, \quad \theta_2 = 0 \tag{12}$$

Eqs. (11) and (12), result in:

$$T_2 = T_1 + \frac{\dot{\theta}_0}{H} \tag{13}$$

Substituting Eq. (13) into Eq. (11), and a little algebra, results in,

$$T_1^2 + \frac{2\dot{\theta}_0}{H}T_1 + \frac{\dot{\theta}_0^2}{2H^2} + \frac{\theta_0}{H} = 0 \tag{14}$$

and consequently, T_1 is found as follows:

$$T_1 = -\frac{\dot{\theta}_0}{H} + \sqrt{\frac{1}{2}\left(\frac{\dot{\theta}_0}{H}\right)^2 - \frac{\theta_0}{H}} \tag{15}$$

Substituting Eq. (15) into Eq. (13) gives following expression for T_2.

$$T_2 = \sqrt{\frac{1}{2}\left(\frac{\dot{\theta}_0}{H}\right)^2 - \left(\frac{\theta_0}{H}\right)} \tag{16}$$

Negative sign of the square root in Eq.(15) is ruled out as it would result in negative T_2. Interestingly for the case below,

$$\dot{\theta}_0 = 0, \;\; \theta_0 < 0 \quad \Rightarrow \quad T_1 = T_2 = \sqrt{-\frac{\theta_0}{H}} \tag{17}$$

two equal and opposite moments are applied during equal periods. At the end of the first period, $\theta = -\theta_0/2$, a negative moment is applied to decelerate the body and bring it to steady state at $\theta = \dot{\theta} = 0$. From Eqs. (5) and (8), the time histories for θ and $\dot{\theta}$ are shown in Fig. 3. We discuss the case of $\theta_0 > 0$ soon.

In the above derivation, we assumed that a positive moment (positive thrust) is applied first. However, nothing tells us which one should start first. Firstly, if we argue that for positive θ apply a negative moment and vice versa, then we need to consider the initial condition at which $\dot{\theta}_0$ is not zero. Secondly and most importantly we need to monitor the thrust profile: what if the SV do not follow our mathematical model due to perturbations or nonlinearities?

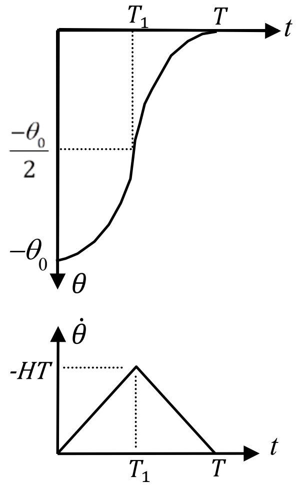

Fig. 3 Time histories for θ and $\dot{\theta}$.

Well, let's investigate the conditions under which time T_1, is a positive number. Elementary algebra shows that Eq. (14) has two positive solutions under following conditions:

$$\frac{\dot{\theta}_0}{H} < 0, \qquad \frac{1}{2}\left(\frac{\dot{\theta}_0}{H}\right)^2 + \frac{\theta_0}{H} > 0, \qquad \frac{1}{2}\left(\frac{\dot{\theta}_0}{H}\right)^2 - \frac{\theta_0}{H} > 0 \tag{18}$$

The above expression can be written in short form as below:

$$\frac{\dot{\theta}_0}{H} < 0, \qquad \frac{1}{2}\left(\frac{\dot{\theta}_0}{H}\right)^2 > \left|\frac{\theta_0}{H}\right| \tag{19}$$

From Eq. (14), another possible condition for a positive T_1 is given as:

$$\frac{1}{2}\left(\frac{\dot{\theta}_0}{H}\right)^2 < -\frac{\theta_0}{H} \tag{20}$$

The regions of the above two equations can be depicted via the phase portrait in Fig. 4. The shape comprises two back-to-back parabolas given by:

$$\frac{1}{2}\left(\frac{\dot{\theta}_0}{H}\right)^2 \pm \frac{\theta_0}{H} = 0 \tag{21}$$

The phase portrait in Fig. 4 is divided into four regions, each has specific signs for T_1 as follows:

Region *I*: T_1 has two positive solutions.
Region *II*: T_1 has one positive and one negative solution.
Region *III*: T_1 has two negative solutions.
Region *IV*: T_1 has two complex conjugate solutions.

