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Optimum Operation of a Thermal Plant with Cogeneration and Heat Pumps*

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Abstract

In this paper, a complex thermal plant is analyzed. The plant under analysis is basically made up of four gas-fueled engines with heat recovery. Each engine can drive, simultaneously, an electric generator and the compressor of a heat-pump/chiller. The plant is interconnected to the electric utility grid, both to receive additional power and to deliver excess power. In addition, each heat-pump/chiller can be driven electrically, using the electric generator as a motor. For any given amount of power and heat required by the users, a large number of operating conditions are possible. So, the problem arises of selecting, at any given time, the operational mode that involves the lowest instantaneous cost of operating the system. This cost can be regarded as the objective function to be minimized in a typical constrained optimization problem. In the paper, after a short description of the plant, this problem is dealt with. Some examples of the results provided by the optimization algorithm are presented and commented on. The economic savings achievable through an optimized operation are highlighted too.

Key words: cogeneration; heat pump; cost minimization.

1. Introduction

When operating a complex thermal plant, it can be very important to define, for any possible load condition, the operational mode that minimizes the overall economic cost of the energy delivered by the system. In general, for complex and flexible plants, many different configurations are available to fulfill a given set of energy demands. Therefore, the problem arises in selecting, at any given time, the operational mode that implies the lowest instantaneous cost of operating the system. This can be regarded as a typical thermoeconomic optimization problem. The operation cost represents the objective function; the constraints are given, in general, by energy and entropy balance equations and by relations that express all of the technical conditions that must be fulfilled so that all devices operate correctly (El-Sayed et al., 1970; Frangopoulos, 1983, 1980a and 1980b). From the point of view of thermodynamics, the analysis of a thermal system can be performed in terms of properties relevant to first-law analysis (energy) and/or second-law analysis (exergy). Since in this specific case in point no cost accounting is required for any physical flow but those purchased or sold at prices fixed by the market, the choice between first- and second-law based variables is not significant. Therefore, a first-law approach has been followed here, due to its simplicity. However, it is useful to remember that in other problems, such as product costing, this choice would be more important (Tsatsaronis, 1987; Bejan et al., 1996). In this paper, the optimization problem described above is analyzed with reference to a real and very interesting plant built to supply electricity

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and thermal energy to the new campus of the University of Naples, Italy.

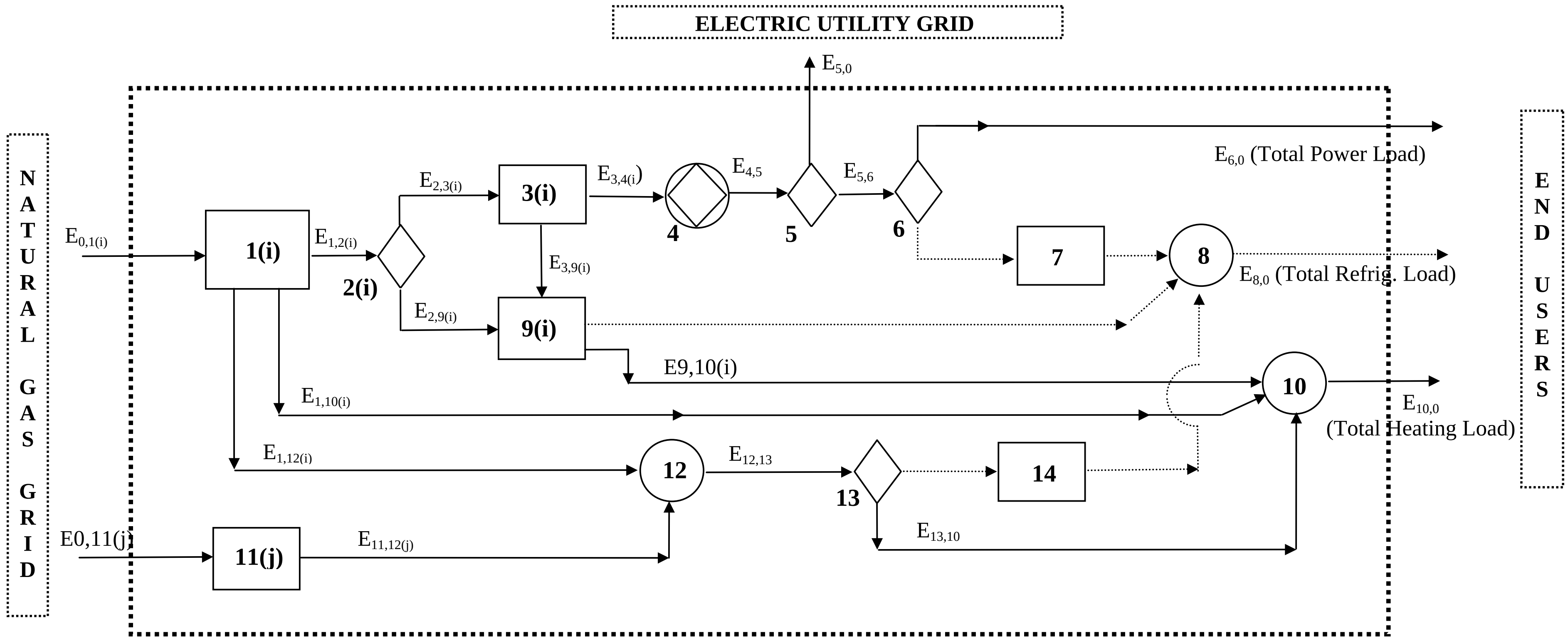
2. Plant Description and Main Assumptions

