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Entropy Generation in Periodic Regenerative Heat Exchanger due to Finite Temperature Difference

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Abstract

This paper describes the second law of thermodynamics analysis of a regenerative heat exchanger. The analysis is based on the fact that the dimensionless parameters, known as the reduced periods and reduced length, are the characteristic variables to describe the heat exchanger. The solid matrix in the heat exchanger passage is discretized using trapezoidal rule and the elemental matrix is taken as a thermodynamic system. The second law of thermodynamics is applied to the system and the entropy generation equation is obtained using the dimensionless numbers Reduced period ($\prod$) and Reduced length (Λ) in each element. In the present paper, the variation of entropy generation due to reduced length and reduced period is studied. The influence of the effectiveness of the heat exchanger on entropy generation is also highlighted.

Keywords: Periodic flow heat exchanger, entropy generation, solid matrix, reduced period, reduced length, regenerative heat exchanger

1. Introduction

The operation of a thermal regenerator can be considered to be the continuous, alternate passage of hot and cold fluid streams over a solid matrix or packing. The length of time for which each fluid flows is known as a period. The solid matrix facilitates heat transfer between the hot and cold fluids by absorbing thermal energy during the hot period and releasing some of this stored energy during the cold period, in order to warm the cold fluid. At the end of each period it is assumed that any remaining fluid in the channels of the matrix is expelled before the start of the next period in what is known as a reversal period.

The temperatures of the fluid streams and the solid matrix in the periodic flow heat exchanger are the functions of distance and time. The combination of the hot and cold periods together with the reversal periods forms a cycle. After a sufficient number of such cycles the regenerator reaches a state of dynamic equilibrium, where the chronological variation of the fluid and solid temperatures is identical over successive cycles. The directions of the hot and cold fluid streams define the mode of operation of a regenerator. If both streams flow in the same direction the mode is co-current or parallel-flow and in opposite directions is counter-current or counter-flow. The latter is more common and is discussed in greater detail in this paper.

2. Entropy Generation

Entropy generation is the parameter to quantify the extent of irreversibility and it can be calculated as the difference between the entropy change of the system and the net entropy transport into the system. According to the second law of thermodynamics, entropy generation is always a positive quantity. It is a unique quantity to measure the extent of irreversibility of the thermodynamic processes (Bejan 1996). Thermal pollution of any thermodynamic system is directly proportional to, and only to, the extent of irreversibility and hence to entropy generation (EG). So it becomes necessary to study the mechanism of entropy generation and the influence of various parameters on it.

There are two sources of entropy generation in heat exchangers. One is due to heat transfer between two streams of finite temperature

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differences, and the second one is due to friction between the heat transfer surfaces and the moving fluids.

$$S_{gen} = S_{gT} + S_{gP}$$

The aim of this work is to identify the effect of the two non-dimensional parameters Λ and Π on the entropy generation due to finite temperature difference (S_{gT}) in the regenerative heat exchanger.

3. Approach

The heat transfer surface of the regenerative heat exchanger is taken as the thermodynamic system. The heat lost by the hot fluid is taken as a positive quantity since it is the amount of heat that is supplied into the system. The heat gain by the cold fluid is taken as a negative quantity since it is the amount of heat that is rejected from the system. The main assumption in this stage is the amount of heat lost by the hot fluid is equal to the amount of heat gained by the cold fluid. In this study, both the fluids are assumed to be ideal gases. For simplicity, the properties, particularly specific heat, viscosity and thermal conductivity of the hot and cold fluids are assumed to be constant for this work in the entire temperature range of the heat exchanger.

The entropy generation is calculated in each element of the regenerative heat exchanger over the entire period of the fluid flow. The summation of all entropy generation thus calculated will give us the net entropy generation in the particular period. Similarly the entropy generation for the other period is also calculated and the sum will yield us the entropy generation in the regenerative heat exchanger during one cycle.

