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Converse Tournaments

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ABSTRACT

This paper proposes two alternatives to Clark and Riis (1998b)'s sequential model of nested multiple-prize contests. First, we consider winning prizes endogenously determined by exerted contest efforts. Second, we extend the model to infinite-horizon. We characterize the unique subgame perfect Nash equilibrium in both models and compare the equilibrium strategies with those in the original model.

Keywords: Converse tournaments, Nested multiple-prize contests, Tullock contests, Endogenous prizes, Subgame perfect Nash equilibrium

ÖZET

Bu makalede Clark ve Riis (1998b)'in iç içe geçmiş çok-ödüllü yarışma modelinin dinamik versiyonuna iki alternatif önerilmiştir. İlk olarak, kazananların alacağı ödüllerin sarf edilen yarışma eforları tarafından endojen olarak belirlendiği bir model çalışılmıştır. İkinci olarak, model genişletilerek bir sonsuz ufuk modeli elde edilmiştir. Önerilen modellerin alt-oyun mükemmel Nash dengeleri karakterize edilmiş olup, ilgili denge stratejileri temel alınan modeldeki denge stratejileri ile karşılaştırılmıştır.

Anahtar Kelimeler:

1. Introduction

Job search is an important topic in labor economics. This has led many scholars to analyze the strategic dimension of job search, mostly concentrating on wage bargaining between the employer and workers (see Shaked and Sutton, 1984; Wolinsky, 1987; Shimer, 2006 among others). Such analyses are often utilized as complements to a number of micro-founded models in the labor economics literature. Although this link between game theory and labor economics was formed more than thirty years ago, to the best of our knowledge, the strategic competition between several job seekers has not yet been investigated. In this paper, we propose new game-theoretic models that can fill this gap. In particular, we analytically investigate a job search scenario in which a number of individuals compete with each other to fill a specific job vacancy. The winner is hired, so that she leaves the market; i.e., when another job vacancy is posted in the following period, she does not apply for that vacancy. As a result, in period $t+1$, only the losers in period t compete with each other. Here we model the competition between job seekers at each period as a *contest* game; so that this work can be considered a contribution to the literature on contests/tournaments.[1]

A *contest* game is a strategic interaction where players exert costly efforts to win a valuable prize. Each player incurs effort costs independent of the contest outcome. Common examples are sports, warfare, R&D competition, election campaign, etc. A *tournament* is a dynamic contest game in which there is either one or multiple component contests in each period, such that the losers are eliminated and the winners proceed to the next period. The tournament champion is revealed after the final component contest is played. Some real-life examples are NBA playoffs, Champions Cup after the group stages, presidency elections in Turkey, etc. As mentioned earlier, in this paper we are interested in a case where several economic agents compete to fill a specific job vacancy. The strategic interaction between job seekers at each period can be modeled as a static contest game; and given that the winner in each period is eliminated, the whole interaction can be labeled as a *converse tournament*.

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This idea is closely related to the sequential version of the nested multiple-prize contests investigated by Clark and Riis (1998a, 1998b).[2] In both of these papers, the authors studied a model where n agents compete for k_1 prizes in the first period, after which the winners leave the contest; then the remaining $n-k_1$ agents compete for k_2 prizes in the second period, after which the winners leave the contest; and so on. They concentrated on the total equilibrium rent-seeking effort, without explicitly characterizing the individual strategies in the equilibrium. The difference between these papers is that Clark and Riis (1998a) used an all-pay auction contest success function, whereas Clark and Riis (1998b) used a Tullock-type contest success function. As we are interested in Tullock contests including only one prize at each period, our baseline model turns out to be a special case of Clark and Riis (1998b)'s model.

Presenting our baseline model along similar lines with Clark and Riis (1998b), we explicitly characterize the equilibrium strategies for each player. Next, we turn to two extensions specific to the above-mentioned job search interpretation: (i) Wages are endogenously determined. In particular, as job seekers exert more effort to get hired, the employer becomes more impressed and offers a higher wage to the winner. This implies a winning prize (i.e., wage) increasing in contest efforts. (ii) In period $t+1$, although the winner in period t leaves the market, a new player joins in the pool of job seekers. Given that the number of job seekers at each period remains constant, this assumption is then utilized to convert the baseline model into an infinite-horizon model. In both extensions, our main objective is to compare the equilibrium strategies with those in the baseline model. Our results indicate that (i) job seekers might play more aggressively if wages are determined endogenously, and $(i\ )$ when the game is extended to infinite-horizon, job seekers become more aggressive as they attach a higher value to future earnings.

