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Local T_2 Constant Filter Convergence Spaces

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Abstract. The aim of this paper is to characterize local Hausdorff constant filter convergence spaces and show that they are hereditary, productive and coproductive.

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1. Introduction

Sequences are sufficient to describe topological properties in metric spaces or, more generally, topological spaces having a countable base for the topology. However, filters are needed in more abstract spaces.The filters are the tool which allows us to reframe topological properties in terms of convergence. Filters can be seen as generalization of sequences.The concept of a filter was introduced by H. Cartan [18, 19] in 1937. In later times, N. Bourbaki [17] used filters in his works. Since topological spaces are inadequate for the investigation of certain interesting limit operations, the idea of using the concept of convergence itself as a primitive term arises naturally. In other words, notions such as convergence, continuity, and separation holds an important place in general topology. It is possible, however, to introduce them in a much more abstract way, based on axioms for convergence instead of neighborhood. Filters are of central importance in the field of general topology and in the theory of convergence spaces filters are the essential object of study. In 1948, Choquet [20] presented the concept of convergence of a filter is axiomatized. In 1954 [29], Kowalsky used filter description of convergence in his works. In 1959, Fischer [25] used category theory in his work, methods. In 1964, Kent [28] introduced Kent convergence spaces (there it is called convergence functions) by further weakening of the convergence axioms. In 1965, Cook and Fisher [21] proved that continuous convergence on the collection of continuous maps from one topological space to another is the coarsest admissible convergence structure and conc1uded that continuous convergence, in general, is not topological. In 1975, Robertson [35] introduced the categories ***GRILL*** (resp. ***FILTER***), the category of grill spaces (resp. the category of filter spaces) and showed that it is a subcategory of Kent convergence spaces [28]. In 1979, Schwarz [36] introduced the category ***ConFCO*** of constant filter convergence spaces and showed that ***ConFCO*** is isomorphic to ***GRILL*** and ***FILTER***. Schwarz [36] showed that ***ConFCO*** is the natural link between ***FILTER*** and ***FCO*** (the category of filter convergence spaces).

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Baran, in [2], introduced local separation properties in set-based topological categories and then, they are generalized to point free definitions by using the generic element method of topos theory [27] or [30]. One of the use of local separation properties is to define the notion of (strong) closedness [3] in set-based topological categories and it is shown, in [12, 13, 15], and [16] that they form appropriate closure operators in the sense of Dikranjan and Giuli [22] in the category convergence spaces, limit spaces [23, 33], and semi uniform convergence spaces [34]. One of the other uses of local T_0 and T_1 properties is to define local Hausdorff [8], local regular, completely regular, and local normal objects in [10, 11].

In this paper, we characterize local Hausdorff constant filter convergence spaces and show that they are hereditary, productive and coproductive.

2. Preliminaries

Let A be a set, $F(A)$ set of all filters on A and K be a function from A to $P(F(A))$. If K satisfies the following conditions, then (A, K) is called a constant filter convergence space.
(1) $[x] \in K$ for each $x \in A$, where $[x] = \{B \subset A : x \in B\}$.
(2) If $\alpha \subset \beta$ and $\alpha \in K$ implies $\beta \in K$ for any filter β on A.

A map $f : (A, K) \to (B, L)$ between constant filter convergence spaces is called continuous if and only if $\alpha \in K$ implies $f(\alpha) \in K$ (where $f(\alpha)$ denotes the filter generated by $\{f(D)|D \in K\}$ i.e., $f(\alpha) = \{U \subset X : D such that f(D) \subset U\}$. The category of constant filter convergence spaces and continuous maps is denoted by ***ConFCO*** [36].

A functor $U : \mathcal{E} \to \mathcal{B}$ is said to be topological or that $\mathcal{E}$ is a topological category over $\mathcal{B}$ if U is concrete (i.e., faithful and amnestic (i.e., if $U(f) = id$ and f is an isomorphism, then $f = id$)), has small (i.e., sets) fibers, and for which every U-source has an initial lift or, equivalently, for which each U-sink has a final lift, see [1, 26, 33]. A topological functor $U : \mathcal{E} \to \mathcal{B}$ is said to be normalized if constant objects, i.e., subterminals, have a unique structure. Note also that U has a left adjoint called the discrete functor D. Recall, in [1, 33] that an object $X \in \mathcal{E}$ is discrete iff every map $U(X) \to U(Y)$ lift to map $X \to Y$ for each object $Y \in \mathcal{E}$. Note that the category ***ConFCO*** is normalized topological category.