4. Mathematical Model

The schematic diagram of the fluid flow in regenerative heat exchanger is shown in *Figure 1*. The differential equations, which model regenerator behavior, are:

$$hA\left(\phi - \Phi\right) = M_m c \frac{\partial \Phi}{\partial \theta} \quad (1)$$

$$hA\left(\Phi - \phi\right) = \dot{m} c_p L \frac{\partial \phi}{\partial y} + M_f c_p \frac{\partial \phi}{\partial \theta} \quad (2)$$

The following assumptions are made to obtain the equations (1) and (2).

1. The effects of the residual fluid in the matrix channels during the reversal periods are ignored.
2. The thermal conductivity of the fluids and matrix is zero in a direction parallel to that of the fluid stream.

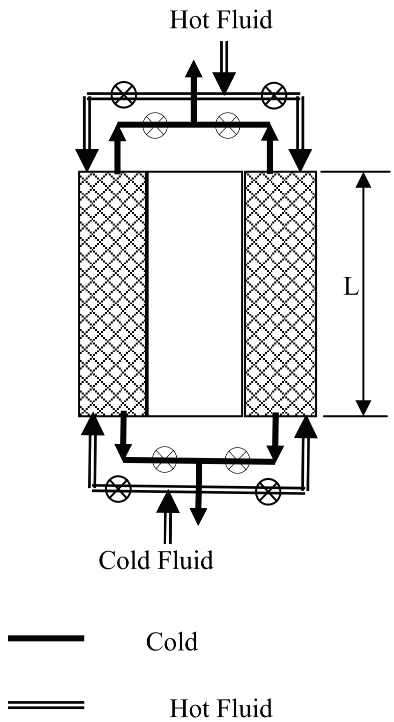


Figure. 1. Schematic Diagram of the fluid flow in regenerative heat exchanger

3. The solid temperature variation in the radial direction is not considered. It is assumed that the thermal conductivity in the radial direction is either infinite, in which case the solid will be isothermal in the radial direction, or finite. In the latter case a bulk heat transfer coefficient is used (Willnott 1969).
4. The heat transfer coefficient and thermal properties of both fluid and solid are regarded as temperature independent.
5. The mass flow rate of the fluid in each period does not vary with time but may be different in the hot and cold periods.

By using the dimensionless parameter introduced by Hausen (1929):

$$\xi = \frac{hAy}{\dot{m} c_p L} \quad (3)$$

$$\eta = \frac{hA}{M_m c}\left(\theta - \frac{M_f y}{\dot{m} L}\right) \quad (4)$$

and the dimensionless temperatures given by:

$$T = \frac{\Phi - \phi'_{min}}{\phi_{max} - \phi'_{min}} \qquad t = \frac{\phi - \phi'_{min}}{\phi_{max} - \phi'_{min}}$$

Equations (1) and (2) can be non-dimensionalized by using equations (3) and (4) to take the form as equations (5) and (6) which are the same as that of the partial differential equations developed by Hausen (1929).

$$\frac{\partial T(\xi,\eta)}{\partial \eta} = t(\xi,\eta) - T(\xi,\eta) \tag{5}$$

$$\frac{\partial t(\xi,\eta)}{\partial \xi} = T(\xi,\eta) - t(\xi,\eta) \tag{6}$$

The values of ξ and η at $y = L$ and $\theta = P$ are:

$$\Lambda = \frac{hA}{\dot{m}c_p} \tag{7}$$

$$\Pi = \frac{hA}{M_m c}\left(P - \frac{M_f}{\dot{m}}\right) \tag{8}$$

which are named as reduced length and reduced period respectively by Hausen (1929). The importance of the term $\left(\frac{M_f y}{\dot{m}L}\right)$ in equation (4) is discussed in detail by Willmott and Hinchcliffe (1976). Any solution to equations (5) and (6) must also take into account the following boundary conditions:

i. The inlet fluid temperature is predefined as some function of time, namely

$$t_0(\eta) = t(0,\eta) \tag{9}$$

ii. Distances within a regenerator are measured from the fluid entrance in both periods. Further, the temperature distribution of the matrix at the end of one period is equal to the temperature distribution of the matrix at the beginning of the opposite period. For counter-current operation this results in the reversal conditions:

