Optimal contract pricing of load aggregators for direct load control in smart
distribution systems

Ali SHAyEGAN-RAD†, Ali ZANGENEH‡∗∗

1MAPNA Electric and Control, Engineering and Manufacturing Co. (MECO), MAPNA Group, Karaj, Iran
2Department of Electrical Engineering, Shahid Rajaee Teacher Training University, Tehran, Iran

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Abstract: Distribution system operators (DSOs) are interested in demand side participation programs as an efficient and secure resource to manage electricity supply and demand. However, it is usually difficult for DSOs to aggregate demand response of large/small consumers. Thus, in some electricity markets, an entity called an aggregator is defined to aggregate the load response of consumers. In this paper a bilevel scheduling model is proposed to determine the long-term optimal contract price between the DSO and aggregator for executing direct load control in smart distribution systems. The DSO and aggregator are considered as two different agents with individual objectives in the proposed bilevel scheduling model. On the one hand, the aggregator maximizes its profit by bidding load reduction of the large consumers to the DSO by executing a direct load control (DLC) mechanism, and on the other hand, the DSO tries to minimize its overall cost to supply all consumers. The DSO has two options to follow the variation of its consumers’ demand: purchasing energy from the electricity market and executing DLC programs. The bilevel programming formulation is transferred into an equivalent single level programming problem using its Karush–Kuhn–Tucker optimality conditions. Moreover, the uncertainties of the electricity market price, demand of consumers, and generation of a wind power plant are modeled via point estimate method. Two typical case studies are implemented to demonstrate the effectiveness of the proposed scheduling model.

Key words: Bilevel optimization, Karush–Kuhn–Tucker, load aggregator, optimal contract pricing, point estimate method

1. Introduction

Consumers have shown great interest in modifying the profile of their load consumption by participating in demand response (DR) programs to reduce their electricity bills. In DR programs, consumers sign agreements with a local utility or an aggregator to reduce their electricity consumption as soon as it is requested. The success of DR programs mainly depends on having smart metering, regulatory policies, and electricity pricing policies [1, 2]. DR programs benefit utilities for managing demand, saving costly generation, restoring quality of services, and ensuring reliability [3].

One type of DR program is direct load control (DLC), by which the utility or aggregator can control consumers’ consumption, especially during critical hours [4]. In DLC, consumers reduce their consumption in exchange for incentive payments and indirectly participate in the operation of the smart distribution network. In [5], an optimization algorithm based on DLC was presented to manage a virtual power plant composed of

‡Correspondence: a.zangeneh@srttu.edu

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a large number of controllable consumers. In other word, besides other generation resources, a share of total
demand of the smart distribution system can be supplied using DLC during peak or emergency hours. The
participation amount of the consumers is determined in such a way that minimizes the overall cost of the smart
distribution networks and also satisfies its technical constraints. Mohsenian-Rad developed an optimization
framework to help load-serving entities. The model comprises small time-shiftable loads, which are able to
submit their demand bids to the electricity market and minimize their energy procurement costs [6]. Large
industrial consumers have great potential to participate in the demand response programs. In [7], an energy
procurement problem was presented for large consumers with multiple available energy sources to assess the
effects of DR programs and energy storage system in the minimizing of total cost. In this regard, Zarnikau
and Hallett also tried to design a market structure in order to facilitate economic demand response for large
industrial consumers [8].

The previous papers only considered either aggregator or distribution system operators (DSO) behaviors.
Since both the DSO and aggregator follow different goals from their own viewpoints, the corresponding objective
functions and the relation between aggregator and DSO can be taken into account via a bilevel programming
problem (BPP) [9]. A BPP is a single-round Stackelberg game problem that contains two types of agents: the
first agent is the leader and the second type is followers [10]. First, the leader moves based on anticipating the
reaction of followers, and subsequently the followers move by knowing the movement of the leader.

Various methodologies, techniques, and algorithms have been investigated to solve BPPs. Recent studies
have used particle swarm optimization [11], evolutionary algorithms [12], branch and bound algorithms [13],
extact penalty functions [14], fuzzy logic [15], and the interior point method [16] to solve BPPs. Among these
methods, the Karush–Kuhn–Tucker (KKT) approach stands out for reducing the BPP into a mathematical
program with equilibrium constraints (MPEC) using KKT optimality conditions of its inner optimization level
[17, 18].

