Proceedings of International Conference on Mathematics
and Mathematics Education (ICMME 2018)
Turk. J. Math. Comput. Sci.
10(2018) 153–159

http://dergipark.gov.tr/tjmcs

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On Contact CR-Submanifolds of a Kenmotsu Manifold

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Received: 15-08-2018 • Accepted: 10-12-2018

Abstract. The object of the present paper is to study the differential geometry of contact CR-submanifolds of a Kenmotsu manifold. Necessary and sufficient conditions are given for a submanifold to be a contact CR-submanifold in Kenmotsu manifolds. Finally, the induced structures on submanifolds are investigated, these structures are categorized and we discuss these results.

2010 AMS Classification: 53C15; 53C44, 53D10.

Keywords: Kenmotsu manifold, contact cr-submanifold.

1. Introduction

Kenmotsu [13] introduced a class of almost contact Riemannian manifolds known as Kenmotsu manifolds. The study of the differential geometry of contact CR-submanifolds, as a generalization of invariant(holomorphic) and anti-invariant(totally real) submanifolds of an almost contact metric manifold was initiated by A. Bejancu [6, 7] and was followed by several researchers. Some authors [1, 7, 8, 12, 14] studied contact CR-submanifolds of different classes of almost contact metric manifolds given in the references of this paper. Recently, in different studies M. Atçeken et al. [2–5] and S. Uddin et al. [15, 16] studied contact CR-submanifold and warped product CR-submanifolds in various type manifolds. The contact CR-submanifolds are rich and interesting subject. Therefore, it was continued to work in this subject matter. This study the present paper is organized as follows.

In this paper, contact CR-submanifolds of a Kenmotsu manifold were studied. In Section 2, basic formulas and definitions for a Kenmotsu manifold and their submanifolds were reviewed. In Section 3, the definition and some basic results of a contact CR-submanifold of a Kenmotsu manifold was recalled. Finally, some new results for contact CR-submanifolds in a Kenmotsu manifold was given.

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2. Preliminaries

In this section, we give some terminology and notations used throughout this paper. We recall some necessary fact and formulas from the theory of Kenmotsu manifolds and their submanifolds.

Let $\widetilde{M}$ be a $(2n+1)$-dimensional almost contact metric manifold with structure (ϕ, ξ, η, g), where ϕ is a $(1,1)-$type tensor field , ξ a vector field, η is a 1-form and g is the Riemann metric on $\widetilde{M}$,such that

$$\phi^2 X = -X + \eta(X)\xi, \;\; \phi\xi = 0, \; \eta(\phi X) = 0, \; \eta(\xi) = 1, \eta(X) = g(X, \xi) \tag{2.1}$$

and

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \;\; g(\phi X, Y) = -g(X, \phi Y) \tag{2.2}$$

for any $X, Y \in \Gamma(\widetilde{M})$, where $\Gamma(\widetilde{M})$ denotes the set differentiable vector fields on $\widetilde{M}$. If in addition to above relations

$$(\widetilde{\nabla}_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi Y \quad \textit{and} \quad \widetilde{\nabla}_X \xi = X - \eta(X)\xi \tag{2.3}$$

for any $X, Y \in \Gamma(\widetilde{M})$, then, $\widetilde{M}$ is called a Kenmotsu manifold, where $\widetilde{\nabla}$ is the Levi-Civita connection of g. Now, let M be an isometrically immersed submanifold in a Kenmotsu manifold $\widetilde{M}$. Then the formulas Gauss and Weingarten for M in $\widetilde{M}$ given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y) \tag{2.4}$$

and

$$\widetilde{\nabla}_X V = -A_V X + \nabla^{\perp}_X V, \tag{2.5}$$

for any vector fields X, Y tangent to M and V normal to M, where, ∇ denotes the induced Levi-Civita connection on M, $\nabla^{\perp}$ is the normal connection , A_V is the shape operator of M with respect to V and σ is second fundamental form of M in $\widetilde{M}$. The second fundamental form σ and shape operator A_V are related by