The plant under study was designed to serve a new complex of buildings of the Federico II University of Naples recently built in Monte Sant'Angelo, Naples, Italy. The overall built volume is about 400,000 m^3, distributed among 8 buildings. The peak energy loads were estimated as follows:

- maximum electric load: about 3.5 MW;
- maximum heating load (winter): about 7.5 MW;
- maximum refrigerating load (summer): about 8.0 MW

A simplified schematic representation of the plant designed to meet these loads is shown in *Figure 1*. All energy flow rates of interest are shown too, with the exception of those dissipated and lost to the environment. Some of the energy flows shown in *Figure 1* can assume a direction opposite to that indicated; in this case, they will be assigned a negative value. Four gas-fueled reciprocating engines (1(i), i = 1 to 4), with heat recovery from jacket water and oil (low-temperature thermal energy) and from exhaust gases (medium temperature thermal energy), constitute the heart of the system. A synchronous electric generator (3(i), i = 1 to 4) and the compressor of a vapor-compression heat-pump/chiller (9(i), i = 1 to 4) are coupled on the shaft of each engine. Thus, at any time, the mechanical work supplied by an engine, $E_{1,2(i)}$, can be distributed between the electric generator and the compressor in variable and complementary rates, depending on the amount of heat and power required by the users and the operation strategy selected for the system. However, in one case (i = 4), the vapor-compression system can operate as a chiller only. In *Figure 1*, to save space, only one of the four groups, each consisting of gas engine, heat pump and electric generator, is shown. The same is done for the auxiliary boilers, 11(1) and 11(2). The plant is interconnected to the electric utility grid, in order to buy additional power ($E_{5,0} < 0$) or to deliver excess power ($E_{5,0} > 0$) whenever it is necessary. When the cost of purchasing electric energy from the utility grid is low, due to the time-dependency of electric power rates, each heat-pump/chiller can be driven electrically, using the electric generator as a synchronous motor ($E_{3,4(i)} < 0$ and $E_{3,9(i)} > 0$). In this case, the corresponding engine will be shut down and disconnected at 2(i) from the electric generator and compressor. Therefore, the fictitious point 4 can be either a junction point and/or a brunching point, depending on the actual direction of the flow $E_{4,5}$. In the cooling season, a single-stage absorption chiller, device 14 in *Figure 1*, can be fed by medium temperature thermal energy obtained from auxiliary boilers and/or high temperature thermal energy recovered from the engines. In addition, an electrically driven chiller (unit 7) is available too. The characteristics of the plant are summarized in TABLE I. It should be noted that the system is described at a rather high "aggregation level", that is, with a low level of resolution: many subsystems, which could be regarded as single components in a more detailed representation, are grouped here into a unique set of devices. Similarly, at this level of resolution, many operational variables that affect the performance of the system, for example, the temperature at which heated or refrigerated water are supplied, will not be taken into account. This is due to the lack of information available at the moment to model and simulate the performance of the plant. In fact, it would be clearly useless to develop a complex and detailed model when little data is available to evaluate the model. For this reason, the statement and the solution of the optimization problem, more than the detailed simulation of the plant, are the focus of the following. At this first level, the performance curves provided for each component by the manufacturer are sufficient in order to approximately describe the behavior of the plant at various load conditions. As soon as more data are available about the real performance of the plant, a more detailed model will be developed. Then it will be possible to up-grade the present study introducing into the problem all of the variables that actually affect the performance of the system, with no significant complications to the calculations and the approach used here. The most important assumptions on which our analysis is based are summarized below. For brevity, only configurations related to the heating season are considered.

- All devices operate under steady-state conditions.
- The performance of each engine is described in terms of thermomechanical efficiency (η_m, ratio of mechanical work to primary energy input, Eq. (1)) and thermal efficiency (η_t, ratio of thermal energy recovered to primary energy input, Eq. (2). These efficiencies are considered to depend only on the percentage load of the engine, $F_{1(i)}$, which is the ratio of the actual mechanical work flow rate $E_{1,2(i)}$ to the maximum value of 480 kW, given by the partial-load performance curves (Eqs. (3) to (5)). A minimum value of 50% is assumed as the lower limit for the percentage load of each engine, when it is turned on (Eq. (6)). For i = 1 to 4, Eqs. (1) to (6) are written as:



LEGEND

Device

Fictitious branching point

Fictitious junction point

Energy Flow Rates, Winter

Energy Flow Rates, Summer

List of devices

1(i) = Gas-Fueled Reciprocating Engine, i = 1 to 4

3(i) = Electric Generator / Motor, i = 1 to 4

7 = Electric Vapor Compression Chiller

9(i) = Vapor Compression Heat Pump / Chiller, i = 1 to 4 (i = 4 => Chiller Only)

11(j) = Gas-Fueled Auxiliary Boiler, j = 1 to 2

14 = Absorption Chiller

List of main energy flow rates (winter)

$E_{0,1}$, $E_{0,11}$ = primary energy (natural gas)

$E_{1,2}$, $E_{2,3}$, $E_{2,9}$, $E_{3,9}$ = mechanical energy

$E_{3,4}$, $E_{4,5}$, $E_{5,6}$, $E_{5,0}$, $E_{6,0}$ = electrical energy

$E_{9,10}$ = thermal energy from the heat pumps

$E_{1,10}$, $E_{1,12}$ = thermal energy from heat recovery (low and medium temperature)

$E_{11,12}$ = thermal energy from the auxiliary boilers

$E_{12,13}$, $E_{13,10}$, $E_{10,0}$ = thermal energy

Figure 1. Simplified schematic representation of the plant under consideration

TABLE I. MAIN COMPONENTS OF THE MONTE SANT'ANGELO THERMAL PLANT.