A bilevel model to focus on the interaction between a retailer and a partially flexible consumer was
presented in [17]. The retailer decides about the optimal price that maximize its profit, while the consumer
optimizes its consumption based on the specified price. Mahmoudi et al. [17] presented bilevel programing in
which a wind power producer determines its favorable price to buy DR for covering its stochastic generation
and then the aggregator determines its DR involvement. The authors of [18] proposed an interesting bilevel
model for implementing DR programs between retailers and customers. The paper assessed the customers’
behavior truly based on different pricing mechanisms. It was concluded that fixed and time-of-use pricing is
more convenient than real-time pricing based on the customer’s viewpoint.

In this paper, a DSO sets a long-term contract with an aggregator to use the aggregated DR of customers
while it has economic and technical benefits. The aim of this paper is to propose a stochastic bilevel scheduling
model to determine the optimal price of the long-term contract between the DSO and aggregator for executing
direct load control (DLC), a typical DR program, in a day-ahead energy market. The DSO buys energy from
either the electricity market or DLC programs to supply its consumers by optimizing the economic cost and
technical conditions of the smart distribution system. On the other hand, the aggregator intends to bid for
the optimal price to the DSO in order to sell more aggregated DLC and maximize its expected profit. In fact,
if the aggregator bids a high price for DLC programs, the DSO might not accept the offer; conversely, if the
aggregator bids a low price, it might not be guaranteed to achieve its maximum expected profit. Hence, the
aggregator determines its optimal bidding price of DLC programs by taking into account the DSO behavior in
a bilevel programming model.
The difference between the proposed model of this paper and [18] is to model the interaction between the DSO and load aggregator to aggregate the load reduction of large customers by executing the DLC mechanism. In other words, it is assumed that large industrial customers declare their maximum load reduction under a predetermined time-of-use tariff and thus the benefit model of the customer side is not considered. The main contributions of this paper are highlighted as follows:

1- An aggregator profit-based model is proposed to aggregate load reduction of the individual large consumers through the DLC program.
2- This paper proposes a bilevel programming problem to model an optimal decision-making interaction between the load aggregator and the DSO. The bilevel model is transformed into an equivalent single level programming problem using its KKT optimality conditions.
3- PEM, known as an efficient and robust technique, is used to model uncertainties in the proposed model.

The rest of this paper is organized as follows: Section 2 describes the proposed BPP scheduling model. Section 3 presents the mathematical formulation of the proposed BPP scheduling model. PEM is explained in Section 4 to model uncertain parameters of the proposed model and Section 5 presents the case studies and the numerical results to verify the effectiveness of the proposed model and to illustrate the obtained results. Finally, the findings of this work are summarized as conclusions in Section 6.

2. Bilevel DLC scheduling model

In this paper, the load aggregator and DSO are defined as two different agents that try to make their own optimal decisions. The aggregator aggregates the participation of large consumers in executing DLC and bids the optimal price to the DSO by anticipating the reaction of the DSO. Consequently, the DSO, which has received the proposed DLC price, determines the required DLC amount to balance between its demand and supply. This problem represents two distinct objective functions: maximizing the aggregator’s expected profit and minimizing the DSO’s operation costs. These objective functions and the relation between the aggregator and DSO are shown in Figure 1. It should be mentioned that besides the executing of the DLC program by the load aggregator, the DSO can provide its required energy through either the electricity market or a wind power plant (WPP).

3. Bilevel problem formulations

3.1. Outer level: Expected profit function of the aggregator

The expected profit function of the load aggregator is maximized in the outer level of the BPP using Eq. (1).

$$\max_{\lambda} \sum_{h} \left[ \lambda \sum_{i} P_i(h) - \sum_{i} \pi_i(h) P_i(h) \right]$$

The first term of Eq. (1) is income of selling energy to the DSO and the second term represents the cost of executing DLC programs.