We denote by $\alpha \cup \beta$ the smallest filter (proper or not) containing both α and β for filters α and β, i.e., $\alpha \cup \beta = \{W \subset A : U \cap V \subset W$ for some $U \in \alpha$ and $V \in \beta\}$.
2.1. A source $\{f_i : (A, K) \to (A_i, K_i), i \in I\}$ in ***ConFCO*** is initial iff $\alpha \in K$ precisely when $f_i(\alpha) \in K_i$ [31, 32].
2.2. The discrete structure (A, K) on A in ***ConFCO*** is given by $K = \{P(A) = [\emptyset], [a] : a \in A\}$.
2.3. An epimorphism $f : (A, K) \to (B, L)$ in ***ConFCO*** is final iff $\alpha \in L$ implies there exists $\beta \in K$ such that $f(\beta) \subset \alpha$ [32, 36].
2.4. An epi sink $\{f_i : (A_i, K_i) \to (A, K), i \in I\}$ is final if and only if $\alpha \in L$ implies there exists $\beta_i \in K_i$ such that $f_i(\beta_i) \subset \alpha, i \in I$ [32, 36].

3. Local T_2 Constant Filter Convergence Spaces

In this section, we give the characterization of pre-Hausdorff objects in the category of constant filter convergence spaces at a point p.

Let B be set and $p \in B$. Let $B \vee_p B$ be the wedge at p [2], i.e., two disjoint copies of B identified at p, or in other words, the pushout of $p : 1 \to B$ along itself (where 1 is the terminal object in **Set**, the category of sets and functions). More precisely, if i_1 and $i_2 : B \to B \vee_p B$ denote the inclusion of B as the first and second factor, respectively, then $i_1 p = i_2 p$ is the pushout diagram [14]. A point x in $B \vee_p B$ will be denoted by $x_1(x_2)$ if x is in the first (resp. second) component of $B \vee_p B$. Note that $p_1 = p_2$.

The principal $p-$ axis map, $A_p : B \vee_p B \to B^2$ is defined by $A_p(x_1) = (x, p)$ and $A_p(x_2) = (p, x)$, the skewed $p-$ axis map, $S_p : B \vee_p B \to B^2$ is defined by $S_p(x_1) = (x, x)$ and $S_p(x_2) = (p, x)$, and the fold map at p, $\triangledown_p : B \vee_p B \to B$ is given by $\triangledown_p(x_i) = x$ for $i = 1, 2$ [2].

Definition 3.1. Let (X, τ) be a topological space and $p \in X$.
(1) For each point x distinct from p, if there exists a neighborhood of p missing x or there exists a neighborhood of x missing p, then (X, τ) called T_0 at p [9].
(2) For each point x distinct from p, if there exists a neighborhood of p missing x and there exists a neighborhood of x missing p, then (X, τ) called T_1 at p [9].

(3) (X, τ) is called pre-Hausdorff space ($PreT_2$) [9] for each point x distinct from p, the set $\{x, p\}$ is not indiscrete, then the points x and p have disjoint neighborhoods.
(4) For each point x distinct from p, if there exists a disjoint neighborhoods of p and x, then (X, τ) called T_2 at p [9].

Theorem 3.2. *Let (X, τ) be a topological space and $p \in X$.*
(1)*Then the followings are equivalent.*
(a) (X, τ) is T_0 at p
(b) The initial topology induced by $A_p : X \vee_p X \to (X^2, \tau_*)$ and $\nabla_p : X \vee_p X \to (X, P(X))\}$ is discrete, where τ_* is the product topology on X^2.
(c) The initial topology induced by $id : X \vee_p X \to (X \vee_p X, \tau^*)$ and $\nabla_p : X \vee_p X \to (X, P(X))$ is discrete, where τ^* is the final topology on $X \vee_p X$ induced by the canonical injections $\{i_1, i_2 : (X, \tau) \to X \bigvee_p X\}$ and $id : X \vee_p X \to (X \vee_p X$ is the idendity function.
(2) *(X, τ) is preT_2 at p if and only if the initial topology induced from $A_p : X \vee_p X \to (X^2, \tau_*)$ and $S_p : X \vee_p X \to (X^2, \tau_*)$ are the same.*
(3) *(X, τ) is T_2 at p if and only if (X, τ) is T_0 at p and preT_2 at p.*