The rest of the paper is organized as follows. Section 2 briefly reviews the relevant literature. In section 3, we analyze subgame perfect Nash equilibrium in the baseline model. In section 4, we introduce our extensions and investigate how individual behavior changes in equilibrium. Section 5 concludes.

2. Literature Review

In labor economics, most of the early papers investigate the dynamics of unemployment, while focusing on job search behavior (see Stigler, 1962; Mortensen, 1970; Jovanovic, 1984; Pissarides, 1985 among others). These models are mostly interested in how a worker is matched with a firm and how wage is determined between these two economic agents. In 1980s, following Rubinstein (1982)'s bargaining model, there has been several contributions utilizing a game-theoretic approach in this literature (see Shaked and Sutton, 1984; Wolinsky, 1987; Cahuc et al., 2006; Shimer, 2006 among others). These papers focus on the wage bargaining process that took place after a worker searches for a job and is matched with a firm. Although the competition between several job seekers is another important strategic factor in job search, to the best of our knowledge, such competitive behavior has not yet been investigated in this literature. Here we argue that this strategic interaction between job seekers can be modeled as a contest game.

The analysis of contest games dates back to the seminal contribution by Tullock (1980). Among other early studies, we can mention the contributions by Lazear and Rosen (1981), Nalebuff and Stiglitz (1983), Dye (1984), Rosen (1986), and Dixit (1987). The extensive literature that emerged from these important papers provides insights in a wide range of application areas: sports, warfare, election campaign, promotion tournament, litigation, etc. In recent years, this branch of game theory has become quite popular, which multiplied the number of contributions on contests and tournaments (see Yildirim, 2005; Münster, 2007; Konrad and Kovenock, 2009; Sela, 2012; Fu et al., 2015; Brown and Chowdhury, 2017; Keskin and Sağlam, 2017; Chowdhury et al., 2018; Doğan et al., 2018; Mago and Sheremeta, 2018 among others).

In this review section, we specifically concentrate on the nested multiple-prize contest model proposed by Clark and Riis (1998a, 1998b), which constitutes our baseline model. Accordingly, there are n agents competing for a number of prizes in a given period; and whichever agents win those prizes leave the game, so that the remaining players compete with each other in the next period. Barut and Kovenock (1998) analyzed a similar model with a major difference that players may have strict preference between any pair of prizes distributed in a given period. Later, Fu et al. (2014) analyzed a multi-prize "reverse" nested lottery contest, which can be viewed as the *mirror image* of the model by Clark and Riis (1998a, 1998b). The contestants choose a single contest effort at the beginning, and the model determines winners by eliminating losers through a sequence of lotteries. Chowdhury and Kim (2014) studied a similar model, considering a sequential elimination of losers; and they showed that the model is equivalent to Berry (1993)'s model in which winners are selected simultaneously. As summarized above, none of these existing papers studied an endogenous prize framework or an infinite-horizon version.

In this paper, we analytically investigate several versions of *converse tournament*: (i) a component contest takes place at each period, (ii) the winner earns some prize and leaves the game, and (iii) the losers proceed to the next period for another round of competition. Building on the model studied by Clark and Riis (1998a, 1998b), we propose two alternative models: (i) with endogenous prizes and (ii) with infinite-horizon. Labeling the contestants as job seekers, we motivate our analysis referring to a job search scenario in labor market. We argue that these converse tournaments can be utilized as complements to the micro-founded models in labor economics, and we hope that the insights gained from our analysis will be helpful in understanding the strategic nature of job search and in implementing job seekers' competitive behavior into the existing models.

3.The Baseline Model

There are three symmetric players in the player set $N=\{1,2,3\}$. They participate in a two-period *converse tournament*. In the first period, there exists a single job vacancy. Players compete with each other to get hired by choosing a non-negative effort level. The winner gets a wage $W_1>0$ and leaves the game; whereas the losers do not receive any payoff. In the second period, another job vacancy becomes available. The remaining players compete with each other to get hired by choosing a non-negative effort level. The winner gets a wage $W_2>0$, whereas the loser ends up with 0. The game ends.[3]

In this game, let $e_i\in[0,\infty)$ denote the contest effort exerted by player $i\in N$ in period 1. Moreover, let $e_{ij}\in[0,\infty)$ denote the contest effort exerted by player $i\in N$ in the contest against player $j\in N\setminus\{i\}$ in the second period. We assume that the winner is determined by a Tullock contest success function in both periods: own effort divided by the total effort exerted by all job seekers. This means that the probability of player $i\in N$ winning the contest in the first period is

$$\frac{e_i}{e_1+e_2+e_3},$$

whereas the probability of player i winning the contest in the second period against player $j\in N\setminus\{i\}$ is[4]

$$\frac{e_{ij}}{e_{ij}+e_{ji}}.$$

We assume a linear cost of effort for every player $i\in N: c_i(e)=e$.