$$g(A_V X, Y) = g(\sigma(X, Y), V) \tag{2.6}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$.
The mean curvature vector H of M is given by $H = \frac{1}{m}\sum_{i=1}^{m} \sigma(e_i, e_i)$, where m is the dimension of M and $\{e_1, e_2, ..., e_m\}$ is a local orthonormal frame of M. A submanifold M of an contact metric manifold $\widetilde{M}$ is said to be totally umbilical if

$$\sigma(X, Y) = g(X, Y)H, \tag{2.7}$$

for any $X, Y \in \Gamma(TM)$. A submanifold M is said to be totally geodesic if $\sigma = 0$ and M is said to be minimal if $H = 0$.
Now, let M be a submanifold of an almost contact metric manifold $\widetilde{M}$. Then for any $X \in \Gamma(TM)$, we can write

$$\phi X = TX + NX, \tag{2.8}$$

where TX is the tangential component and NX is the normal component of ϕX. Similarly for $V \in \Gamma(T^{\perp}M)$, we can write

$$\phi V = tV + nV, \tag{2.9}$$

where tV is the tangential component and nV is also the normal component of ϕV.
Furthermore, for any $X, Y \in \Gamma(TM)$, we have $g(TX, Y) = -g(X, TY)$ and $V, U \in \Gamma(T^{\perp}M)$, we get $g(U, nV) = -g(nU, V)$. These show that T and n are also skew-symmetric tensor fields. Moreover, for any $X \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$, we have

$$g(NX, V) = -g(X, tV), \tag{2.10}$$

which gives the relation between N and t.
Now, applying ϕ to (2.8) and (2.9), we respectively, obtain

$$T^2 X = -X + \eta(X)\xi - tNX, \quad NTX + nNX = 0 \tag{2.11}$$

and

$$TtV + tnV = 0, \quad NtV + n^2 V = -V \tag{2.12}$$

for any vector fields X tangent to M and V normal to M.

We define the covariant derivatives of the tensor field T, N, t and n by $(\nabla_X T)Y = \nabla_X TY - T\nabla_X Y$, $(\nabla_X N)Y = \nabla_X^{\perp} NY - N\nabla_X Y$, $(\nabla_X t)V = \nabla_X tV - t\nabla_X^{\perp} V$ and $(\nabla_X n)V = \nabla_X^{\perp} nV - n\nabla_X^{\perp} V$ respectively.
Since M is tangent to ξ, making use of (2.4), (2.6) and (2.8), we obtain

$$\nabla_X \xi = X - \eta(X)\xi, \quad \sigma(X,\xi) = 0, \quad A_V \xi = 0 \tag{2.13}$$

for all $V \in \Gamma(T^{\perp}M)$ and $X \in \Gamma(TM)$.

Let X and Y be vector fields tangent to M. Then we obtain

$$(\nabla_X T)Y = A_{NY}X + t\sigma(X,Y) + g(TX,Y)\xi - \eta(Y)TX \tag{2.14}$$

and

$$(\nabla_X N)Y = n\sigma(X,Y) - \sigma(X,TY) - \eta(Y)NX. \tag{2.15}$$

Similarly, for any vector field X tangent to M and any vector field V normal to M. Then we have

$$(\nabla_X t)V = A_{nV}X - TA_V X + g(NX,V)\xi \tag{2.16}$$

and

$$(\nabla_X n)V = -\sigma(tV,X) - NA_V X. \tag{2.17}$$

3. Contact CR-Submanifold of a Kenmotsu Manifold

In this section, we shall define contact CR-submanifolds in a Kenmotsu manifold and research fundamental properties of their from theory of submanifold.
Let M be submanifold of an almost contact metric manifold $\widetilde{M}$, then M is called invariant submanifold if $\phi(T_pM) \subseteq T_pM$, $\forall p \in M$. Further, M is said to be anti-invariant submanifold if $\phi(T_pM) \subseteq T_p^{\perp}M$, $\forall p \in M$. Similarly, it can be easily seen that a submanifold M of an almost contact metric manifolds $\widetilde{M}$ is said to be invariant(anti-invariant), if N (or T) are identically zero in (2.8). Now we give definition of contact CR-submanifold which is a generalization of invariant and anti-invariant submanifolds.