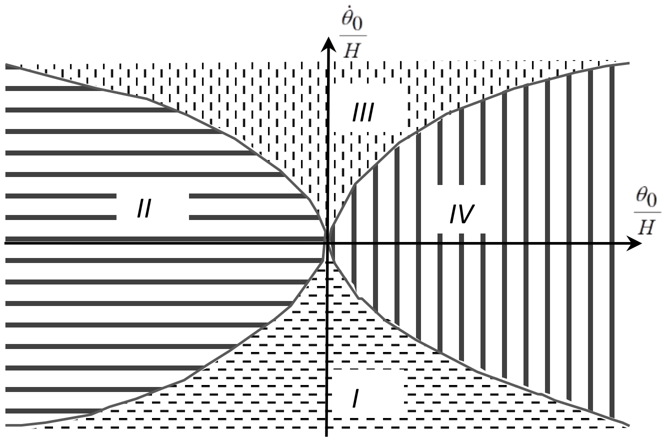

Fig. 4 Phase portrait.

Eq. (19) implies that the initial condition resides outside the parabolas in region I; while Eq. (20) implies that the initial condition resides inside the left parabola in region II. Thus, if the initial condition resides in the region $I \oplus II$, then T_1 will be real and positive. It is in this combined region that the attitude control system will be required to enact a positive thrust (i.e. positive moment). Intuitively, one can guess that in the remainder region of the phase portrait, the ACS must enact a negative thrust (*i.e.* negative moment). Indeed this is the case. If Eqs. (5)-(16) were re-derived with the negative thrust enacted first, one could have arrived that in region $III \oplus IV$, T_1 will be real and positive. Actually, we do not need to do the lengthy derivation of all these equations. All is needed is replacing H with $-H$ in these equations to arrive to the conclusion. Thus for the negative thrust, the activation regions are as follows:

Region I: T_1 has two complex conjugate solutions.
Region II: T_1 has two negative solutions.
Region III: T_1 has two positive solutions.
Region IV: T_1 has one positive and one negative solution.

From the above, one may realize that non-positive or complex solutions are an indication that we cannot restore the attitude of the SV, if we enacted the wrong thruster first in these regions. Likewise, we can be faced with the case of two positive solutions of T_1, in Region I. Eq. (14) tells that these two solutions are as follows:

$$T_1 = \frac{\dot{\theta}_0}{H} \pm \sqrt{\frac{1}{2}\left(\frac{\dot{\theta}_0}{H}\right)^2 + \frac{\theta_0}{H}} \tag{22}$$

When the root is positive the solution of T_2, given by Eq. (16), is positive, but when the root is negative T_2 is negative. Thus one solution (with positive root) is only viable and therefore a point in the combined region $III \oplus IV$ has only one viable solution for T_1 and T_2. Despite the apparent complexity of the above analysis, the thruster sign law is so simple:

$$disc = \frac{1}{2}\left(\frac{\dot{\theta}_0}{H}\right)^2 - \frac{|\theta_0|}{H} \tag{23}$$

if $(disc > 0)$ $\Rightarrow$ SV is outside the parabola borders

$sgnThr = sgn(-\dot{\theta}_0)$ $\quad i.e$ in regions I or III
$else$ $\quad \Rightarrow$ SV is inside the parabola borders
$sgnThr = sgn(-\theta_0)$ $\quad i.e$ in regions II or IV

where "$sgnThr$" denote the thrust sign. From Eqs. (15), (16) and (22) pulse durations are given by:

$$T_2 = \sqrt{\frac{1}{2}\left(\frac{\dot{\theta}_0}{H}\right)^2 - \frac{\theta_0}{H} sgnThr} \tag{24}$$

$$T_1 = T_2 - \frac{\dot{\theta}_0}{H} sgnThr \tag{25}$$

The real payoff, from a control system point of view, is that the ACS can monitor the attitude of the space vehicle continually. It can determine the region of the phase portrait it resides in, and then it can decide on which thruster it should fire first.

3. Computer simulations

A one-degree of freedom flight simulation is used to examine the dynamic performance of the new ACS. The base line parameter is: $H = 100$ deg/s^2.

When applying the control law we have two options: The first is to apply the thruster doublet during T_1 and T_2 in an open loop manner; the second is to apply the control law given by Eq. (23) by computing the parameter '$disc$' continually and based on its value we decide on the thruster sign. In ideal conditions both strategies gave the same results as depicted in Figures 5(a)-5(d) which show the trajectories of SV under four scenarios that originate from Regions I, II, III and IV respectively.