Device	*Number of units*	*Main Characteristics*
Reciprocating Gas Engine	4	- max mechanical output = 480 kW - max thermal output = 600 kW - nominal thermomechanical efficiency = 0.378
Synchronous Electric Gen./Motor	4	- max electric output = 448 kW - max mechanical output = 400 kW
Vapor compression heat-pump/chiller (gas-engine driven or electric driven)	3	- winter: air to water heat pump -summer: water to water chiller - max thermal output (7 °C) = 1380 kW - max work input (7 °C) = 382 kW - max refr. output (32 °C) = 2050 kW - max work input (32 °C) = 395 kW
Vapor comp. chiller (gas-engine driven or electric driven)	1	- water to water - max refr. output (32 °C) = 2050 kW - max work input (32 °C) = 395 kW
Electric vapor Comp. chiller	1	- water to water - max refr. output (32 °C) = 2030 kW - max electric input (32 °C) = 450 kW
Single-stage absorption chiller	1	- max. refr. output (32 °C) = 650 kW - max. thermal input (water at 110 °C) = 1110 kW
Gas fueled aux. boiler	2	- max thermal output = 2300 kW - nominal efficiency = 0.90

$$\eta_{m,1(i)} = E_{1,2(i)} / E_{0,1(i)} \quad (1)$$

$$\eta_{t,1(i)} = (E_{1,10(i)} + E_{1,12(i)}) / E_{0,1(i)} \quad (2)$$

$$0 \le F_{1(i)} = E_{1,2(i)} / 480 \le 1.0 \quad (3)$$

$$\eta_{m,1(i)} = 0.255 + 0.211 \times F_{1(i)} - 0.088 \times F_{1(i)}^{2} \quad (4)$$

$$\eta_{t,1(i)} = 0.626 - 0.208 \times F_{1(i)} + 0.48 \times F_{1(i)}^{2} \quad (5)$$

$$F_{1(i)} > 0 \Rightarrow 0.50 \le F_{1(i)} \le 1.0 \quad (6)$$

-The performance of each vapor-compression heat pump is described in terms of its maximum thermal output, $E^*_{9,10(i)}$, and the corresponding maximum mechanical work required by the compressor, $E^*_{2,9(i)}$, both considered as depending on the outdoor temperature, T_{out} (Eqs. (7), (7b) and (8)).

However, it is assumed that the actual thermal output $E_{9,10(i)}$ depends on the mechanical work flow rate provided by the engine, $E_{2,9(i)}$, or by the electric motor, $E_{3,9(i)}$, given by the partial-load performance curve (Eq. (11)) for these devices. A minimum value of 58% is assumed as the lower limit for the percentage heat-pump load, $F_{9(i)}$ (Eq. (12)). Thus, for i = 1 to 3, we have that:

$$T_{out} < 7.0\ °C \Rightarrow E^*_{9,10(i)} = 838.67 + 58.25 \times T_{out} + 72.63 \times T_{out}^{2} \quad (7)$$

$$T_{out} \ge 7.0\ °C \Rightarrow E^*_{9,10(i)} = 929.49 + 70.04 \times T_{out} - 0.826 \times T_{out}^{2} \quad (7b)$$

$$E^*_{2,9(i)} = E^*_{3,9(i)} = 296.22 + 15.31 \times T_{out} + 0.436 \times T_{out}^{2} \quad (8)$$

$$E_{2,9(i)} > 0 \Rightarrow E_{3,9(i)} = 0 \quad (9)$$

$$E_{3,9(i)} > 0 \Rightarrow E_{2,9(i)} = 0 \quad (9b)$$

$$E_{2,9(i)} > 0 \Rightarrow 0 \le F_{9(i)} = (E_{2,9(i)} / E^*_{2,9(i)}) \le 1.0 \quad (10)$$

$$E_{3,9(i)} > 0 \Rightarrow 0 \le F_{9(i)} = (E_{3,9(i)} / E^*_{3,9(i)}) \le 1.0 \quad (10b)$$

$$E_{9,10(i)} = (-1.034 + 2.773 \times F_{9(i)} - 0.7429 \times F_{9(i)}^{2}) \times E^*_{9,10(i)} \quad (11)$$

$$F_{9(i)} > 0 \Rightarrow 0.58 \le F_{9(i)} \le 1.0 \quad (12)$$

-When a heat pump is driven electrically using the electric generator as a synchronous motor ($E_{3,4(i)} < 0$ and $E_{3,9(i)} > 0$), the corresponding engine is shut down and disconnected from its coupling with the electric motor and compressor. So, for i = 1 to 3, we have that:

$$E_{3,9(i)} > 0 \Rightarrow E_{3,4(i)} < 0 \text{ and } E_{1,2(i)} = 0 \quad (13)$$

-The performance of each auxiliary boiler is described in terms of first-law efficiency, $\eta_{11(j)}$, which depends on the amount of thermal energy actually supplied with respect to the maximum of 2300 kW, given by the partial-load performance curve of the boilers. A minimum value of 30% is assumed as the lower limit for the percentage load of each boiler when in operation. Thus, for j = 1 to 2:

$\eta_{11(j)} = E_{11,12(j)} / E_{0,11(j)}$ (14)

$0 \leq F_{11(j)} = (E_{11,12(j)} / 2300) \leq 1.0$ (15)