Definition 3.1. A submanifold M of a Kenmotsu manifold. $\widetilde{M}$ is called contact CR-submanifold if there exists on M a differentiable invariant distribution D whose orthogonal complementary $D^{\perp}$ is anti-invariant, i.e.,
i) $TM = D \oplus D^{\perp}$, $\xi \in \Gamma(D)$
ii) $\phi D_p = D_p$
iii) $\phi D_p^{\perp} \subseteq T_p^{\perp}M$, for each $p \in M$ [13].
A contact CR-submanifold is called anti-invariant(or, totally real) if $D_p = 0$ and invariant(or, holomorphic) if $D_p^{\perp} = 0$, respectively, for any $p \in M$. It is called proper contact CR-submanifold if neither $D_p = 0$ nor $D_p^{\perp} = 0$.

Anti-invariant and invariant submanifolds are the special case of contact CR-submanifolds.
If we denote dimensions of the distributions D and $D^{\perp}$ by m_1 and m_2, respectively. Then M is called anti-invariant (resp. invariant) if $m_1 = 0$ ($resp.$ $m_2 = 0$).

Let us denote the orthogonal projections on D and $D^{\perp}$ by $P_1 : \Gamma(TM) \to D$ and $P_2 : \Gamma(TM) \to D^{\perp}$ respectively. Then we have

$$X = P_1X + P_2X + \eta(X)\xi$$

for any $X \in \Gamma(TM)$, where $P_1X \in \Gamma(D)$ and $P_2X \in \Gamma(D^{\perp})$. From (2.8) and (2.9), we have
and

$$\phi X = TX + NX = \phi P_1X + \phi P_2X = TP_1X + NP_1X + TP_2X + NP_2X$$

it is clear that

$$NP_1 = 0 \text{ and } TP_2 = 0,$$
$$N = NP_2 \text{ and } T = TP_1.$$

Proposition 3.2. *Let M be an isometrically immersed submanifold of a Kenmotsu manifold $\widetilde{M}$. Then the invariant distribution D has an almost contact metric structure (T,ξ,η,g) and so $dim(D_p)$ =odd for each $p \in M$ [5].*

We denote the orthogonal subbundle $\phi D^{\perp}$ in $T^{\perp}M$ by υ, then we have direct sum

$$T^{\perp}M = \phi D^{\perp} \oplus \nu \text{ and } \phi D^{\perp} \perp \nu.$$

Here we note that ν is an invariant subbundle with respect to ϕ and so dim(ν)=even.
Also,

$$t(T^{\perp}M) = D^{\perp} \text{ and } n(T^{\perp}M) \subset \nu.$$

Let M be a contact CR-submanifold of a Kenmotsu manifold $\widetilde{M}$. Then for any $X, Y \in \Gamma(D^{\perp})$ and $U \in \Gamma(TM)$, also by using (2.3), (2.4) and (2.6), we have

$$\begin{aligned} g(A_{NX}Y - A_{NY}X, U) &= g(\sigma(Y, U), NX) - g(\sigma(X, U), NY) \\ &= g(\widetilde{\nabla}_U Y, \phi X) - g(\widetilde{\nabla}_U X, \phi Y) \\ &= g(\phi\bar{\nabla}_U X, Y) - g(\phi\bar{\nabla}_U Y, X) \\ &= -g(A_{NX}U, Y) + g(A_{NY}U, X) \\ &= g(A_{NY}X - A_{NX}Y, U). \end{aligned}$$

It follows that

$$A_{NX}Y = A_{NY}X. \tag{3.1}$$

Proposition 3.3. *Let M be a contact CR-submanifold of a Kenmotsu manifold $\widetilde{M}$. Then, we have*

$$\nabla^{\perp}_X \phi Y - \nabla^{\perp}_Y \phi X \in \phi(D^{\perp})$$

for any $X, Y \in \Gamma(D^{\perp})$.