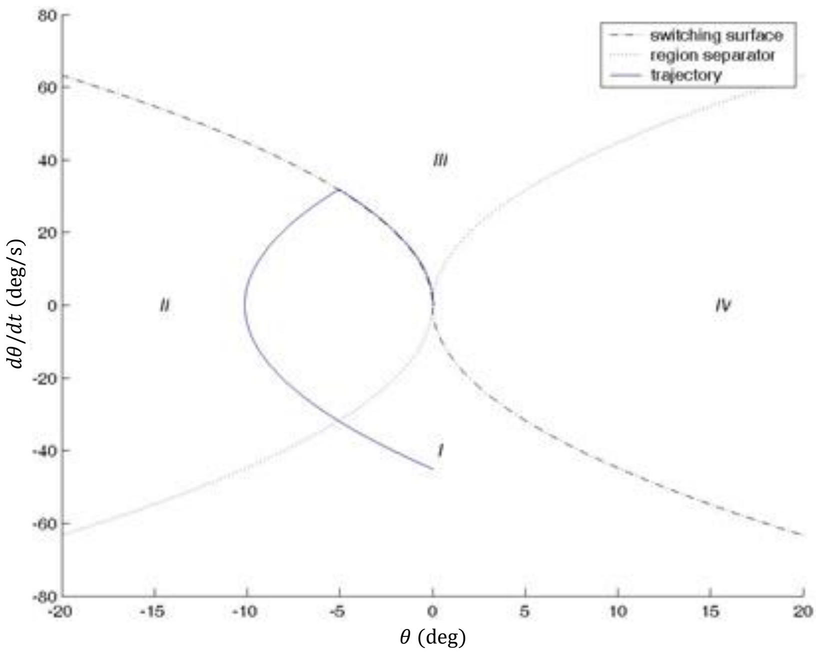

Fig. 5(a) Region I trajectory.

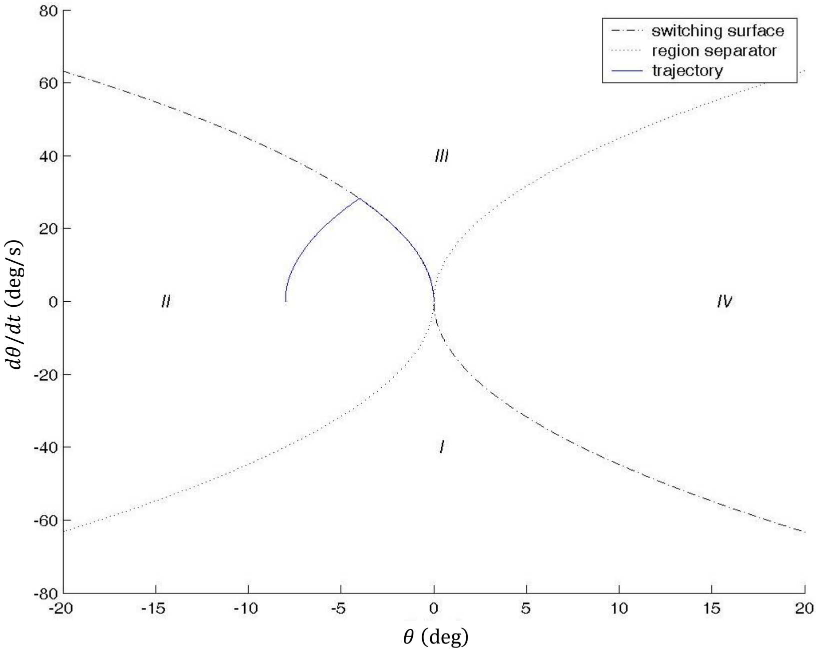

Fig. 5(b) Region *II* trajectory.