$E_{11,12(j)} > 0 \Rightarrow 0.30 \leq F_{11(j)} \leq 1.0$ (16)

$\eta_{11(j)} = (0.0951 + 1.525\times F_{11(j)} - 0.6249 \times F_{11(j)}{}^{2}) \times 0.90$ (17)

-For any branching or junction point a balance equation can be written such that for point 2(i), for i = 1 to 3:

$E_{1,2(i)} = E_{2,3(i)} + E_{2,9(i)} \Leftrightarrow 0 \leq Z_{2,3(i)} = (E_{2,3(i)}/E_{1,2(i)}) \leq 1.0$ (18)

point 4: $\sum_{i=1}^{4} E_{3,4(i)} = E_{4,5}$ (19)

point 5: $E_{4,5} = E_{5,0} + E_{5,6}$ (20)

point 6 (winter): $E_{5,6} = E_{6,0}$ (21)

point 10: $E_{13,10} + \sum_{i=1}^{4} (E_{1,10(i)} + E_{9,10(i)}) = E_{10,0}$ (22)

point 12: $\sum_{i=1}^{4} E_{1,12(i)} + \sum_{j=1}^{2} E_{11,12(j)} = E_{12,13}$ (23)

point 13 (winter): $E_{12,13} = E_{13,10}$ (24)

-Unit 9(4) can only operate as a chiller. Therefore, in the heating season, it is:

$Z_{2,3(4)} = E_{2,3(4)} / E_{1,2(4)} > 0 \Rightarrow Z_{2,3(4)} = 1.0$ (25)

-A constant overall electrical/mechanical efficiency of 0.95 is assumed for each electric generator/motor. Thus, for i = 1 to 4, we have that:

$E_{3,4(i)} = 0.95 \times E_{2,3(i)}$ (26)

-The electric consumption of all auxiliary devices is included in the consumption of its corresponding principal equipment.

-Additional electric energy can be purchased from the utility grid at a known unit price, $c_{E,\vartheta}$, that includes taxes and depends on the time, ϑ, at which it is purchased and the time-dependent structure of national electric-power rates. According to these rates' structure, the hours of a year are divided into four categories, varying from ϑ = H1 (peak-load hours) to ϑ = H4 (minimum-load hours). If the power required is greater than the maximum value fixed by contract, $E^*_{E,\vartheta}$, at time ϑ, an economic penalty, C_P, must be added to the cost of the electricity. This penalty depends on ϑ and is given in Eq. (28). In addition, it is assumed that the additional power required from the grid must not exceed the value fixed by contract by more than 25%, provided that this is possible through a proper selection of plant configuration. This hypothesis is in agreement with the rules of the national electric utility. Finally, a tax for consumption of self-produced electric energy, $t_{E,SC}$, must be considered. Therefore, the overall cost, C_E, of purchasing the electric energy $(E_{6,0} - E_{5,0})$ from the utility grid and internally consuming the remaining part $E_{5,0}$, is as follows:

$(E_{6,0}-E_{5,0}) \geq 0$ and $E_{5,0} \geq 0 \Rightarrow C_E = c_{E,\vartheta}\times(E_{6,0} - E_{5,0}) + t_{E,SC}\times E_{5,0} + C_P$ (27)

$(E_{6,0}-E_{5,0}) \geq 0$ and $E_{5,0} < 0 \Rightarrow C_E = c_{E,\vartheta} \times (E_{6,0} - E_{5,0}) + C_P$ (27b)

where $(E_{6,0} - E_{5,0}) > E^*_{E,\vartheta} \Rightarrow C_P = 3 \times c_{P,\vartheta} \times (E_{6,0} - E_{5,0})$ (28)

and $(E_{6,0} - E_{5,0}) \leq E^*_{E,\vartheta} \Rightarrow C_P = 0$ (28b)

-When the electric power supplied by the system is greater than that required by the users, including possible electricity consumption by the heat pumps, the power in excess can be sold to the public utility at a price, $p_{E,\vartheta}$, depending on the time ϑ at which it is delivered to the grid. In this case, the corresponding cost may become negative, that is, it may represent an income, i.e.:

$(E_{6,0} - E_{5,0}) < 0 \Rightarrow C_E = p_{E,\vartheta} \times (E_{6,0} - E_{5,0}) + t_{E,SC} \times E_{6,0}$ (29)

-A fixed unit cost, c_G, is assumed for natural gas. This cost does not include unit taxes, t_G. The latter must consequently be added to c_G for every kJ of gas consumed, with the exception of 2.4 kJ of gas for each kJ of electric energy produced by the plant, which are tax free, according to a national law for the promotion of gas-fired cogeneration power plants. Therefore, the overall cost due to gas consumption is:

$$C_G = c_G\times(\sum_{i=1}^{4} E_{0,1(i)} + \sum_{j=1}^{2} E_{0,11(j)}) + t_G\times[\sum_{i=1}^{4} E_{0,1(i)} + \sum_{j=1}^{2} E_{0,11(j)} - 2.4\times E_{4,5}] \quad (30)$$

-The following control variables are used to minimize, at each given time, the cost of operating the system. In other words, they are assumed as the independent decision variables in the optimization problem to be solved:

* percentage load of each engine, $F_{1(i)}$;

* distribution factors at the branching point 2(i), defined as:

$$Z_{2,3(i)} = E_{2,3(i)} / E_{1,2(i)} \quad (31)$$

* percentage load of each heat pump when electrically driven, $F_{9(i)}$;
* percentage load of each auxiliary boiler, $F_{11(j)}$.