$$T(\xi,0) = T'[\Lambda'(1-\xi/\Lambda),\Pi'] \tag{10}$$

$$T'(\xi',0) = T[\Lambda(1-\xi/\Lambda'),\Pi] \tag{11}$$

5. Analysis

In 1979 Razelos presented a closed solution to the two linear partial differential equations, which describe regenerative heat exchanger behaviour. In this solution, equation (6) is discretized using Euler's rule. An analytic approach is then used to solve the resultant set of ordinary differential equations in the independent variable η. This gives rise to a set of simultaneous, linear, algebraic equations. The number of linear equations is directly proportional to the number of steps required in the discretization.

Euler's rule is the least accurate of the finite-difference formulae (Lambert 1981) and, due to stability considerations, can only be implemented using a sufficiently small step length. In order to achieve sufficient accuracy, and implicitly to avoid the effects of instability, Razelos (1979) found it was necessary to use a step length of the order 0.01 when representing equation (6); consequently the solution of up to 1000 linear equations was required. An approximated analytical expression for the entropy generation is given by Das and Sahoo (1991). In this paper, a new robust method explained by Hill and Willmott (1987), who use the trapezoidal rule to discretize equation (6) is used to establish the temperature field in the regenerative heat exchanger. The trapezoidal rule is more accurate than Euler's and has no stability problems when used to replace equation (6) (Lambert 1981). This approach not only substantially reduces the number of linear equations to be solved but also offers a solution to the long regenerator problem (Willnott and Thomas 1974). A significant advantage of this method for cyclic equilibrium calculations is that both the fluid and solid temperature distributions can be computed for any instant of time. This is not the case in the closed methods of Iliffe (1948) and Nahavandi and Weinstein (1961) where only the solid temperatures are available at the beginning and end of the hot and cold periods of operation.

Discretizing equation (6) employing the trapezoidal rule and considering the temperatures of fluid and solid at (N + 1) equidistant points or nodes along the regenerator yields:

$$\frac{dT_n}{d\eta} = t_n - T_n \tag{12}$$

$$t_{n+1} = bt_n + a(T_n + T_{n+1}) \quad 0 \le n \le N-1 \tag{13}$$

with

$$a = \frac{\Delta\xi}{2+\Delta\xi}; \quad b = \frac{2-\Delta\xi}{2+\Delta\xi}; \quad (2a+b) = 1 \tag{14}$$

and

$$\Delta\xi = \frac{\Lambda}{N} \qquad \xi = n\Delta\xi \tag{15}$$

The reversal conditions (10) and (11) become:

$$T_n(0) = T'_{N-n}(\Pi') \qquad (0 \le n \le N) \tag{16}$$

$$T'_n(0) = T_{N-n}(\Pi) \qquad (0 \le n \le N) \tag{17}$$

The transformation Ψ is now introduced and equation (12) is used to give:

$$\Psi_n = \exp(\eta)T_n \qquad (0 \le n \le N) \tag{18}$$

$$\frac{d\Psi_n}{d\eta} = \exp(\eta)t_n \qquad (0 \le n \le N) \tag{19}$$

Which, using equation (13), becomes:

$$\frac{d\Psi_n}{d\eta} = a\Psi_n + a\Psi_{n-1} + b\frac{d\Psi_{n-1}}{d\eta}$$

$$(1 \le n \le N) \tag{20}$$

Any solution to equation (19) must satisfy this auxiliary equation (20).