Proof. The proof is given in [9]. □

Definition 3.3. Let $U : \mathcal{E} \to \boldsymbol{Set}$ be a topological functor, X an object in $\mathcal{E}$ with $U(X) = B$, and $p \in B$.
(1) If the initial lift of the U-source $\{A_p : B \bigvee_p B \to U(X^2) = B^2$ and $\nabla_p : B \bigvee_p B \to UD(B) = B\}$ is discrete, then X is called $\overline{T}_0$ at p [7].
(2) If the initial lift of the U-source $\{id : B \bigvee_p B \to U(B \bigvee_p B)' = B \bigvee_p B$ and $\nabla_p : B \bigvee_p B \to UD(B) = B\}$ is discrete, then X is called T_0' at p, where $(B \bigvee_p B)'$ is the final lift of the U-sink $\{i_1, i_2 : U(X) = B \to B \bigvee_p B\}$, i_1 and i_2 are the canonical injections [7].
(3) If the initial lift of the U-source $S_p : B \vee_p B \to U(X^2) = B^2$ and the initial lift of the U-source $A_p : B \vee_p B \to U(X^2) = B^2$ coincide, then X is said to be $PreT_2$ at p [14].
(4) If X is both $\overline{T}_0$ at p and $pre\overline{T}_2$ at p, then X is said to be $\overline{T}_2$ at p [2, 8].
(5) If X is both T_0' at p and $pre\overline{T}_2$ at p, then X is said to be KT_2 at p [8].

Theorem 3.4. *A constant filter convergence space (B, K) is $\overline{T}_2$ at p if and only if the following conditions are satisfied.*
(1) *$[x] \cap [p] \notin K$ for all $x \in X$ with $x \neq p$.*
(2) *$K_p = \{\alpha : \alpha \subset [p]\}$ is closed under finite intersection.*
(3) *For any $\alpha \in K_p$ and $\beta \in K$ if $\alpha \cup \beta$ is proper and $\beta \cap [p] \subset \alpha$, then $\beta \cap [p] \in K$.*

Proof. Suppose (B, K) is $\overline{T}_2$ at p, that is, by Definition 3.3, (B, K) is both $\overline{T}_0$ and $pre\overline{T}_2$ at p. If $[x] \cap [p] \in K$ for some $x \in X$ with $x \neq p$, then let $\sigma = [\{(x, p), (p, x)\}]$. We get

$$\pi_1 A_p \sigma = [x] \cap [p] = \pi_2 A_p \sigma \in K$$

and

$$\nabla_p \sigma = [x] \in K_d$$

where K_d is the discrete structure on B. This is a contradiction since (B, K) is $\overline{T}_2$ at p. Hence, we must have $[x] \cap [p] \notin K$ for all $x \in X$ with $x \neq p$, i.e condition (1) is hold. Since (B, K) is $pre\overline{T}_2$ at p, then by the proof of Theorem 3.6 in [24], the Parts (2) and (3) holds.

Conversely, suppose the conditions hold and σ is a filter on the wedge $B \vee_p B$ which satisfies

$$\pi_1 A_p \sigma \in K, \pi_2 A_p \sigma \in K$$

and

$$\nabla_p \sigma = [x], [\emptyset]$$

for some $x \in X$. If $\nabla_p\sigma = [\emptyset]$, then $\sigma = [\emptyset]$. If $\nabla_p\sigma = [x]$, then it follows easily that $\sigma = [(x, p)], [(p, x)]$ or $\sigma \supset [\{(x, p), (p, x)\}]$. We show that the last case can not occur. If $\sigma = [\{(x, p), (p, x)\}]$, then

$$\pi_1 A_p\sigma = [x] \cap [p] = \pi_2 A_p\sigma \in K$$

contradicting to $[x] \cap [p] \notin K$. If

$$\sigma \supset [\{(x, p), (p, x)\}]$$

and

$$[\emptyset] \neq \sigma \neq [\{(x, p), (p, x)\}]$$

then, there exists $U \in \sigma$ such that $U \neq \{(x, p), (p, x)\}$ and $U \neq \emptyset$. Since $\{(x, p), (p, x)\} \in \sigma$ and σ is a filter, it follows

$$U \cap \{(x, p), (p, x)\} = \{(x, p)\}, \{(p, x)\}$$

is in σ i.e $\sigma = [(x, p)]$ or $[(p, x)]$. Therefore $\sigma = [(x, p)], [(p, x)]$ or $[\emptyset]$ i.e (B, K) is $\overline{T}_0$ at p.