Accordingly, without loss of generality, the expected payoff function for player 1 can be written as

$$\frac{e_1}{e_1+e_2+e_3}W_1+\frac{e_2}{e_1+e_2+e_3}\left(\frac{e_{13}}{e_{13}+e_{31}}W_2-e_{13}\right)+\frac{e_3}{e_1+e_2+e_3}\left(\frac{e_{12}}{e_{12}+e_{21}}W_2-e_{12}\right)-e_{11}$$

Player 1's objective is to maximize the expected payoff function by choosing the effort levels e_1, e_{12}, and e_{13}. Below we analyze subgame perfect Nash equilibrium of this model.

Proposition 1 *Consider the three-player converse tournament described above. Assume that* $4W_1\geq W_2$. *The unique subgame perfect Nash equilibrium suggests that each player exerts* $(4W_1-W_2)/18$ *in the first period; and the losers exert* $W_2/4$ *in the second period. The expected payoff for each player is*

$$\frac{W_1+2W_2}{9}.$$

In case $4W_1<W_2$, however, players choose not to exert any effort in the first period. Then, the losers exert $W_2/4$ in the second period, so that the expected payoff for each player becomes

$$\frac{2W_1+W_2}{6}.$$

Proof. See the Appendix.

As mentioned earlier, Clark and Riis (1998b) chose not to provide an explicit characterization of the equilibrium strategies. Proposition 1 fills this gap by explicitly characterizing the unique subgame perfect Nash equilibrium. We observe that if W_1 is sufficiently high, all players exert positive efforts in both periods; however, if $W_1<W_2/4$, it is not beneficial for players to exert any effort in the first period. In the latter case,

players rather wait for the second period to have a chance of receiving a higher wage.

We can also see that the equilibrium strategies in the first period is increasing in W_1 and decreasing in W_2, whereas the equilibrium strategies in the second period is increasing in W_2. These results are expected: players exert more effort if the current winning prize is higher, but exert less effort if their outside option (which is, for the first period, the possibility of getting W_2 next period) is higher.

Furthermore, the expected equilibrium effort in the second period can be calculated as

$$\frac{2}{3}\frac{W_2}{4}=\frac{W_2}{6}.$$

Thus, if $W_1 > W_2$, the symmetric equilibrium effort in the first period is greater than the expected equilibrium effort in the second period.

4. Alternative Models

In this section, we study two extensions of the baseline model. First, we assume that winning prizes are endogenously determined. Second, we assume that a new player joins in the player set each period replacing the winner in the previous period. In both of these extensions, we characterize subgame perfect Nash equilibrium and compare the equilibrium strategies with those in the baseline model.

4.1. Endogenous Prizes

Here we assume that winning prizes are endogenously determined by the exerted contest efforts. The only difference from the baseline model is that the prizes in the first and second periods are described by the functions $W_1 : [0,\infty)^3 \to \mathbb{R}_+$ and $W_2 : [0,\infty)^2 \to \mathbb{R}_+$, respectively. Both functions are increasing and concave in all of their arguments. Intuitively, as job seekers exert more effort in the contest, the employer becomes more impressed and offers a higher wage to the winner, but the respective increase in wage diminishes as the effort levels increase. We further assume that the prize functions are *symmetric*, meaning that when two players exchange their searching efforts, their wage levels would be exchanged as well.

The expected payoff functions can be written similarly as in the baseline model. Below we analyze subgame perfect Nash equilibrium of this model.

Proposition 2 *Consider the three-player converse tournament with endogenously determined prizes described above. The unique subgame perfect Nash equilibrium suggests that each player exerts* $\mathbf{e}_1^*$ *in the first period, which is implicitly defined by*

$$\frac{2}{9\mathbf{e}_1^*}=\frac{1-\frac{1}{3}\frac{\partial W_1(\cdot,\cdot,\cdot)}{\partial e_{i1}}\Big|_{(\mathbf{e}_1^*,\mathbf{e}_1^*,\mathbf{e}_1^*)}}{W_1(\mathbf{e}_1^*,\mathbf{e}_1^*,\mathbf{e}_1^*)-EW_2^*}, \tag{1}$$

where EW_2^* denotes the equilibrium expected payoff for any player in the second period; and the losers exert $\mathbf{e}_2^*$ in the second period, which is implicitly defined by

$$\frac{1}{4\mathbf{e}_2^*}=\frac{1-\frac{1}{2}\frac{\partial W_2(\cdot,\cdot)}{\partial e_{i2}}\Big|_{(\mathbf{e}_2^*,\mathbf{e}_2^*)}}{W_2(\mathbf{e}_2^*,\mathbf{e}_2^*)}. \tag{2}$$

Proof. See the Appendix.