Solution of equation (19)

The general solution to the equation (19) (when $t_0 = 0$) is given by:

$$K_n = (-1)^n A_0 + \exp(a\eta)\left[A_n + \sum_{j=1}^{n-1} A_j \alpha_{n-j}\right] \quad (21)$$

with

$$K_0 = A_0 \quad (22)$$

and

$$\alpha_r = b^r \sum_{k=1}^{r} \binom{r-1}{k-1} \left(\frac{a(1+b)\eta}{b}\right)^k \frac{1}{k!} \quad (23)$$

The A_j values are constants of integration. When $t_0 = t_0(\eta)$ the particular solution to equation (19) is given by:

$$R_n = \exp(a\eta) \sum_{r=0}^{n-1} \left[a(1+b)\right]^{n-1-r} b^r \binom{n-1}{r} \times \int^{n-r} \exp(-a\eta) G_0(\eta)\, d\eta \quad (24)$$

where

$$G_0(\eta) = a\int \exp(\eta) t_0(\eta) d\eta + b\exp(\eta) t_0(\eta) \quad (25)$$

$$R_0 = \int \exp(\eta) t_0(\eta) d\eta \quad (26)$$

and the symbol $\int^{n-r}$ indicates (n-r) indefinite integrations of the function $\exp(-a\eta)G_0(\eta)$. Given the general and particular solutions to equation (19), together with equation (18), all the required temperatures can be calculated.

Solution for constant inlet temperatures

In this case we have $t_0(\eta) = 1$ and $t'_0(\eta') = 0$ which yields:

$$G'_0(\eta') = 0$$

$$G_0(\eta) = (a+b)\exp(\eta) = (1-a)\exp(\eta) \quad (27)$$

The particular solution R, now takes the form:

$$R'_n = 0 ;\ R_n = \exp(\eta) \quad (28)$$

From equations (18), (21) and (28) we obtain the temperatures of the matrix elements as

$$T_n = \exp(-\eta)(-1)^n A_0 + \exp\left[(a-1)\eta\right]\left\{A_n + \sum_{j=1}^{n-1} A_j \alpha_{n-j}\right\} + 1 \quad (29)$$

$$T'_n = \exp(-\eta')(-1)^n A'_0 + \exp\left[(a'-1)\eta'\right]\left\{A'_n + \sum_{j=1}^{n-1} A'_j \alpha'_{n-j}\right\} \quad (30)$$

$$T_0 = \exp(-\eta) A_0 + 1 \qquad T'_0 = \exp(-\eta') A'_0 \quad (31)$$

For the fluid temperatures equations (21), (23) and (30) are used to give:

$$t_n = a\exp\left[(a-1)\eta\right] \times \left[A_n + \sum_{j=1}^{n-1} A_j\left\{\alpha_{n-j} + (1+b)\beta_{n-j}\right\}\right] + 1 \quad (32)$$

$$t'_n = a'\exp\left[(a'-1)\eta'\right] \times \left[A'_n + \sum_{j=1}^{n-1} A'_j\left\{\alpha'_{n-j} + (1+b')\beta'_{n-j}\right\}\right] \quad (33)$$

where

$$\beta_r = b^{r-1} \sum_{k=0}^{r-1} \binom{r-1}{k} \left(\frac{a(1+b)\eta}{b}\right)^k \frac{1}{k!} \quad (34)$$

Applying equations (29 – 31) to the reversal conditions (16) and (17) yields 2(N + 1) equations in the 2(N + 1) unknowns A_j, A'_j:

$$(-1)^n A_0 + A_n + 1 = \exp(-\Pi')(-1)^{N-n} A'_0 + \exp\left[(a'-1)\Pi'\right]\left\{A'_{N-n} + \sum_{j=1}^{N-n-1} A'_j \alpha'_{N-n-j}\right\} \quad (35)$$

$$(-1)^n A'_0 + A'_n = 1 + \exp(-\Pi)(-1)^{N-n} A_0 + \exp\left[(a-1)\Pi\right]\left\{A_{N-n} + \sum_{j=1}^{N-n-1} A_j \alpha_{N-n-j}\right\} \quad (36)$$