If the conditions (2) and (3) hold, then by Theorem 3.6 of [24], (B, K) is $pre\overline{T}_2$ (in [24] $pre\overline{T}_2$ is called $preT_2$ at p). Hence, (B, K) both is $\overline{T}_0$ and $pre\overline{T}_2$ at p and by Definition 3.3, (B, K) is $\overline{T}_2$. □

Theorem 3.5. *A constant filter convergence space (B, K) is KT_2 at p if and only if K_p is closed under finite intersection and for any $\alpha \in K_p$ and $\beta \in K$ if $\alpha \cup \beta$ is proper and $\beta \cap [p] \subset \alpha$, then $\beta \cap [p] \in K$.*

Proof. Suppose constant filter convergence space (B, K) is KT_2 at p, that is, by Definition 3.3, (B, K) is both $T_0^{'}$ and $pre\overline{T}_2$ at p. In particular, since (B, K) is $pre\overline{T}_2$ at p, then by the proof of Theorem 3.6 in [24], the conditions hold.
Conversely, suppose the conditions hold. By Definition 3.3, we need to show that (B, K) is both $T_0^{'}$ and $pre\overline{T}_2$ at p. Let σ be any filter on the wedge $B \vee_p B$ with $\sigma \supset i_k\alpha$ for some $\alpha \in K$, $k = 1, 2$ and $\nabla_p\sigma = [x]$ or $[\emptyset]$ for some $x \in B$.
If $\nabla_p\sigma = [\emptyset]$, then $\sigma = [\emptyset]$.
If $\nabla_p\sigma = [x]$ for some $x \in X$, then

$$\sigma = [(x, p)], [(p, x)], [(x, p), (p, x)]$$

or

$$\sigma \supset [[(x, p), (p, x)]]$$

If $\sigma \supset [\{(x, p), (p, x)\}]$, then σ lies both component of the wedge which is impossible since $\sigma \supset i_k\alpha$ $k = 1, 2$. Therefore,

$$\sigma = [(x, p)], [(p, x)]$$

and as a result, if σ is a filter on the wedge $B \vee_p B$ with

$$\sigma \supset i_k\alpha$$

and

$$\nabla_p\sigma = [\emptyset], [x]$$

then

$$\sigma = [\emptyset], [(x, p)], [(p, x)]$$

Hence, (B, K) is $T_0^{'}$ at p.
If the conditions hold, then by Theorem 3.6 of [24], (B, K) is $pre\overline{T}_2$ (in [24] $pre\overline{T}_2$ is called $preT_2$ at p). Hence, (B, K) is both $T_0^{'}$ and $pre\overline{T}_2$ at p.

□

Theorem 3.6. .
(1) *If a constant filter convergence space (B, K) is $\overline{T}_2$ at p and $M \subset B$ with $p \in M$, then M is $\overline{T}_2$ at p.*
(2) *For all $i \in I$ and $p_i \in B_i$, (B_i, K_i) $\overline{T}_2$ at p_i if and only if $(B = \prod_{i\in I} B_i, K)$ is $\overline{T}_2$ at p, where K is the product structure on B and $p = (p_1, p_2, ...)$.*
(3) *If (B_i, K_i) T_1 at p_i for all $i \in I$ and $p_i \in B_i$, then $(B = \coprod_{i\in I} B_i, K)$ is $preT_2$ at (i, p), where K is the coproduct structure on B and $(i, p) \in B$.*

Proof. **(1)** Let K_M be the initial structure on M induced by the inclusion map $i : M \subset B$ and $[x] \cap [p] \in K_M$ for $x \in M$ with $x \neq p$. By 2.1,

$$i([x] \cap [p]) = i([x]) \cap i([p]) = [x] \cap [p] \in K$$

for $x \in X$ with $x \neq p$, a contradiction since (B, K) is $\overline{T}_2$ at p. Hence, $[x] \cap [p] \notin K_M$ for all $x \in M$ with $x \neq p$.
Suppose $\alpha, \beta \in (K_M)_p$. By 2.1, $i(\alpha), i(\beta) \in K$ and by Theorem 3.4, $i(\alpha \cap \beta) \in K$ and by 2.1, $\alpha \cap \beta \in K_M$.
Suppose $\alpha \in (K_M)_p$ and $\beta \in K_M$ for which $\alpha \cup \beta$ is proper and $\beta \cap [p] \subset \alpha$. It follows from 2.1 that

$$i(\alpha), i(\beta) \in K_p, i(\alpha) \cup i(\beta) = i(\alpha \cup \beta)$$

is proper and

$$i(\beta) \cap i([p]) = i(\beta \cap [p]) \subset i(\alpha)$$

By Theorem 3.4, $i(\beta \cap [p]) \in K$ and by 2.1, $\beta \cap [p] \in K_M$. Hence, (M, K_M) is $\overline{T}_2$ at p.