Proposition 2 implicitly characterizes the unique subgame perfect Nash equilibrium for any pair of wage functions. In order to obtain more concrete ideas about the equilibrium strategies, we now specify certain functional forms for the wage functions W_1 and W_2. Let

$$W_1(e_1,e_2,e_3)=\sqrt{3(e_1+e_2+e_3)}$$

and

$$W_2(e_{ij},e_{ji})=\sqrt{2(e_{ij}+e_{ji})} \quad \textit{for every} \quad i,j\in N.$$

Given these wage functions, which depend on the aggregate effort levels, equation (1) becomes

$$\frac{1}{4\mathbf{e}_2^*}=\frac{1-\frac{1}{4\sqrt{\mathbf{e}_2^*}}}{2\sqrt{\mathbf{e}_2^*}}.$$

This implies

$$\mathbf{e}_2^*=\frac{9}{16}=0.5625,$$

which in turn yields

$$W_2(\mathbf{e}_2^*,\mathbf{e}_2^*)=\frac{3}{2}=1.5 \quad \textit{and} \quad EW_2^*=\frac{3}{16}=0.1875.$$

Then equation (2) becomes

$$\frac{2}{9\mathbf{e}_1^*} = \frac{1-\dfrac{1}{6\sqrt{\mathbf{e}_1^*}}}{3\sqrt{\mathbf{e}_1^*}-0.1875}.$$

This implies

$$\mathbf{e}_1^* = \frac{22+5\sqrt{19}}{72} \approx 0.6082,$$

which in turn yields

$$W_1(\mathbf{e}_1^*,\mathbf{e}_1^*,\mathbf{e}_1^*) \approx 2.3397 \quad and \quad EW_1^* \approx 0.2967.$$

The first observation is that $W_1^* > W_2^*$ and $\mathbf{e}_1^* > \mathbf{e}_2^*$ in the equilibrium. This indicates that the contest is more intense in the first period compared to the second period. Another observation is that if these endogenously determined prizes are implemented directly into the baseline model as exogenous prizes, the respective equilibrium efforts would be

$$e_i^{base} = 0.4366 \quad and \quad e_j^{base} = 0.375$$

for any players $i, j \in N$, yielding the following expected values:

$$EW_1^{base} = 0.5933 \quad and \quad EW_2^{base} = 0.375.$$

Since $\mathbf{e}_1^* > e_i^{base}$ and $\mathbf{e}_2^* > e_{ij}^{base}$, we conclude that job seekers try harder for the job in this alternative model with endogenous prizes. Accordingly, if an employer prefers to see higher total efforts, this alternative model would be a better option. On the other hand, if contest efforts are considered social waste (as in the standard rent-seeking contests), then the socially efficient model is the baseline model with exogenous prizes.

The endogenous wage functions above are such that each player would receive the same wage amount in case of winning even when players choose asymmetric effort levels. One can also consider a case where a player's wage is a function of his/her own effort plus a joint component. This way, when there is asymmetry between players' contest efforts, each player would be offered a different wage level in case of winning.[5] Notice that the same analysis would apply for the symmetric equilibrium (as in Proposition 2). Let

$$W_{1,i}(e_1,e_2,e_3) = \sqrt{e_i} + \sqrt{3(e_1+e_2+e_3)} \quad for\ every \quad i \in N$$

and

$$W_{2,i}(e_{ij},e_{ji}) = \sqrt{e_{ij}} + \sqrt{2(e_{ij}+e_{ji})} \quad for\ every \quad i,j \in N.$$

Given these wage functions, equation (1) becomes

$$\frac{1}{4\mathbf{e}_2^*} = \frac{1-\dfrac{1}{2\sqrt{\mathbf{e}_2^*}}}{3\sqrt{\mathbf{e}_2^*}}.$$

This implies

$$\mathbf{e}_2^* = \frac{25}{16} = 1.5625,$$

which in turn yields

$$W_{2,i}(\mathbf{e}_2^*,\mathbf{e}_2^*) = \frac{15}{4} = 3.75 \quad and \quad EW_2^* = \frac{5}{16} = 0.3125.$$