Proof. For any $X, Y \in \Gamma(D^{\perp})$, $V \in \Gamma(\nu)$. Then (2.3), Gauss and Weingarten formulas, we have

$$\begin{aligned} g(\nabla^{\perp}_Y \phi X - \nabla^{\perp}_X \phi Y, V) &= g(A_{\phi X}Y + \widetilde{\nabla}_Y \phi X - A_{\phi Y}X - \widetilde{\nabla}_X \phi Y, V) \\ &= g(\widetilde{\nabla}_Y \phi X - \widetilde{\nabla}_X \phi Y, V) \\ &= g((\widetilde{\nabla}_Y \phi)X + \phi\widetilde{\nabla}_Y X - (\widetilde{\nabla}_X \phi)Y - \phi\widetilde{\nabla}_X Y, V) \\ &= g(g(\phi Y, X)\xi - \eta(X)\phi Y + \phi\widetilde{\nabla}_Y X - g(\phi X, Y)\xi + \eta(Y)\phi X - \phi\widetilde{\nabla}_X Y, V) \\ &= g(\phi\widetilde{\nabla}_Y X - \phi\widetilde{\nabla}_X Y, V) = g(\widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X, \phi V) \\ &= g(\sigma(X, Y) - \sigma(Y, X), \phi V) = 0. \end{aligned}$$

Thus the proof is complete. □

Theorem 3.4. *Let M be a contact CR-submanifold of a Kenmotsu manifold $\widetilde{M}$. Then the tensor n is parallel if and only if the shape operator A_V of M satisfies the condition*

$$A_V tY = A_Y tV, \tag{3.2}$$

for all $Y, V \in \Gamma(T^{\perp}M)$.

Proof. For all $Y, V \in \Gamma(T^{\perp}M)$, and for all $X \in \Gamma(TM)$. By using (2.6), (2.10) and (2.17), we have

$$\begin{aligned} g((\nabla_X n)V, Y) &= -g(\sigma(tV, X), Y) - g(NA_V X, Y) \\ &= -g(A_Y tV, X) + g(A_V X, tY) \\ &= g(A_V tY - A_Y tV, X). \end{aligned}$$

The proof is complete. □

Theorem 3.5. *Let M be a contact CR-submanifold of a Kenmotsu manifold $\widetilde{M}$. Then the anti-invariant distribution $D^{\perp}$ is completely integrable and its maximal integral submanifold is an anti-invariant submanifold of $\widetilde{M}$.*

Proof. For any $Z, W \in \Gamma(D^{\perp})$ and $X \in \Gamma(D)$, By using (2.2) and (2.3), we have

$$\begin{aligned} g([Z,W],X) &= g(\widetilde{\nabla}_Z W, X) - g(\widetilde{\nabla}_W Z, X) \\ &= g(\widetilde{\nabla}_W X, Z) - g(\widetilde{\nabla}_Z X, W) \\ &= g(\phi\widetilde{\nabla}_W X, \phi Z) - g(\phi\widetilde{\nabla}_Z X, \phi W) \\ &= g(\widetilde{\nabla}_W \phi X - (\widetilde{\nabla}_W \phi)X, \phi Z) - g(\widetilde{\nabla}_Z \phi X - (\widetilde{\nabla}_Z \phi)X, \phi W) \\ &= g(\widetilde{\nabla}_W \phi X - g(\phi W, X)\xi + \eta(X)\phi W, \phi Z) - g(\widetilde{\nabla}_Z \phi X - g(\phi Z, X)\xi + \eta(X)\phi Z, \phi W). \end{aligned}$$