When the corresponding gas engine drives a heat pump, its load factor $F_{9(i)}$ is not independent of $Z_{2,3(i)}$, so it can be regarded as a dependent variable.

3. Optimal Operating Modes

3.1 Statement of the optimization problem

As stated above, our aim is to minimize the instantaneous cost of operating the plant with respect to the following independent decision variables:

- $F_{1(i)}$, i = 1 to 4;
- $Z_{2,3(i)}$, i = 1 to 4;
- $F_{9(i)}$, i = 1 to 3, when $E_{3,9(i)} > 0$ (heat pump electrically driven);
- $F_{11(j)}$, j = 1 to 2.

All these quantities vary in the range from 0 to 1.0, subject to the limitations stated above. Referring to the heating season only, the energy needs of the whole complex at a given time will be characterized by just two values:

- the overall power load, $E_{6,0}$, not including the possible power required to electrically drive some of the heat pumps;
- the overall thermal load, $E_{10,0}$.

The values of these two variables are obviously assigned externally. Therefore, they represent two fixed parameters in the optimization. Thus, the optimization problem consists of minimizing the following objective function for given values of $E_{6,0}$, $E_{10,0}$, ϑ and T_{out}:

$$\text{minimize } OF = C_G + C_E = f(E_{6,0}, E_{10,0}, T_{out}, \vartheta, \underline{X}) \quad (32)$$

with respect to:

$$\underline{X} = [F_{1(1)}, \ldots F_{1(4)}, Z_{2,3(1)}, \ldots Z_{2,3(4)}, F_{9(1)}, \ldots F_{9(3)}, F_{1,1(1)}, F_{1,1(2)}] \quad (33)$$

subject to a set of equality and inequality constraints.

$\underline{X}$ is the vector of the independent variables. C_G and C_E are the overall instantaneous costs of purchasing gas and electricity, respectively, as defined in Eqs. (27) to (30). The constraints are represented by all the equations and logical expressions numbered from 1 to 31.

3.2 Solving algorithm

Due to the strong non-linearity of the objective function and the constraints, the problem cannot be solved analytically. Therefore, a heuristic combinatorial method, with adaptive range reduction, has been selected as the simplest solution algorithm in spite of its low rate of convergence (Reklaitis et al. 1983, Kuester et al. 1973). In a first step, a randomized selection of values of the independent variables is generated. In this phase, dividing its range of variation using a step size of 0.10 discretizes each independent variable. This search attempts to differentiate the types of configurations able to lead to an optimum operating mode. Four different groups of configurations are defined: in the first one, all heat pumps in operation are driven by a gas engine, while in the last one a maximum of three heat pumps can be driven electrically. Each configuration that is not compatible with the energy flows to be supplied and/or with a constraint, is rejected. Once the most promising group of configurations has been selected, it is further investigated in the second step, with a step's size of 0.05. Here, starting from the best point selected in the first phase, each variable is optimized by random search, holding the others fixed. When the last variable has been optimized, the second step re-starts from the new base point, until the termination criterion is met, namely, insufficient improvement in the value of the objective function. All tests conducted to evaluate the optimization algorithm described above have confirmed that it is sufficient to individuate the global optimum, in spite of the rather simplified approach used. Discretization of the independent variables with steps smaller than 0.05 is possible but not of real interest. In fact, even if the plant were supplied with a sophisticated control system, it would not plausibly be possible, to adjust the values of these variables with sensitivities better than 5%. In all tests conducted on a PC with a 333 MHz processor, time for convergence has ranged from a minimum of 10 s to a maximum of 20 s.

3.3 Numerical example

An example of the results obtained for various values of the parameters is provided in *Figures 2* to *5*. Data used for the calculations are summarized in TABLE II. In *Figures 2* to *4*, the optimal values of all independent variables are shown for given values of $E_{10,0}$, $E_{6,0}$, T_{out}, and for each value of ϑ. The values assumed by the boiler-related variables are not shown, for brevity. Labels 1 to 4 refer to the value of index i in the independent variables $F_{1(i)}$, $Z_{2,3(i)}$ and $F_{9(i)}$. Starting with *Figure 2*, it should be noted how the optimal operating mode varies while passing from peak and high-load hours (H1 and H2, respectively) to medium and minimum-load hours (H3 and H4, respectively). In H1 and H2, three gas engines operate at full load ($F_{1(i)}$ = 1.0, i = 2 to 4), and all of them are fully devoted to the production of electric energy ($Z_{2,3(i)}$ = 1.0, i = 2 to 4). A part of the power produced is used by heat pump 9(1), which is driven electrically ($F_{9(1)}$ = 0.90). In H3, the production of electricity is not advantageous; so two engines are shut down, and the others are used to drive their heat pumps. Finally, in H4, only engine 1(4) is operating, at 50% load, and the electric power supplied is used to cover part of the power load due to the operation of three heat pumps. Similar comments may be made at *Figures 3* and *4* in which greater heating loads are assumed, while the value assumed for the power load is the same as before (1000 kW). Comparing the results of *Figures 2, 3* and *4*, one can observe how, in H1 and H2, the fraction of mechanical work devoted to the heat pumps increases with the need for heat. For the same reason, in H3, one more heat pump - with respect to the case shown in *Figure 2* - is turned on when $E_{10,0}$ is equal to 5000 and 7000 kW. For $E_{10,0}$ = 5000 kW, the additional heat pump is operated electrically, whereas, in the case of *Figure 4*, the operation using a gas engine is preferred, resulting in a greater availability of thermal energy due to heat recovery. The same can be said about the minimum-load hours, H4. In *Figure 5*, the minimum values of the objective function are shown as a function of the heating load, for fixed values of the power load and outdoor temperature. In the same figure, these values are compared with the economic cost calculated for a conventional thermal plant, with electric energy supplied by the utility grid. As shown in *Figure 5*, economic savings ranging from 40 to 50% can be achieved.