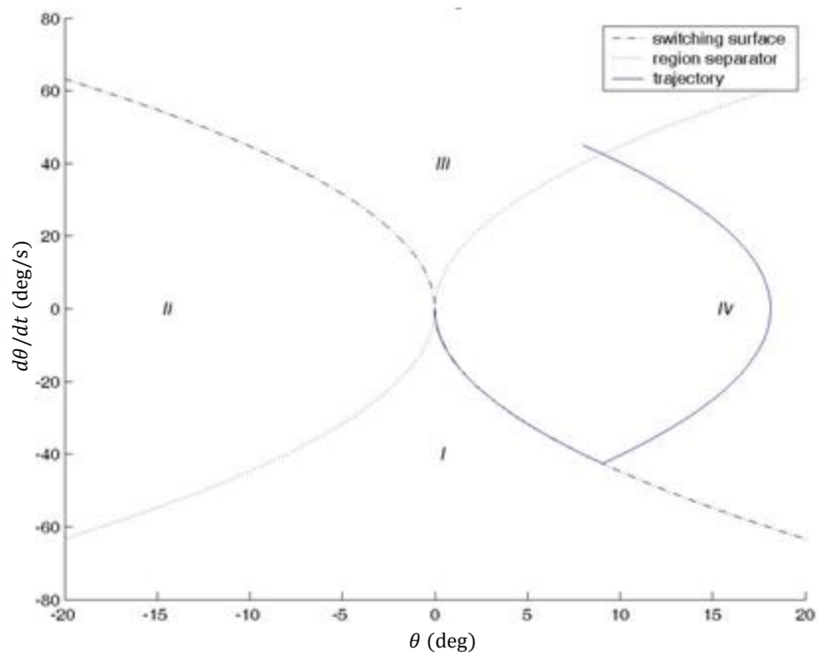

Fig. 5(c) Region *III* trajectory.

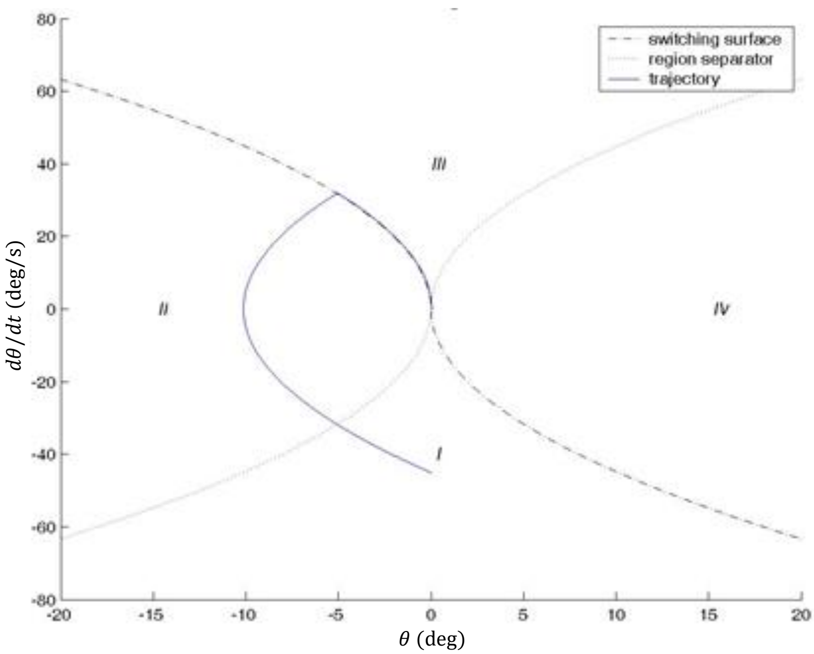

Fig. 5(d) Region *IV* trajectory.

Some practical issues remain to be discussed. In the above analysis we did not consider sensor inaccuracies, thruster model uncertainties or SV parameter errors. These error sources will influence the values of the sensor readings, the computation of the parameter H and consequently on '$disc$' Eq. (23) parameter validity. A way to obviate these problems is to monitor the region in which $|\theta - \theta_f| < \varepsilon_1$ and $|\dot{\theta} - \dot{\theta}_f| < \varepsilon_2$, where $(\theta, \dot{\theta})$ is the current state, $(\theta_f, \dot{\theta}_f)$ is the desired final state, and $\varepsilon_1, \varepsilon_2$ denote two small acceptable errors. The control law is then implemented so long as the state is outside the region. Once the state crosses the region, thrust forces are cutoff. This avoids needless thruster limit cycle.

4. Conclusions

Bang-bang is indeed a simple method for attitude control system of a SV and other similar systems. The immediate benefits of this design is short response time which results in larger fuel margin, less fuel consumption and less weight. The amount of computations needed for implementing the algorithm is minimal. Even though the algorithm presented herein is described for a 2-dimensional case, it can be easily extended to handle 3-dimensional systems. The algorithm can be modified to account for system modeling inaccuracies.

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