3.2. Inner level: Cost function of distribution system operator

The DSO optimization problem in the inner level of the BPP is the minimized total cost of the DSO in supplying its consumers. The incurred cost of the DSO is represented by Eq. (2), subject to the constraints of Eqs. (3)–(8).
The first and second terms of Eq. (2) represent the cost of purchasing electricity from the aggregator and day-ahead electricity market, respectively. The aggregator provides available DLC among permissible upper and lower bounds using Eq. (3). Approximate power flow equations are considered to model distribution system behaviors [19]. The active power flow in the line connecting nodes n and m is calculated by Eq. (4). Network active power balance for each bus is insured in Eq. (5), which represents that power generation minus power consumption is equal to distribution system losses. Upper and lower power flow limits of distribution lines are considered using Eq. (6). The minimum and maximum bus voltage limits are applied in Eq. (7). Finally, Eq. (8) is implemented to consider the maximum capacity of the substation transformer connecting the distribution system to the main grid.

\[
\begin{align*}
\min_{P_i, P_{sub}, U_n} \sum_h & \left[ \lambda \sum_i P_i(h) + \rho(h)P_{sub}(h) \right] \\
\text{subject to:} & \\
& P_i^\text{min} \leq P_i(h) \leq P_i^\text{max} \quad \forall i \in I, \forall h \in H \\
& P_{nm}(h) = \frac{|U_n||U_n - U_m|}{Z_{nm}} \quad \forall n \in N, \forall m \in M, \forall h \in H \\
& P_{gen}^n(h) - P_{dem}^n(h) = \sum_{m \in M} \frac{|U_n||U_n - U_m|}{|Z_{nm}|} \quad \forall n \in N, \forall h \in H \\
& \frac{|U_n||U_n - U_m|}{|Z_{nm}|} \leq P_{nm}^\text{max} \quad \forall n \in N, \forall m \in M, \forall h \in H \\
& U_n^\text{min} \leq U_n(h) \leq U_n^\text{max} \quad \forall n \in N, \forall h \in H \\
& P_{sub}(h) \leq P_{\text{sub}}^\text{max} \quad \forall n \in N, \forall h \in H
\end{align*}
\]
3.3. Equivalent single level optimization problem

The BPP can be substituted into an equivalent single level optimization problem through KKT optimality conditions of the inner optimization problem. According to the KKT approach [17,18], Eq. (9) represents the obtained MPEC objective function subject to the previous constraints of Eqs. (3)–(8) and also the new constraints of Eqs. (10)–(20). Note that the constraints of Eq. (10)–(20) are obtained by taking the derivative of the Lagrange function with respect to each of the independent variables of the inner level problem \((\lambda, P_{\text{sub}}, U_n)\) and setting the derivatives equal to zero. The constraints of Eqs. (13)–(20) are complementarity conditions.

\[
\max_{\lambda, P_i, P_{\text{sub}}, U_n} \sum_h \left[ \lambda \sum_i P_i(h) - \sum_i \pi_i(h)P_i(h) \right]
\]

s.t. Eq.(3-8) and

\[
\lambda - \omega_i(h) + \zeta_i(h) = 0 : \forall i \in I, \forall h \in H
\]

\[
\rho(h) - \omega_{\text{sub}}(h) + \bar{\pi}_{\text{sub}}(h) - \underline{\alpha}_{\text{sub}}(h) = 0 : \forall \text{sub} \in \text{SUB}, \forall h \in H
\]

\[
\omega_n(h) \sum_{m \in M} \frac{2[U_n(h)] - |U_m(h)|}{|Z_{nm}|} - \sum_{m \in M} \omega_m(h) \frac{|U_m(h)|}{|Z_{nm}|} + \sum_{m \in M} \left( \varphi_{nm}(h) - \varphi_{mn}(h) \right) \left( \frac{2[U_n(h)] - |U_m(h)|}{|Z_{nm}|} \right)
\]

\[
+ \sum_{m \in M} \left( \varphi_{nm}(h) - \varphi_{mn}(h) \right) \frac{|U_m(h)|}{|Z_{nm}|} + \beta_n(h) - \underline{\beta}_n(h) : \forall n \in N, \forall h \in H
\]

\[
\zeta_i(h) - (P_i(h) - P_i^{\text{max}}(h)) = 0; \zeta_i(h) \geq 0 : \forall i \in I, \forall h \in H
\]