It is very important to analyze the sensitivity of the objective function to the values of all the variables of influence. A very small sensitivity would make the solution of the optimization problem not very important. In contrast, a too-high sensitivity would lead to the impossibility of reaching, at any time, the optimum plant configuration due to the limited accuracy achievable in measurements of all the physical variables of influence. In regard to this problem, more than 50 cases have been investigated, varying the parameters of the problem, taking into account for each case five configurations in addition to the optimal one. The difference in the values of OF ranged from 2 to more than 50%. The lowest values are always observed when the cost of operating the plant is low (ϑ = H3 and ϑ = H4). Thus, it can be concluded that interesting economic savings are achievable with an optimized operation. Nonetheless, a slight sensitivity was found with respect to the uncertainty of the independent as well as the dependent variables. For example, a maximum error of 1% was evaluated for the optimum value of the objective function for a 5% error in the value of a control variable, holding the others fixed, and a 5% error for a 10% difference in the efficiency of all the engines or heat pumps.

TABLE II. DATA FOR THE NUMERICAL EXAMPLE

	ϑ =*H1*	ϑ =*H2*	ϑ =*H3*	ϑ =*H4*
Cost of electric energy, tax included, $c_{E,\vartheta}$ *(Euro/GJ)*	30.3	24.6	18.6	15.0
Cost of excess power, $c_{P,\vartheta}$ *(Euro/GJ)*	9.1	4.5	1.7	0.44
Price of electric energy sold to grid, $p_{E,\vartheta}$ *(Euro/GJ)*	14.3	14.3	14.3	6.7
Tax for internal consumpt. of el. energy, $t_{E,SC}$ *(Euro/GJ)*	3.4	3.4	3.4	3.4
Max. power available by contract, $E^*_{E,\vartheta}$ *(kW)*	1300	1600	1600	1600
Cost of natural gas (not including taxes), c_G *(Euro/GJ)*	5.74	5.74	5.74	5.74
Tax on natural gas consumption, t_G *(Euro/GJ)*	4.3	4.3	4.3	4.3

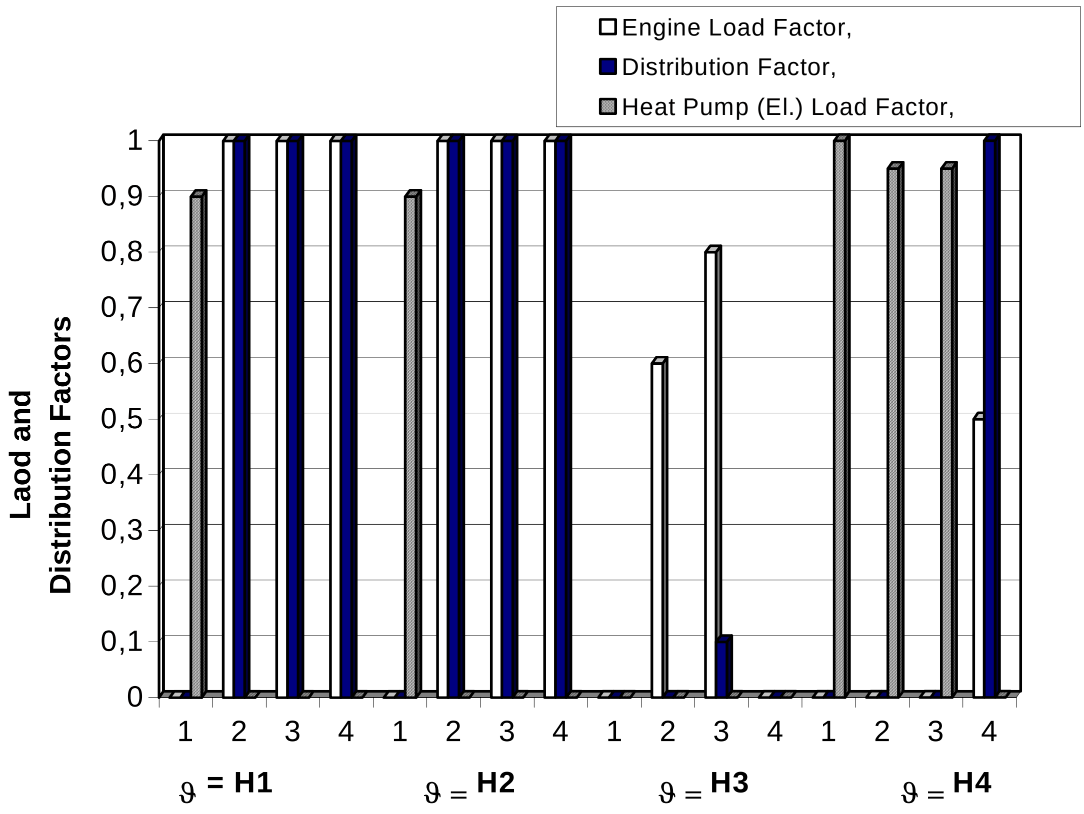


Figure 2. Optimal configuration for $E_{6,0}$ =1000 kW, $E_{10,0}$ =3000 kW, T_{out} =7 °C.