Then equation (2) becomes

$$\frac{2}{9\mathbf{e}_1^*} = \frac{1-\dfrac{1}{3\sqrt{\mathbf{e}_1^*}}}{4\sqrt{\mathbf{e}_1^*}-0.3125}.$$

This implies

$$\mathbf{e}_1^* = \frac{439+2\ \sqrt{394}}{648} \approx 1.3514,$$

which in turn yields

$$W_{1,i}(\mathbf{e}_1^*,\mathbf{e}_1^*,\mathbf{e}_1^*) \approx 4.6499 \quad and \quad EW_1^* \approx 0.4069.$$

Once again, we start with the observation that $W_{1,i}^* > W_{2,i}^*$ and $\mathbf{e}_1^* < \mathbf{e}_2^*$ in the equilibrium. Interestingly, under these functions, the contest becomes more intense in the second period compared to the first period. Be that as it may, since all players contribute to endogenous wage in the first period, the equilibrium wage turns out to be higher in the first period. All equilibrium values increase compared to those implied by the previously-considered wage functions. This is due to the additional incentives created by $\sqrt{e_i}$ and $\sqrt{e_{ij}}$ in the wage functions. Another observation is that if these endogenously determined prizes are implemented directly into the baseline model as exogenous prizes, the respective equilibrium efforts would be

$$e_i^{base} = 0.825 \quad and \quad e_{ij}^{base} = 0.9375$$

for any players $i, j \in N$, yielding the following expected values:

$$EW_1^{base} = 1.35 \quad and \quad EW_2^{base} = 0.9375.$$

Seeing that $\mathbf{e}_1^* > e_i^{base}$ and $\mathbf{e}_2^* > e_{ij}^{base}$, also under these endogenous wage functions, we conclude that job seekers try harder for the job in this model with endogenous prizes.

4.2. Infinite-horizon

Here we first present a small extension to our baseline model. Consider a two-period converse tournament such that whichever player wins the contest in the first period is replaced by a symmetric player in the second period. This implies that another three-player contest takes place in the second period. We consider a common wage $W > 0$, which is to be received at every period after being hired. There is also a common discount factor $\delta \in (0,1)$.

From this point onward, let $e_{it} \in [0,\infty)$ denote the contest effort exerted by player $i \in N$ in period $t = 1,2$. Accordingly, the expected payoff function for player $i \in N$ can be written as[6]

$$\frac{e_{i1}}{e_{11}+e_{21}+e_{31}}(W+\delta W)+\delta\frac{e_{j1}+e_{k1}}{e_{11}+e_{21}+e_{31}}\left(\frac{e_{i2}}{e_{12}+e_{22}+e_{32}}W-e_{i2}\right)-e_{i1}$$

where $j,k \in N \setminus \{i\}$. Player i's objective is to maximize this expected payoff function by choosing the effort levels e_{i1} and e_{i2}. Below we analyze subgame perfect Nash equilibrium of this model.

Proposition 3 *Consider the two-period three-player converse tournament with replacement described above. The unique subgame perfect Nash equilibrium suggests that each player exerts* $(18+16\delta)W/81$ *in the first period; and the losers exert* $2W/9$ *in the second period. The expected payoff for each player is*

$$\frac{(9+17\delta)W}{81}.$$

Proof. See the Appendix.

The equilibrium efforts in both periods increase in W. Since the discount factor positively affects the wage in the first period, the respective equilibrium effort increases also in δ. Moreover, in any case, the contest is more intense in the first period compared to the second period. Using the insights obtained from the baseline model, we can state that this result is caused by wage being higher in the first period.

A direct comparison to the baseline model does not seem possible. Accordingly, we concentrate on the equilibrium efforts relative to the *prize spread* (i.e., winning payoff minus the expected payoff from losing):

$$(W+\delta W)-\frac{\delta W}{9}=\frac{(9+8\delta)W}{9}.$$

Interestingly, the equilibrium effort corresponds to the $2/9$ th of the prize spread, which is the same in the baseline model.

This type of modeling allows us to convert the baseline model into an infinite-horizon model. We assume that if some player wins the contest in period $t \in \mathbb{N}$, then in the following period $t+1$, she is replaced by another symmetric player. Accordingly, we obtain a converse tournament with a three-player contest taking place at each period $t \in \mathbb{N}$. The common wage and common discount factor assumptions are preserved as above.