At the hot and cold fluid entrances, the corresponding equations are:

$$A_0 + 1 = \exp(-\Pi')(-1)^N A'_0 + \exp\left[(a'-1)\Pi'\right]\left\{A'_N + \sum_{j=1}^{N-1} A'_j \alpha'_{N-j}\right\} \quad (37)$$

$$A'_0 = 1 + \exp(-\Pi)(-1)^N A_0 + \exp\left[(a-1)\Pi\right]\left\{A_N + \sum_{j=1}^{N-1} A_j \alpha_{N-j}\right\} \quad (38)$$

$$(-1)^N A_0 + A_N + 1 = \exp(-\Pi') A'_0 \quad (39)$$

$$(-1)^N A'_0 + A'_N = 1 + \exp(-\Pi) A_0 \quad (40)$$

In equations (35) - (38) the α_r, α'_r values are calculated at $\eta = \Pi$ and $\eta' = \Pi'$, respectively. In the important case of the symmetric regenerative heat exchanger, where $\Lambda = \Lambda'$ and $\Pi = \Pi'$, it can be shown that $(A_j + A'_j)$ is equal to zero and the number of equations required is halved.

The first and second laws of thermodynamics, taken together, state that the entropy generated by any engineering system is proportional to the work lost (destroyed) irreversibly by the system. This truth is expressed concisely as the Gouy-Stodola Theorem (Szargut 1980).

$$W_{lost} = T_o \sum_{\substack{\text{all} \\ \text{system} \\ \text{components}}} S_{gen} \qquad (41)$$

Here W_{lost} is the lost available work (lost availability, or lost exergy) (Poulikakos and Bejan 1982), T_o is the absolute temperature of the environment, and S_{gen} is the entropy generated in each compartment of the system. Equation (41) implies that the thermodynamic irreversibility (entropy generation) of each system component contributes to the aggregate loss of available work in the system (W_{lost}).

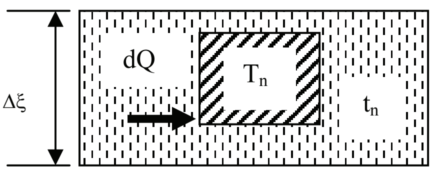
$$W_{lost} = \int_0^{\Pi} \delta Q \left[\left(1-\frac{T_o}{\phi_n}\right)-\left(1-\frac{T_o}{\Phi_n}\right)\right] \qquad (42)$$

Heat flow in regenerative heat exchanger during hot and cold periods are given in the Figure 2. Hence during the hot period,

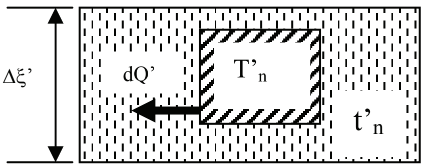
$$S_{genh} = \frac{1}{T_o}\times\int_0^{\Pi} \delta Q \left[\left(1-\frac{T_o}{\phi_n}\right)-\left(1-\frac{T_o}{\Phi_n}\right)\right] \qquad (43)$$

similarly during the cold period,

$$S_{genc} = \frac{1}{T_o}\times\int_0^{\Pi'} \delta Q' \left[\left(1-\frac{T_o}{\Phi'_n}\right)-\left(1-\frac{T_o}{\phi'_n}\right)\right] \qquad (44)$$



(a) – During hot period dη



(b) – During cold period dη'

Figure. 2. Schematic Diagram of the heat flow in regenerative heat exchanger during hot and cold periods

Adding the S_{genc} and S_{genh} yields the total entropy generation during one cycle (hot and cold period),

$$\begin{aligned} S_{gen} &= \frac{1}{T_o}\times\int_0^{\Pi} \delta Q \left[\left(1-\frac{T_o}{\phi_n}\right)-\left(1-\frac{T_o}{\Phi_n}\right)\right] \\ &+ \frac{1}{T_o}\times\int_0^{\Pi'} \delta Q' \left[\left(1-\frac{T_o}{\Phi'_n}\right)-\left(1-\frac{T_o}{\phi'_n}\right)\right] \end{aligned} \qquad (45)$$

In the calculation of entropy generation (equation 45) the effect of reversal period is not taken into the account. T_o is independent of the system variables and it is constant. Therefore equation (45) gives

$$S_{gen} = \int_0^{\Pi} \delta Q\left(\frac{1}{\Phi_n}-\frac{1}{\phi_n}\right)+\int_0^{\Pi'} dQ'\left(\frac{1}{\phi'_n}-\frac{1}{\Phi'_n}\right) \qquad (46)$$