Here, By using (2.4), (2.6) and (3.1), we obtain

$$\begin{aligned} g([Z,W],X) &= g(\widetilde{\nabla}_W \phi X, \phi Z) - g(\widetilde{\nabla}_Z \phi X, \phi W) \\ &= g(\sigma(\phi X, W), \phi Z) - g(\sigma(\phi X, Z), \phi W) \\ &= g(A_{\phi Z} W - A_{\phi Z} W, \phi X) = 0. \end{aligned}$$

Thus $[Z, W] \in \Gamma(D^{\perp})$ for any $Z, W \in \Gamma(D^{\perp})$, that is, $D^{\perp}$ is integrable. Thus the proof is complete. □

Definition 3.6. A contact CR-submanifold M of Kenmotsu manifold $\widetilde{M}$ is said to be D-geodesic (resp. $D^{\perp}$-geodesic) if $\sigma(X, Y) = 0$ for $X, Y \in \Gamma(D)$ (resp. $\sigma(Z, W) = 0$ for $Z, W \in \Gamma(D^{\perp})$). If $\sigma(X, Z) = 0$, the M is called mixed geodesic submanifold, for any $X \in \Gamma(D)$ and $Z \in \Gamma(D^{\perp})$.

Theorem 3.7. *Let M be a contact CR-submanifold of a Kenmotsu manifold $\widetilde{M}$. Then the anti-invariant distribution $D^{\perp}$ is totally geodesic in M if and only if $\sigma(Z, X) \in \Gamma(\nu)$ for any $Z \in \Gamma(D^{\perp})$ and $X \in \Gamma(D)$.*

Proof. For any $Z, Y \in \Gamma(D^{\perp})$ and $X \in \Gamma(D)$, we have

$$\begin{aligned} g(\nabla_Z Y, \phi X) &= -g(\widetilde{\nabla}_Z \phi X, Y) \\ &= -g((\widetilde{\nabla}_Z \phi)X + \phi\widetilde{\nabla}_Z X, Y) \\ &= -g(g(\phi Z, X)\xi - \eta(X)\phi Z + \phi\widetilde{\nabla}_Z X, Y) \\ &= g(\widetilde{\nabla}_Z X, \phi Y) = g(\sigma(Z, X), \phi Y). \end{aligned}$$

Thus $\nabla_Z Y \in \Gamma(D^{\perp})$ if and only if $\sigma(Z, X) \in \Gamma(\nu)$. □

Theorem 3.8. *Let M be a contact CR-submanifold of a Kenmotsu manifold $\widetilde{M}$. Then the invariant distribution D is totally geodesic in M if and only if $\sigma(Z, Y) \in \Gamma(\nu)$ for any $Z, Y \in \Gamma(D)$.*

Proof. For any $Z, Y \in \Gamma(D)$ and $X \in \Gamma(D^{\perp})$, we have

$$\begin{aligned} g(\nabla_Z \phi Y, X) &= g((\widetilde{\nabla}_Z \phi)Y + \phi\widetilde{\nabla}_Z Y, X) \\ &= g(g(\phi Z, Y)\xi - \eta(Y)\phi Z + \phi\widetilde{\nabla}_Z Y, X) \\ &= -g(\widetilde{\nabla}_Z Y, \phi X) = -g(\sigma(Z, Y), \phi X), \end{aligned}$$

thus $\nabla_Z Y \in \Gamma(D)$ if and only if $\sigma(Z, Y) \in \Gamma(\nu)$. This completes of the prof. □

Theorem 3.9. *Let M be a proper contact CR-submanifold of a Kenmotsu manifold $\widetilde{M}$. If N is parallel on D, then either M is a $D-$geodesic submanifold or $\sigma(X, Y)$ is an eigenvector of n^2 with eigenvalue -1, for any $X, Y \in \Gamma(D)$.*

Proof. If N is parallel, then from (2.15), we have

$$n\sigma(X, Y) - \sigma(X, TY) - \eta(Y)NX = n\sigma(X, Y) - \sigma(X, TY) = 0. \tag{3.3}$$

for any $X, Y \in \Gamma(D)$.