(2) Suppose that $(B = \prod_{i \in I} B_i, K)$ is $\overline{T}_2$ at p. Since each (B_i, K_i) is isomorphic to a subspace of (B, K), it follows from part (1) that (B_i, K_i) is $\overline{T}_2$ at p_i for all $i \in I$ and $p_i \in B_i$.
Suppose that (B_i, K_i) is $\overline{T}_2$ at p_i for all $i \in I$, $p_i \in B_i$ and $[x] \cap [p] \in K$ for all $x = (x_1, x_2, ...) \in X$ with $x \neq p$. It follows that there exists $j \in J$ such that $x_j \neq p_j$. Since $[x] \cap [p] \in K$, by 2.1,

$$\pi_j([x] \cap [p]) = \pi_j([x]) \cap \pi_j([p]) = [x_j] \cap [p_j] \in K_j$$

for $x_j \neq p_j$ which contradicts to (B_j, K_j) being $\overline{T}_2$ at p_j. Hence, $[x] \cap [p] \notin K$ for $x \in X$ with $x \neq p$.
Suppose $\alpha, \beta \in K_p$, where $p = (p_1, p_2, ...)$. By 2.1, $\pi_i(\alpha), \pi_i(\beta) \in ((K_i)_{p_i}$ for all $i \in I$. Since (B_i, K_i) is $preT_2$ at p_i for all $i \in I$, by Theorem 3.4,

$$\pi_i(\alpha) \cap \pi_i(\beta) = \pi_i(\alpha \cap \beta) \in (K_i)_{p_i}$$

and by 2.1,

$$\alpha \cap \beta \in K_p$$

Suppose that $\alpha \in K_p$ and $\beta \in K$ with $\alpha \cup \beta$ is proper and

$$\beta \cap [p] \subset \alpha \pi_i(\alpha) \in (K_i)_{p_i}, \pi_i(\beta) \in K_i$$

for all $i \in I$, $\pi_i(\alpha \cup \beta)$ is proper and

$$\pi_i(\beta \cap [p]) \subset \pi_i(\alpha)$$

By Theorem 3.4,

$$\pi_i(\beta \cap [p]) = \pi_i(\beta) \cap \pi_i[p] = \pi_i(\beta) \cap [p_i] \in K_i$$

since (B_i, K_i) is $\overline{T}_2$ at p_i for all $i \in I$ and by 2.1, $\beta \cap [p] \in K$. Hence, by Theorem 3.4, (B, K) is $\overline{T}_2$ at p.

(3) Suppose that (B_i, K_i) is $\overline{T}_2$ at p_i for all $i \in I$, $p_i \in B_i$, $(B = \coprod_{i \in I} B_i, K)$, where K is the coproduct structure on B and $(i, p) \in B$.
Suppose that (B_i, K_i) is $\overline{T}_2$ at p_i for all $i \in I$, $p_i \in B_i$ and $[(i, x)] \cap [(i, p)] \in K$ for $(i, x) \in X$ with $(i, x) \neq (i, p)$. Since $[(i, x)] \cap [(i, p)] \in K$, by 2.4, there exists $\beta_i \in K_i$ such that $i(\beta_i) \subset [(i, x)] \cap [(i, p)]$. It follows that $\beta_i = [x_i] \cap [p_i] \in K_i$ contradicting to (B_i, K_i) is $\overline{T}_2$ at p_i. Hence, $[(i, x)] \cap [(i, p)] \notin K$ for all $(i, x) \in X$ with $(i, x) \neq (i, p)$.
If $\alpha, \beta \in K_{(i,p)}$, then by 2.4, there exist $\delta, \gamma \in (K_i)_{p_i}$ such that $i(\delta) \subset \alpha$ and $i(\gamma) \subset \beta$. Note that

$$i(\delta \cap \gamma) = i(\delta) \cap i(\gamma) \subset \alpha \cap \beta$$

Since (B_i, K_i) is $\overline{T}_2$ at p_i, by Theorem 3.4, $\delta \cap \gamma \in (K_i)_{p_i}$ and by 2.4, $\alpha \cap \beta \in K_{(i,p)}$.
Suppose that $\alpha \in K_{(i,p)}$ and $\beta \in K$ with $\alpha \cup \beta$ is proper and $\beta \cap [(i, p)] \subset \alpha$. Then there exist $\delta \in (K_i)_{p_i}$ and $\gamma \in K_i$ such that $i(\delta) \subset \alpha$ and $i(\gamma) \subset \beta$. Note that