\[
\zeta_i(h) - (P_i^{\text{min}}(h) - P_i(h)) = 0; \zeta_i(h) \geq 0 : \forall i \in I, \forall h \in H
\]

\[
\bar{\pi}_{\text{sub}}(h) - (P_{\text{sub}}(h) - P_{\text{sub}}^{\text{max}}(h)) = 0; \bar{\pi}_{\text{sub}}(h) \geq 0 : \forall \text{sub} \in \text{SUB}, \forall h \in H
\]

\[
\underline{\alpha}_{\text{sub}}(h).P_{\text{sub}}(h) = 0; \underline{\alpha}_{\text{sub}}(h) \geq 0 : \forall \text{sub} \in \text{SUB}, \forall h \in H
\]

\[
\varphi_{nm}(h) \left( \frac{|U_n(h)|}{|Z_{nm}|} - |U_m(h)| \right) = 0; \varphi_{nm}(h) \geq 0 : \forall n \in N, \forall m \in M, \forall h \in H
\]

\[
\varphi_{mn}(h) \left( - \frac{|U_n(h)|}{|Z_{nm}|} - |U_m(h)| \right) = 0; \varphi_{nm}(h) \geq 0 : \forall n \in N, \forall m \in M, \forall h \in H
\]

\[
\beta_n(h)(U_n(h) - U_n^{\text{max}}) = 0; \beta_n(h) \geq 0 : \forall n \in N, \forall h \in H
\]

\[
\beta_n(h)(U_n^{\text{min}} - U_n(h)) = 0; \beta_n(h) \geq 0 : \forall n \in N, \forall h \in H
\]
4. Hong’s two-point estimate method (HTPEM)

The probabilistic techniques to model uncertainties are classified as Monte Carlo simulation (MCS), approximate methods, and analytical methods [20]. Among these techniques, MCS is the most accurate, but it requires many model runs to attain convergence. Approximate methods provide an approximate description of the statistical properties of random output variables and analytical methods use convolution techniques to simplify the problem. Among these methods, PEM stands out with the main advantages as follows [21]:

1- Similar to MCS, PEM employs deterministic routines to solve probabilistic problems in each model run; however, it has a much lower computational burden.

2- PEM overcomes the difficulties associated with the lack of perfect knowledge by considering the probability functions of the stochastic variables. Since these functions are approximated using only the first few statistical moments of the input random variables (i.e. mean, variance, skewness, and kurtosis), a low level of information is required.

PEM defines C points, which are called the concentration for each input random variable. Using these points and a function F (which relates input and output variables), output random variables can be obtained [21].

The HTPEM is used to model uncertain parameters including power generation variation of the WPP, aggregator demands, and electricity market price in the scheduling model. Suppose a problem includes input random variables \(X\{x_1, x_2, ..., x_j\}\) with a mean value \(\mu_{x_k}\) and standard deviation \(\sigma_{x_k}\). Function \(Z = F(X)\) represents the problem with \(j\) input random variables. \(Z\) is an output random variable as a function of \(X\) and the function \(F\) transfers the uncertainty associated with the input random variables to the output random variables. The HTPEM only employs two concentrations, \((C = 1, 2)\), for each random variable. In the HTPEM, \(F\) needs to be solved only 2 times for each input random variable \(x_k(x_{k,1}, x_{k,2})\). Since this problem has three input random variables, the MPEC would be evaluated 6 times. Table 1 indicates the consequences of evaluations. For example, the first evaluation is at points \(x_{1,1}\) (first input random variable) and \(\mu_2\) and \(\mu_3\) are mean values regarding the second and third variables.

<table>
<thead>
<tr>
<th>Points of evaluation</th>
</tr>
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<tbody>
<tr>
<td>Evaluation 1</td>
</tr>
<tr>
<td>Evaluation 2</td>
</tr>
<tr>
<td>Evaluation 3</td>
</tr>
<tr>
<td>Evaluation 4</td>
</tr>
<tr>
<td>Evaluation 5</td>
</tr>
<tr>
<td>Evaluation 6</td>
</tr>
</tbody>
</table>

The step by step producer of the HTPEM is proposed in Figure 2. The considered stochastic parameters in the MPEC of this paper are as follows: \(X = [\rho(h), P_{dem}(h), P_{WPP}(h)]\).