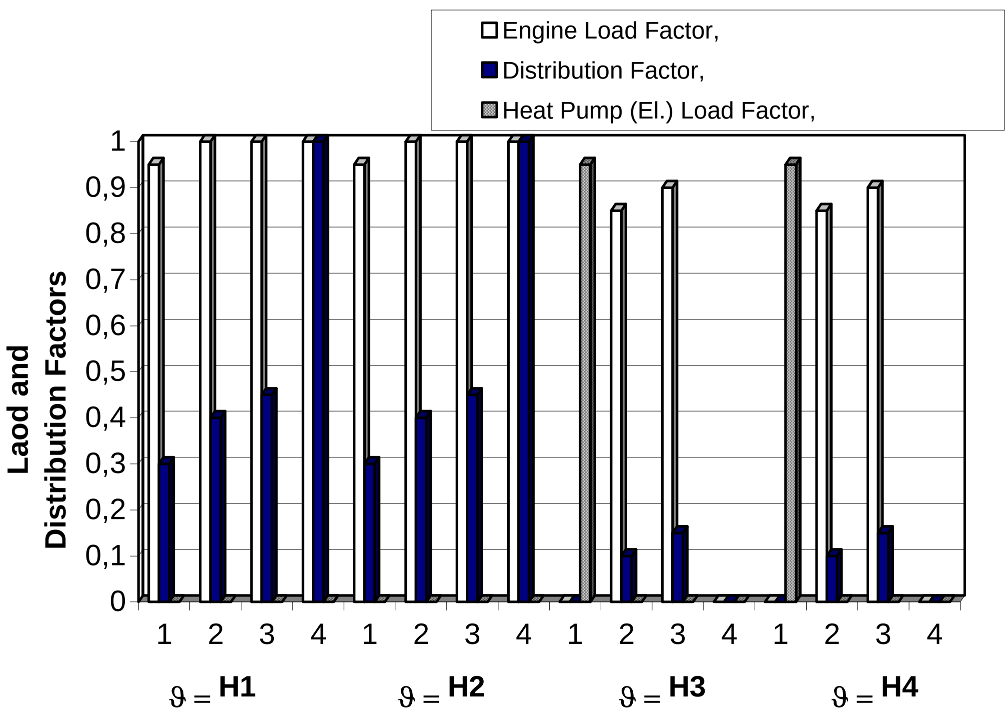


Figure 3. Optimal configuration for $E_{6,0}$ = 1000 kW, $E_{10,0}$ = 5000 kW, T_{out} = 7 °C.

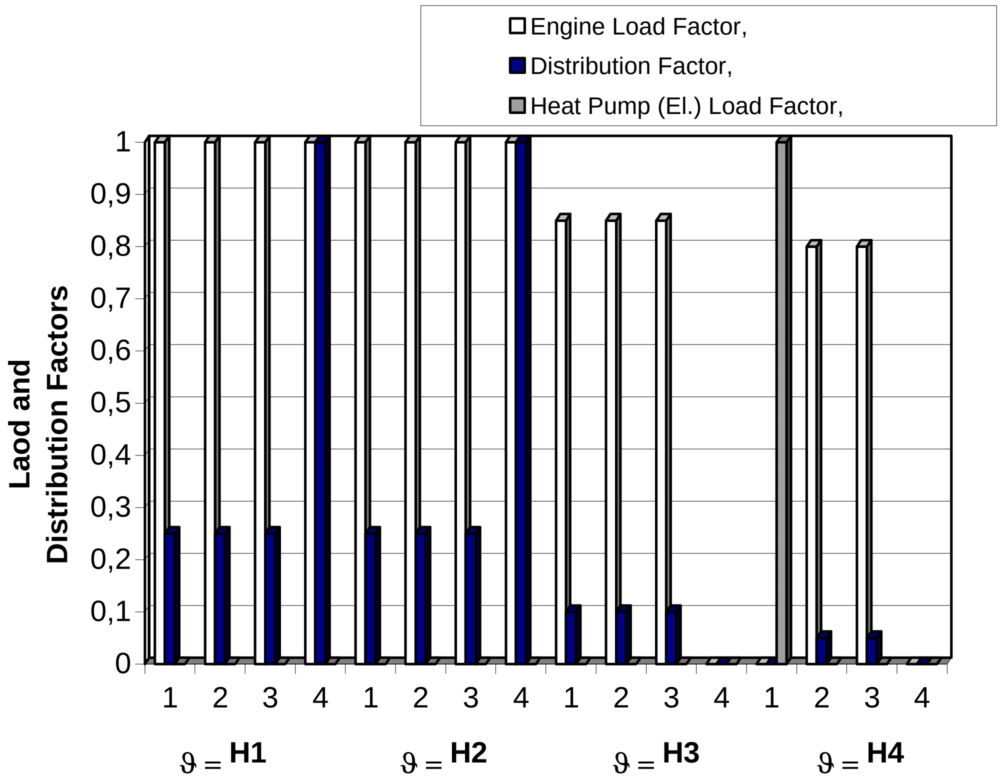


Figure 4. Optimal configuration for $E_{6,0}$= 1000 kW, $E_{10,0}$ = 7000 kW, T_{out} = 7 °C.