Similarly, let $e_{it} \in [0,\infty)$ denote the contest effort exerted by player $i \in N$ in period $t \in \mathbb{N}$. Accordingly, the expected payoff function for player $i \in N$ can be written as

$$V_{i,t}(\cdot,\cdot,\cdot)=\frac{e_{it}}{e_{1t}+e_{2t}+e_{3t}}\frac{W}{1-\delta}+\delta\frac{e_{jt}+e_{kt}}{e_{1t}+e_{2t}+e_{3t}}V_{i,t+1}(\cdot,\cdot,\cdot)-e_{it}$$

where $j,k \in N \setminus \{i\}$. Player i's objective is to maximize this expected payoff function by choosing the effort levels e_{it} for every $t \in \mathbb{N}$. Below we analyze subgame perfect Nash equilibrium of this model. Along this line, an important observation is that if a player could not win the contest at period t, then for that player, the subgame starting at period $t+1$ is exactly the same with the subgame started at period t.

Proposition 4 *Consider the infinite-horizon three-player converse tournament described above. The unique symmetric subgame perfect Nash equilibrium suggests that each player exerts* $2W/(9-8\delta)$ *at each period. The expected payoff for each player is*

$$\frac{W}{(1-\delta)(9-8\delta)}.$$

Proof. See the Appendix.

The equilibrium effort is increasing in W and δ. The response to a change in the discount factor seems counterintuitive, as it indicates that when job seekers become more patient, they play more aggressively. The reason is that the winning payoff $W/(1-\delta)$ is increasing in δ, creating extra incentives to be aggressive. On the other hand, if the discount factor increases in such a way that $W/(1-\delta)$ remains constant (i.e., wage is accordingly adjusted), such extra incentives would disappear, and we would observe a decrease in the equilibrium effort. The reason is that, now, a rise in the discount factor only increases the expected winning prize in later periods, which indicates an improvement on the outside option of players.

Finally, the *prize spread* in this model can be calculated as

$$\frac{W}{1-\delta}-\frac{\delta W}{(1-\delta)(9-8\delta)}=\frac{9W}{9-8\delta}.$$

The equilibrium effort corresponds to the $2/9$ th of the prize spread, which is the same in the baseline model.

5. Conclusion

In this paper, we have started with the investigation of a *converse tournament* originally suggested by Clark and Riis (1998b). Then, we have extended the model in two separate dimensions. After analyzing subgame perfect Nash equilibrium of these alternative models, we compared the equilibrium strategies with those in the baseline model.

For our alternative models, we have provided an interesting job search interpretation. A possible future research direction would be to embed these game-theoretic models into the existing job search models in labor economics.[7] In that regard, generalizations to n players and k_i prizes seem necessary. We believe that generalization to n players is rather straightforward, but generalizations to k_i prizes would be challenging.

Endnotes

[1]For detailed investigations of the works in the contest literature, see Corchón (2007); Konrad (2009); Dechenaux et al. (2015) among others.

[2]See also Berry (1993); Clark and Riis (1996); Barut and Kovenock (1998); among others. Furthermore, for "reverse" nested lottery contests, the interested reader is referred to Chowdhury and Kim (2014) and Fu et al. (2014).

[3]Arguably, $W_1 > W_2$ since the winner in the first period will be employed for two periods. However, as we do not impose any restrictions on W_1 or W_2, our model allows for $W_1 < W_2$ as well.

[4]In case the denominator is zero, each player has an equal chance of winning.

[5]We thank an anonymous reviewer for bringing this to our attention.

[6]For expositional simplicity, we use the same index for the winner and his/her replacement.

[7]Another modeling approach would be to assume that contest effort is a stock variable such that when a player exerts some effort this period, it increases his/her chances of getting hired next period, while its effect depreciates over time. Such an assumption might be particularly relevant in certain contexts.

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Appendix

Proof of Proposition 1: We analyze subga me perfect Nash equilibrium via backward induction. Consider the second period. Assume that some player $k \in N$ won the contest in the first period. Then, the contest in the second period is between players $i, j \in N \setminus \{k\}$. Player i maximizes

$$\frac{e_{ij}}{e_{ij}+e_{ji}} W_2 - e_{ij}.$$

The first order condition with respect to e_{ij} is

$$\frac{e_{ji}}{(e_{ij}+e_{ji})^2} W_2 - 1 = 0.$$

Considering the symmetric first order condition for player j, we have $e_{ij}^* = e_{ji}^*$ at the equilibrium. Then, the first order condition above leads to

$$e_{ij}^* = e_{ji}^* = \frac{W_2}{4};$$

so that the expected payoff for both players are $W_2/4$.