Considering the mean and standard deviation of \(x_k\), Eq. (21) computes the location of \(x_{k,c}\) where \(\zeta_{x_k}\) represents the standard location of input random variable \(x_k\). Eq. (22) denotes the solution of the deterministic
MPEC where $Z_{k,c}$ is the vector of output random variables.

$$x_{k,c} = \mu_{x_k} + \zeta_{k,c}\sigma_{x_k}$$  \hspace{1cm} (21)$$

$$Z_{k,c} = F(x_{k,1}, x_{k,2}, \ldots, x_{k,c}, \ldots, x_{j,c})$$  \hspace{1cm} (22)$$

Eqs. (23) and (24) are applied to compute the standard location $\zeta_{k,c}$ and weight $\omega_{k,c}$ of input random variable $x_k$, respectively.

$$\zeta_{k,1} = \frac{\lambda_{k,3}}{2} + \sqrt{j + \left(\frac{\lambda_{k,3}}{2}\right)^2}, \quad \zeta_{k,2} = \frac{\lambda_{k,3}}{2} - \sqrt{j + \left(\frac{\lambda_{k,3}}{2}\right)^2}$$  \hspace{1cm} (23)$$

$$\omega_{k,1} = -\frac{\zeta_{k,2}}{j(\zeta_{k,1} - \zeta_{k,2})}, \quad \omega_{k,2} = \frac{\zeta_{k,1}}{j(\zeta_{k,1} - \zeta_{k,2})}$$  \hspace{1cm} (24)$$

Here, $\lambda_{k,3}$ is the skewness of the random input variables, which is determined using Eq. (25).

$$\lambda_{k,3} = E\left[(x_k - \mu_{x_k})^3\right]$$  \hspace{1cm} (25)$$

In Eq. (25), $E$ is the row moments of output variables updated using Eq. (26).

$$E(Z) = E(Z) + \sum_c Z_{k,c}\omega_{k,c}$$  \hspace{1cm} (26)$$

5. Numerical results

In order to demonstrate the effectiveness of the proposed scheduling model, the distribution test system shown in Figure 3 is applied in this paper. The system topology is similar to the well-known 34-bus distribution system; however, a modified single-phase version is considered [22]. The distribution system is connected to the electricity market through bus 1. It is assumed that the DSO serves 34 large consumers with mean values of total demand as shown in Figure 4 for two typical days. The standard deviation of the total demand profile is considered as 0.7 MW and it is assumed that large consumers at buses 4–16, 18, 20, 22, 23, 25–28, and 30–34 participate in the DLC program. Each large consumer that participates in the DLC program agrees to reduce its hourly demand by up to 30% of its maximum demand according to the preestablished contracts.

The mean values of the day-ahead market prices for a daily time horizon are shown in Figure 5 [23] and its standard deviation value is considered as 2 €. The daily time horizon is classified into three categories including valley, shoulder, and peak hours. Data of this classification and the cost of load reduction that the aggregator must pay to individual large consumers are presented in Tables 2 and 3, respectively.

**Table 2. Hour classifications.**

<table>
<thead>
<tr>
<th>Working day</th>
<th>Weekend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valley</td>
<td>2–7</td>
</tr>
<tr>
<td>Shoulder</td>
<td>1, 8–10, 15–20, 23, 24</td>
</tr>
<tr>
<td>Peak</td>
<td>11–14, 21, 22</td>
</tr>
</tbody>
</table>
The WPP is installed at bus 24 of the distribution system and its stochastic parameters of power generation are represented in Table 4. Moreover, this paper applies two different case studies as follows to evaluate the effectiveness of the proposed method.
Figure 3. Schematic diagram of the IEEE 34-bus smart distribution system.

Figure 4. Mean value of the total demand for the IEEE 34-bus smart distribution system.

Figure 5. Mean value of hourly day-ahead market price.