δQ and δQ' can be calculated using equations, (47) and (48) respectively. The integration in equation (46) is carried out numerically using Simpson's 3/8th rule, which provides more accuracy than the Simpson's 1/3rd rule (Rice and Do 1995). In the above equation, $T_n \le t_n$ and $t'_n \le T'_n$. Hence the value of S_{gen} is always positive and its lowest limit is 0 if the process is reversible.

$$\delta Q = \frac{M_m c}{N}\times[\phi_n - \Phi_n]d\theta;$$

dθ can be calculated from the equation (4) and δQ becomes

$$\delta Q = \frac{M_m c}{N}\times[\phi_n - \Phi_n]d\eta \qquad (47)$$

Similarly

$$\delta Q' = \frac{M_m c}{N}\times[\Phi'_n - \phi'_n]d\eta' \qquad (48)$$

In the above equations, ϕ and Φ can be calculated from t and T respectively.

6. Results and discussion

The variation of the entropy generation in the regenerative heat exchanger is given in the *Figures* 3 and 4. Regenerator effectiveness is measured in terms of the thermal ratio η_{reg} which describes the ratio of the actual heat transferred during a period to the thermodynamically limited maximum obtainable heat transfer for that period. This results in the two values of η_{reg} shown by Iliffe (1948) to be:

$$\eta'_{reg} = \frac{\Lambda'}{\Pi'}\left[\frac{1}{\Lambda'}\int_0^{\Lambda'} T'(\xi',0)d\xi' - \frac{1}{\Lambda}\int_0^{\Lambda} T(\xi,0)d\xi\right]$$

$$\eta_{reg} = \frac{\Pi'\Lambda}{\Pi\Lambda'} \times \eta'_{reg} \quad (49)$$

In our calculation the symmetric regenerative heat exchanger is selected for the analysis of entropy generation. For the symmetric regenerative heat exchanger $\Pi = \Pi'$; $\Lambda = \Lambda'$. Hence

$$\eta_{reg} = \eta'_{reg} \quad (50)$$

The entropy generation in the regenerative heat exchanger is varying linearly with the reduced period for a particular value of the reduced length (Refer to *Figure 3*). The entropy generation is directly proportional to the reduced period and the minimum entropy generation occurs when the reduced period is a minimum value. The rate of increase in the entropy generation is high for the low reduced length (Refer to Table 1).

For the regenerator, if the reduced period is increased, it means that the actual period P is increased (Refer to equation 8) for the same amount of heat transfer and the heat transfer rate is low. Hence the entropy generation due to the finite temperature difference is increased.

The entropy generation in the regenerative heat exchanger is continuously decreasing with the reduced length for a particular value of the reduced period (Refer to *Figure 4*). The minimum entropy generation occurs when the reduced length is the maximum value possible.

If the reduced length of the heat exchanger is increased, the product of heat transfer coefficient and the heat transfer area of the heat exchanger is increased for the given mass flow rate and the specific heat of the fluid. Hence the entropy generation due to the finite temperature difference is decreased.

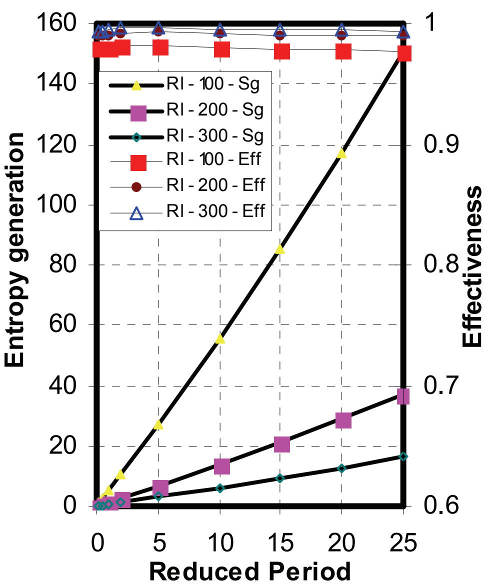


Figure. 3. Variation of entropy generation and effectiveness with reduced period

TABLE 1 THE VARIATION OF THE RATE OF ENTROPY GENERATION INCREASE WITH RESPECT TO THE REDUCED LENGTH

Sl. No	Reduced Length Λ	Slope of the line
1	50	24.539
2	100	5.934
3	200	1.460
4	300	0.647
5	400	0.365
6	500	0.234
7	600	0.163
8	700	0.120
9	800	0.093
10	900	0.074
11	1000	0.060