Anticipating this, in the first period, any player $i \in N$ maximizes

$$\frac{e_i}{e_1+e_2+e_3} W_1 + \frac{e_j+e_k}{e_1+e_2+e_3}\frac{W_2}{4} - e_i$$

where $j, k \in N \setminus \{i\}$. The first order condition with respect to e_i is

$$\frac{e_j+e_k}{(e_1+e_2+e_3)^2} W_1 - \frac{e_j+e_k}{(e_1+e_2+e_3)^2}\frac{W_2}{4} - 1 = 0$$

Since the first order conditions are symmetric for all three players, we have $e_1^* = e_2^* = e_3^*$ at the equilibrium. Then, the first order condition above leads to

$$\frac{2}{9e_i^*} W_1 - \frac{1}{9e_i^*}\frac{W_2}{2} = 1$$

$$e_i^* = \frac{4W_1 - W_2}{18}.$$

The strategy e_i^* is realized as the equilibrium effort only if $4W_1 \geq W_2$. On the other hand, if $4W_1 < W_2$ for some reason, then we would observe $e_1^* = e_2^* = e_3^* = 0$ at the equilibrium. Accordingly, the expected payoff for each player can be written as

$$\begin{array}{lll} \dfrac{W_1+2W_2}{9} & \text{if} & 4W_1 \geq W_2 \\ \dfrac{2W_1+W_2}{6} & \text{if} & 4W_1 < W_2. \end{array}$$

Proof of Proposition 2: We analyze subgame perfect Nash equilibrium via backward induction. Consider the second period. Assume that some player $k \in N$ won the contest in the first period. Then, the contest in the second period is between players $i, j \in N \setminus \{k\}$. Player i maximizes

$$\frac{e_{ij}}{e_{ij}+e_{ji}} W_2(e_{ij}, e_{ji}) - e_{ij}.$$

The first order condition with respect to e_{ij} is

$$\frac{e_{ji}}{(e_{ij}+e_{ji})^2} W_2(e_{ij}, e_{ji}) + \frac{\partial W_2(e_{ij}, e_{ji})}{\partial e_{ij}}\frac{e_{ij}}{e_{ij}+e_{ji}} - 1 = 0.$$

Considering the symmetric first order condition for player j, we have $\mathbf{e}_2^* \equiv e_{ij}^* = e_{ji}^*$ at the equilibrium. Then the first order condition above leads to

$$\frac{1}{4\mathbf{e}_2^*} = \frac{1 - \frac{1}{2}\frac{\partial W_2(e_{ij}, e_{ji})}{\partial e_{ij}}\Big|_{(\mathbf{e}_2^*, \mathbf{e}_2^*)}}{W_2(\mathbf{e}_2^*, \mathbf{e}_2^*)}. \quad (1)$$

The symmetric equilibrium strategy $\mathbf{e}_2^*$ is implicitly characterized by equation (1) above. Accordingly, the symmetric expected payoff from this period can be calculated:

$$EW_2^* = \frac{1}{2}W_2(\mathbf{e}_2^*, \mathbf{e}_2^*) - \mathbf{e}_2^*.$$

Anticipating this, in the first period, any player $i \in N$ maximizes

$$\frac{e_i}{e_1 + e_2 + e_3} W_1(e_1, e_2, e_3) + \frac{e_j + e_k}{e_1 + e_2 + e_3} EW_2^* - e_i$$

where $j, k \in N \setminus \{i\}$. The first order condition with respect to e_i is

$$\frac{e_j + e_k}{(e_1 + e_2 + e_3)^2} W_1(e_1, e_2, e_3) + \frac{\partial W_1(e_1, e_2, e_3)}{\partial e_i} \frac{e_i}{e_1 + e_2 + e_3} - \frac{e_j + e_k}{(e_1 + e_2 + e_3)^2} EW_2^* - 1 = 0.$$

Since the first order conditions are symmetric for all three players, we have $\mathbf{e}_1^* \equiv e_1^* = e_2^* = e_3^*$ at the equilibrium. Then, the first order condition above leads to

$$\frac{2}{9\mathbf{e}_1^*} = \frac{1 - \frac{1}{3}\frac{\partial W_1(e_1, e_2, e_3)}{\partial e_i}\Big|_{(\mathbf{e}_1^*, \mathbf{e}_1^*, \mathbf{e}_1^*)}}{W_1(\mathbf{e}_1^*, \mathbf{e}_1^*, \mathbf{e}_1^*) - EW_2^*}. \quad (2)$$