$$i(\delta) \cup i(\gamma) = i(\delta \cup \gamma)$$

is proper and

$$i(\gamma \cap [p_i]) = i(\gamma) \cap [(i, p)] \subset \alpha$$

implies $\delta \cup \gamma$ is proper and $\gamma \cap [p_i] \subset \delta$. Since (B_i, K_i) is $\overline{T}_2$ at p_i for all $i \in I$, by Theorem 3.4, $\gamma \cap [p_i] \in (K_i)_{p_i}$ and by 2.4, $\beta \cap [(i, p)] \in K_{(i,p)}$. Hence, by Theorem 3.4, (B, K) is $\overline{T}_2$ at p.

□

Let $\boldsymbol{pre\overline{T_2}(\mathcal{E})}$ be the subcategory of a topological category $\mathcal{E}$ whose objects are $pre\overline{T}_2$ objects. By Theorem 3.4 of [14] $\boldsymbol{pre\overline{T_2}(\mathcal{E})}$ is a topological category . Let $\boldsymbol{KT_2ConFCO}$ be the subcategory of $\boldsymbol{ConFCO}$ whose objects are local KT_2 constant filter convergence spaces. By Theorem 3.6 of [24] and by Theorem 3.5, we have the following result:

Theorem 3.7. *$\boldsymbol{KT_2ConFCO}$ and $\boldsymbol{pre\overline{T_2}(ConFCO)}$ are isomorphic categories.*

Theorem 3.8. *The category $\boldsymbol{KT_2ConFCO}$ has all kinds of limits and colimits.*

Proof. Since, by Theorem 3.7, the category $\boldsymbol{KT_2ConFCO}$ is isomorphic to $\boldsymbol{pre\overline{T_2}(ConFCO)}$ and $\boldsymbol{pre\overline{T_2}(ConFCO)}$ is a topological category by Theorem 3.4 of [14], it follows that the category $\boldsymbol{KT_2ConFCO}$ has all kinds of limits and colimits. In particular, $\boldsymbol{KT_2ConFCO}$ has all subspaces, products, coproducts, and quotients. □

Remark 3.9. .

(1) Note that, by Theorem 3.2, for the category $\boldsymbol{Top}$ of topological spaces, $\overline{T_2}$ at p and KT_2 at p are equivalent and reduce to the usual T_2 at p.

(2) Let $U : \mathcal{E} \to \boldsymbol{Set}$ be a topological functor, X an object in $\mathcal{E}$ and $p \in U(X)$ be a retract of X, i.e., the initial lift $h : \bar{1} \to X$ of the U-source $p : 1 \to U(X)$ is a retract, where 1 is the terminal object in $\boldsymbol{Set}$. Then if X is $\overline{T_0}$ at p, then X is T_0' at p [7] and consequently by Definition 3.3, $\overline{T_2}$ at p implies KT_2 [10, 11] but, by Theorems 3.4 and 3.5, the reverse implication is not true. For example let $\boldsymbol{R}$ be the set of reel numbers and $K = F(\boldsymbol{R})$, the indiscrete structure on $\boldsymbol{R}$. Note that by Theorem 3.5, $(\boldsymbol{R}, K)$ is KT_2 at p but, by Theorem 3.4, it is not $\overline{T_2}$ at p since a condition (1) of Theorem 3.4 can not be satified.

(3) If $U : \mathcal{E} \to \boldsymbol{Set}$ is a normalized topological functor, then $\overline{T_2}$ at p implies KT_2 at p [5].

(4) In a topological category over the category $\boldsymbol{Set}$ of sets, by Theorem 3.2, $\overline{T_2}$ at p and KT_2 at p may be equivalent objects. All $\overline{T_0}$ at p, T_0' at p, $pre\overline{T}_2$ at p, $\overline{T_2}$ at p and KT_2 at p objects may be equivalent [6]. $\overline{T_2}$ at p and KT_2 at p may be a point or the empty set [4] or they could be all objects [6].

(5) One of the use of $\overline{T_0}$ at p and T_0' at p is to define the notion of local T_2 objects [2] and the notion of closedness in set-based topological categories [3].

(6) One of the use of $pre\overline{T}_2$ at p, $\overline{T_2}$ at p, and KT_2 at p is to define the notion of local T_3 and local T_4 objects [10], local completely regular objects [11] in set-based topological categories [3].

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