For the increased reduced length, the product of mass flow rate and the specific heat of the fluid will be a lower value for the given product of heat transfer coefficient and the heat transfer area of the heat exchanger. This also will decrease the entropy generation but of course will require a bigger heat exchanger.

7. Conclusion

- The entropy generation due to finite temperature difference is almost invariant with respect to the effectiveness of the regenerative heat exchanger.
- For a particular reduced length, the entropy generation due to finite temperature difference is in linear relation with the reduced period.
- To reduce the entropy generation due to the temperature difference in the regenerative heat exchanger to a minimum, the reduced length

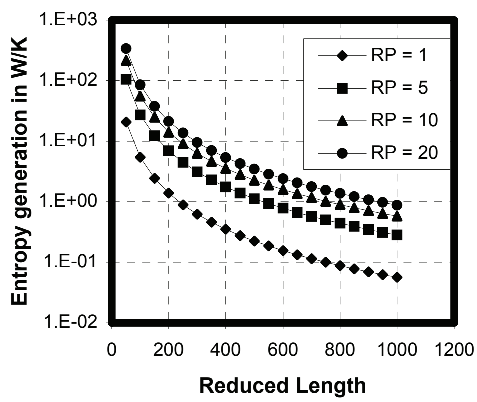


Figure. 4. Variation of entropy generation with reduced length

should be the maximum possible value and the reduced period should be the minimum possible value.

Nomenclature

a	Trapezoidal discretization constant
A	Heat transfer surface area [m^2]
A_n	Constants of integration in general solution to equation (19)
B	Trapezoidal discretization constant
c	Specific heat of matrix [$kJ\ kg^{-1}\ K^{-1}$]
c_p	Specific heat of fluid [$kJ\ kg^{-1}\ K^{-1}$]
H	Heat transfer coefficient [$kJ\ s^{-1} m^{-2}\ K^{-1}$]
K	General solution to equation (19)
L	Length of regenerator [m]
$\dot{m}$	Mass flow rate of fluids [$kg\ s^{-1}$]
M_f	Mass of fluid in regenerator channels [kg]
M_m	Mass of matrix [kg]
N	Total number of regenerator segments
P	Duration of a period [s]
Q	Heat transfer rate in [W]
R	Particular solution to equation (19)
S_{gen}	Entropy generation per period [$kJ\ K^{-1}$]
S_{gT}	Entropy generation due to finite temperature difference
S_{gP}	Entropy generation due to pressure difference
t	Dimensionless [0, l] fluid temperature
T	Dimensionless [0, l] matrix temperature
Y	Distance down the regenerator [m].

Greek symbols

α_j	Series in η for general solution to equation (19)
β_i	Differentiated α_i values
η	Dimensionless time defined by equation (6)
η_{reg}	Thermal ratio
ϕ	Fluid temperature [K]
$\phi_0(\theta)$	Inlet fluid temperature as a function of time [K]
ϕ_{max}	Maximum value of $\phi_0(\theta)$ for $0 < \theta < P$ [K]
ϕ'_{min}	Minimum value of $\phi'_0(\theta')$ for $0 < \theta' < P'$ [K]
θ	Time [sec]
ξ	Dimensionless distance defined by equation (5)
Λ	Reduced length defined by equation (7)
Π	Reduced period defined by equation (8)
Φ	Matrix temperature [K]
Ψ	Transformation of T.

Superscripts

’	Refers to cold period.

Subscripts

N	Refers to nth node.
O	Refers to the ambient condition

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