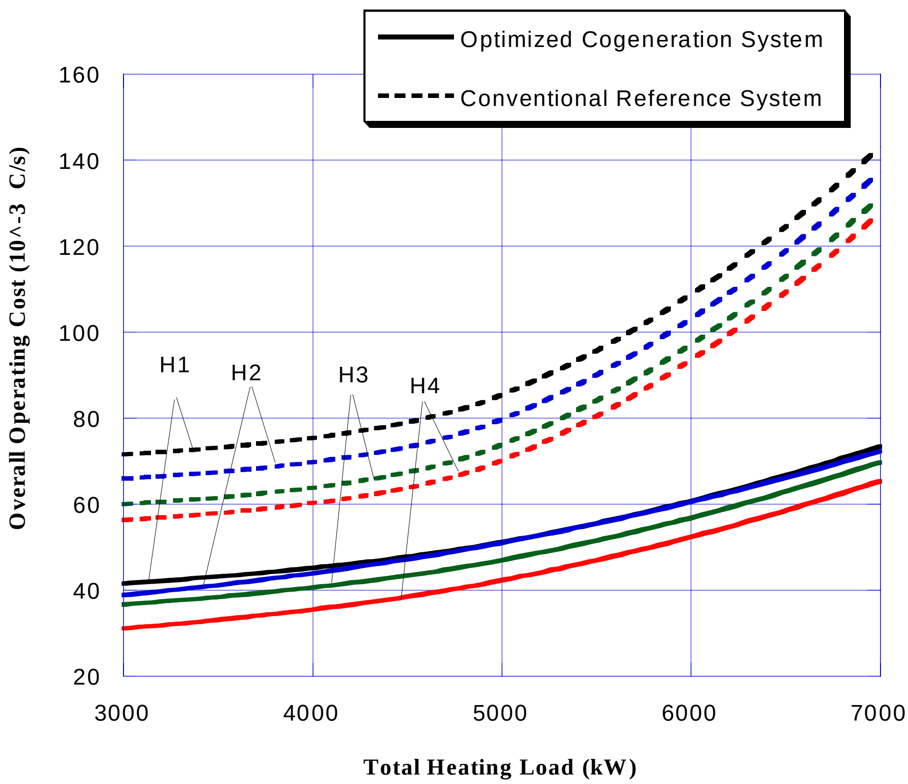


Figure 5. Minimum value of the operating cost (objective function) and corresponding values for a conventional reference system (electric utility grid and gas boilers). $E_{6,0}$ = 1000 kW, T_{out} = 7 °C.

4. Conclusions

With reference to a thermal plant recently built in Naples, Italy, attention has been paid to the problem of defining, for any load condition, the operational mode that minimizes the economic cost of operating the system. The focus of the work presented was the optimization problem rather than the detailed modeling of the plant. A low-level model of the plant, based on the nominal performance curves of each component, has been used as a first approach. With reference to the heating season, some numerical examples have been presented. The economic savings achievable through an appropriate selection of operating configuration have been highlighted. A more complex model of the plant will be developed in the future, as soon as a monitoring and control system will be available, permitting an accurate validation of the model itself. The design of such a system is presently in progress. In addition to plant-related information needed to develop a high-level simulation model, the monitoring system will also allow an evaluation of how the energy requirements vary during a whole year (load profiles). It will then be possible to perform analyses and optimizations on a yearly base.

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Nomenclature

C = economic cost (Euro/s)
c = unit cost (Euro/kJ)
$E_{j,k(i)}$ = energy flow rate (kW)
$F_{k(i)}$ = load factor for unit k(i)
H1 = peak-load hours
H2 = high-load hours
H3 = medium-load hours
H4 = minimum-load hours
OF = objective function (Euro/s)
p =unit price for energy sale (Euro/kJ)
T_{out} = outdoor temperature (°C)
t = unit tax (Euro/kJ)
$\underline{X}$ = vector of independent variables
$Z_{j,k(i)}$ = distribution factor
η = first-law efficiency
ϑ = time-related variable

Superscripts

* = maximum value

Further symbols (used as subscripts)

0 = external environment
E = electric energy
G = natural gas
j(i) = referred to unit j(i)
m = mechanical
P = penalty for power in excess
SC = self-consumed electric energy
t = thermal

References

Bejan, A., Tsatsaronis, G. and Moran, M., 1996, *Thermal Design and Optimization*, John Wiley and Sons, New York.

El-Sayed Y.M. and Evans R.B., 1970, "Thermoeconomics and the Design of Thermal Systems", *Journal of Engineering for Power*; Vol. 92, N. 1, pp. 27-35.

Frangopoulos C.A., 1983, *Thermoeconomic Functional Analysis: a method for optimal design or improvement of complex thermal systems*, Ph.D. Thesis; Georgia Institute of Technology, Atlanta, GA, USA.

Frangopoulos C.A., 1990a, "Optimization of synthesis-design-operation of a cogeneration system by the Intelligent Functional Approach", *Proceedings, Florence World Energy Research Symposium*, pp. 597-609, Florence, Italy.

Frangopoulos C.A., 1990b, "Intelligent Functional Approach: a method for analysis and optimal synthesis-design-operation of complex systems", *Proceedings, Florence World Energy Research Symposium*, pp. 805-815, Florence, Italy.

Kuester J.L. and Mize J.H., 1973, *Introduction to Linear and Nonlinear Programming*, Addison-Wesley Publishing Co., Reading Massachusetts.

Reklaitis G.V., Ravindran A. and Ragsdell K.M., 1983, *Engineering Optimization*, John Wiley and Sons, New York.

Tsatsaronis G., 1987, "A Review of Exergoeconomic Methodologies", *Second Law Analysis of Thermal System*, Eds. Moran and Sciubba, ASME, New York, pp. 81-87.