On the other hand, since D is a invariant distribution and $T\xi = 0$, we have

$$n\sigma(X, -Y + \eta(Y)\xi) = \sigma(X, T(-Y + \eta(Y)\xi)) \tag{3.4}$$

that is,

$$n\sigma(X, Y - \eta(Y)\xi) = \sigma(X, TY).$$

Now, applying n to (3.5), we obtain

$$n^2\sigma(X, Y - \eta(Y)\xi) = n\sigma(X, TY). \tag{3.5}$$

By interchanging of Y and TY in (3.3) , we have

$$n\sigma(X, TY) = \sigma(X, T^2Y). \tag{3.6}$$

Hence, by using (3.5) and (3.6), we obtain

$$n^2\sigma(X, Y - \eta(Y)\xi) = n\sigma(X, TY) = \sigma(X, T^2Y) = -\sigma(X, Y - \eta(Y)\xi + tNY) = -\sigma(X, Y - \eta(Y)\xi).$$

This implies that either σ vanishes on D or σ is an eigenvector of n^2 with eigenvalue -1. □

Example 3.10. From now on, $(\mathbb{R}^9, \phi, \xi, \eta, g)$ will denote the manifold $\mathbb{R}^9$ with its usual an almost contact metric structure given by

$$\eta = \frac{1}{2}(dz - \sum_{i=1}^{4} y_i dx_i), \;\; \xi = 2\frac{\partial}{\partial z}$$

$$g = \eta \otimes \eta + \frac{1}{4}\sum_{i=1}^{4}(dx_i \otimes dx_i + dy_i \otimes dy_i)$$

$$\phi(\sum_{i=1}^{4}(X_i\frac{\partial}{\partial x_i} + Y_i\frac{\partial}{\partial y_i}) + Z\frac{\partial}{\partial z}) = \sum_{i=1}^{4}(Y_i\frac{\partial}{\partial x_i} - X_i\frac{\partial}{\partial y_i}),$$

where $(x_i, y_i, z), i = 1, 2, 3, 4$ are the cartesian coordinates.

Now, let M be a submanifold of $\mathbb{R}^9$ defined by the following equation

$$\chi(w, u, s, v, z) = 2(w, 0, u, 0, s, 0, 0, v, z).$$

We can easily to see that the tangent bundle of M is spanned by the tangent vectors

$$e_1 = 2(\frac{\partial}{\partial x_1} + y_i\frac{\partial}{\partial z}), \; e_2 = 2\frac{\partial}{\partial y_1}\;, e_3 = 2(\frac{\partial}{\partial x_3} + y_3\frac{\partial}{\partial z}), \; e_4 = 2\frac{\partial}{\partial y_4}, e_5 = 2\frac{\partial}{\partial z} = \xi.$$

For the almost contact structure ϕ of $\mathbb{R}^9$. We obtain,

$$\phi e_1 = -e_2, \; \phi e_2 = e_1, \; \phi e_3 = -2\frac{\partial}{\partial y_3}, \; \phi e_4 = 2\frac{\partial}{\partial x_4}, \; \phi e_5 = 2\frac{\partial}{\partial z} = 0.$$

By direct calculations, we can infer $D = span\{e_1, e_2, e_5\}$ is invariant distribution. Since $g(\phi e_4, e_j) = 0, \; j = 1, 2, 3, 5$ and $g(\phi e_3, E_i) = 0, \;\; i = 1, 2, 4, 5, \; \phi e_3, \phi e_4 \in T^{\perp}M, \; D^{\perp} = span\{e_3, e_4\}$ is an anti-invariant distribution. Thus M is a 5-dimensional proper contact CR-submanifold of $\mathbb{R}^9$ with it's usual almost contact metric structure [10].

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