**Case A:** proposed strategy is applied in a typical working day.

**Case B:** proposed strategy is applied in a typical weekend.

The proposed NLP model has been implemented in GAMS [24] software and solved using the SNOPT solver. Table 5 summarizes the obtained optimal daily prices of cases A and B. Both the DRP and the aggregator are satisfied with the obtained prices. Since working day prices are mainly higher than weekend prices in the day-ahead market, the obtained optimal daily price of a working day becomes greater than weekend prices.
Table 4. WPP characteristics.

<table>
<thead>
<tr>
<th>Hour</th>
<th>WPP (MW)</th>
<th>Hour</th>
<th>WPP (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. dev.</td>
<td>Mean</td>
</tr>
<tr>
<td>h=1,…,4</td>
<td>2</td>
<td>0.18</td>
<td>h=13,…,16</td>
</tr>
<tr>
<td>h=5,…,8</td>
<td>2.5</td>
<td>0.22</td>
<td>h=17,…,20</td>
</tr>
<tr>
<td>h=9,…,12</td>
<td>3</td>
<td>0.14</td>
<td>h=21,…,24</td>
</tr>
</tbody>
</table>

Table 5. Optimal price of aggregator.

<table>
<thead>
<tr>
<th>Case</th>
<th>Optimal price (€/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Working day</td>
<td>51.92</td>
</tr>
<tr>
<td>Weekend</td>
<td>36.57</td>
</tr>
</tbody>
</table>

Figure 6 shows the power provided by the aggregator for the two case studies. The DSO prefers to accept the aggregator’s bid when the costs of purchasing power from the day-ahead market are more than purchasing from the aggregator. Furthermore, power purchases from the aggregator lead to less power losses in the distribution system and therefore reduces the total cost of the DSO.

Figure 7 depicts the modified demand patterns of consumers after applying the DLC program in the two case studies. The DSO mainly decreases the initial demand at peak hours more than the others to flatten its demand patterns. Flat demand would cause the DSO not to buy expensive power at peak hours. Figure 8 shows the purchased power from the day-ahead market. As is observed, in addition to the power purchased from the aggregator and power generation of the WPP, the DSO participates in the day-ahead market to meet its remaining power demand at each hour. Power system losses are depicted in Figures 9 and 10, which show that applying load reduction has reduced power system losses in the two case studies.

Hourly costs of the DSO for working days and weekends are shown in Figures 11 and 12, respectively. It is observed that power purchased from the aggregator leads to a decrease in hourly cost of the DSO at each hour. This is because of less power purchasing at peak hours.

Table 6 lists the overall cost of the DSO with and without execution of the DLC program by large consumers. As shown, the DSO saved up to 7% of its total costs by executing the DLC program. This demonstrates the economical merit of utilizing DLC programs. Table 7 details the expected profits of the
Figure 7. The initial and modified demand patterns of the DSO.

Figure 8. Power purchased from day-ahead market.

Figure 9. Power system losses in case A.

Figure 10. Power system losses in case B.

aggregator in the two different case studies. It shows that the proposed model can be profitable for both agents considered here. However, it should be noted that executing DLC programs has three sides: large consumers, aggregator, and DSO. In this paper, the behavior of large consumers has not been considered.

6. Conclusion
This paper proposed a bilevel scheduling model to determine the optimal contract price of aggregating and bidding a DLC program to the DSO in a smart distribution system. It should be noted that executing DLC programs has three sides: large consumers, aggregator, and DSO; however, in this paper, just the interactions between the last two players are modeled. The BPP considers the interests of two agents: the DSO and the
load aggregator. The objectives of the bilevel scheduling model are to minimize the overall cost of the DSO and also to maximize the expected profit of the aggregator. The bilevel formulation is reduced to a single level optimization problem (MPEC) using the KKT optimality conditions of the inner problem. Also, the two-point
estimate method is applied to model the stochastic parameters including WPP generation, day-ahead market price, and consumer demands. Two typical days are considered as case studies to evaluate the efficiency of the proposed model. The obtained results show the capability and robustness of the proposed model. This approach leads to following results:

- Not only does the aggregator earn profit, but also the DSO saves up to 7% of its total costs.
- The DSO modifies its demand consumption.
- Power system losses of the network are reduced by up to 16.2% of its total losses. Hence, this approach is interesting for both the aggregator and DSO. The obtained results show the capability and robustness of the proposed model.

References