The symmetric equilibrium strategy $\mathbf{e}_1^*$ is implicitly characterized by equation (2) above. For the sake of completeness, the symmetric expected payoff from the whole game can be calculated as:

$$EW_1^* = \frac{1}{3}W_1(\mathbf{e}_1^*, \mathbf{e}_1^*, \mathbf{e}_1^*) + \frac{2}{3}\left(\frac{1}{2}W_2(\mathbf{e}_2^*, \mathbf{e}_2^*) - \mathbf{e}_2^*\right) - \mathbf{e}_1^*.$$

Proof of Proposition 3: We analyze subgame perfect Nash equilibrium via backward induction. Consider the second period. Player $i \in N$ maximizes

$$\frac{e_{i2}}{e_{12} + e_{22} + e_{32}} W - e_{i2}.$$

The first order condition with respect to e_{i2} is

$$\frac{e_{j2} + e_{k2}}{(e_{12} + e_{22} + e_{32})^2} W - 1 = 0$$

where $j, k \in N \setminus \{i\}$. Since the first order conditions are symmetric for all three players, we have $e_{12}^* = e_{22}^* = e_{32}^*$ at the equilibrium. Then, the first order condition above leads to

$$e_{i2}^* = \frac{2W}{9};$$

so that the expected payoff for each player is $W/9$.

Anticipating this, in the first period, any player $i \in N$ maximizes

$$\frac{e_{i1}}{e_{11} + e_{21} + e_{31}}(W + \delta W) + \frac{e_{j1} + e_{k1}}{e_{11} + e_{21} + e_{31}} \frac{\delta W}{9} - e_{i1}$$

where $j, k \in N \setminus \{i\}$. The first order condition with respect to e_{i1} is

$$\frac{e_{j1} + e_{k1}}{(e_{11} + e_{21} + e_{31})^2}(W + \delta W) - \frac{e_{j1} + e_{k1}}{(e_{11} + e_{21} + e_{31})^2} \frac{\delta W}{9} - 1 = 0.$$

Since the first order conditions are symmetric for all three players, we have $e_{11}^* = e_{21}^* = e_{31}^*$ at the equilibrium. Then, the first order condition above leads to

$$\frac{2}{9e_{i1}^*}(W + \delta W) - \frac{2}{9e_{i1}^*}\frac{\delta W}{9} = 1$$

$$e_{i1}^* = \frac{(18 + 16\delta)W}{81}.$$

Accordingly, the expected payoff for each player can be written as

$$\frac{(9 + 17\delta)W}{81}.$$

Proof of Proposition 4: Consider a generic period $t \in \mathbb{N}$. Take any symmetric subgame perfect Nash equilibrium, and let EW_t^* denote the respective expected earning for each player at the generic period. Player $i \in N$ maximizes

$$\frac{e_{it}}{e_{1t} + e_{2t} + e_{3t}} \frac{W}{1 - \delta} + \frac{e_{jt} + e_{kt}}{e_{1t} + e_{2t} + e_{3t}} \delta EW_t^* - e_{it}.$$

where $j, k \in N \setminus \{i\}$. The first order condition with respect to e_{it} is

$$\frac{e_{jt} + e_{kt}}{(e_{1t} + e_{2t} + e_{3t})^2} \frac{W}{1 - \delta} - \frac{e_{jt} + e_{kt}}{(e_{1t} + e_{2t} + e_{3t})^2} \delta EW_t^* - 1 = 0.$$

At a symmetric equilibrium in which $e_{1t}^* = e_{2t}^* = e_{3t}^*$, this equation reduces to

$$\frac{2}{9e_{it}^{*}}\left(\frac{W}{1-\delta}-\delta EW_{t}^{*}\right)-1=0,$$

which in turn yields

$$e_{it}^{*}=\frac{2}{9}\left(\frac{W}{1-\delta}-\delta EW_{t}^{*}\right).$$

Given this equilibrium strategy, the expected earning can be re-written as

$$EW_{t}^{*}=\frac{1}{3}\frac{W}{1-\delta}+\frac{2}{3}\delta EW_{t}^{*}-\frac{2}{9}\left(\frac{W}{1-\delta}-\delta EW_{t}^{*}\right)=\frac{1}{9}\frac{W}{1-\delta}+\frac{8\delta}{9}EW_{t}^{*}$$

Accordingly,

$$EW_{t}^{*}=\frac{W}{(1-\delta)(9-8\delta)}\quad and \quad e_{it}^{*}=\frac{2W}{9-8\delta}.$$