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Entropy Generation Minimization as a Design Tool. Part 1: Analysis of Different Configurations of Branched and Non-branched Laminar Isothermal Flow Through a Circular Pipe

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Abstract

The paper presents three examples of a solution of a simple multi-variable optimization problem: the "optimal configuration" of a branch of a pipe of circular cross-section with a given initial radius r_0 and delivering a given mass flow rate m_0. Three cases, two presented in previous papers and a novel one, are used to illustrate two theses: first, that for a given design "task", the configurations (shapes) displaying the minimal entropy generation are compatible with the shapes observed in nature; second, that an EGM analysis not only leads to the identification of a thermodynamically optimal solution, but offers substantial additional insight into the flow characteristics even in simple -but realistic- cases as the ones discussed here, for which an analytical solution to the Navier Stokes equations exists.

The entropy generation rate is due -in all three examples- only to viscous flow effects within the tubes, and several simplifying assumptions are made to reduce the problem to a multi-variable optimization in 2 (for the tube with wall suction) or 3 (for the branchings) independent variables: the aspect ratio of the domain served by the flow, the diameter ratio of the primary and secondary branches, and the length of the secondary branch (the location of both the "source" of the fluid and the "sink", i.e. the place of desired delivery of the fluid, being a datum).

It is shown that the solution is strongly dependent both on the aspect ratio and on the diameter ratio, and in the case of wall suction, to the wall porosity.

The study is divided in two parts: the analysis presented in this first paper is useful from a theoretical point of view, because it sheds some light on the phenomenology of the configurations studied here. The final purpose is twofold: the *a priori* identification of more efficient geometries for the channels of heat exchangers and flow devices through a preliminary EGM analysis, and a better understanding of the teleology of some of the structures observed in nature. The present study and its conclusions are still preliminary, but since the procedure can be easily "falsified", and all numerical experiments on more complex flow geometries to date do not disprove the present findings, it is indeed a topic that warrants further investigation.

Keywords: *Flow bifurcations; entropy generation minimization; shape optimization; mass transport; porous walls.*

1. Introduction

The scope of this paper is to explore the correlation between the entropy generation rate and the flow configuration in three simple laminar isothermal flow geometries: a single bifurcation, a straight tube with wall suction, and a bifurcation with wall suction. The problem is related to three fundamental questions:

a) In natural flow systems (both inert and biological structures), *why does a flow bifurcation* (of the type shown in Figure 1) *occur*?
b) In natural flow systems (both inert and biological structures), *is there a preferred channel shape in the presence of wall suction* (Figure 2)?
c) In man-made applications, *is there an "optimal" (bifurcated or not) pattern for a given design goal, and how can it be identified?*

A quantitative answer to the first and second question necessitates a large database of exactly defined and comparable flow geometries, and such an organised sampling list has never been compiled. Furthermore, for realistic cases the problem involves such a large number of relevant parameters (initial conditions and stochastic noise thereof, degree of interaction with the surroundings, randomly varying boundary conditions) that a satisfactory predictive paradigm is still unthinkable. But if we focus our attention on the third question, we see that the answer suggested here is of invaluable interest for all heat transfer practitioners. Indeed, the approach currently adopted in the design of a new heat transfer device is basically heuristic (Escher et al. 2008, Ramos-Alvarado et al. 2009): the Designer devises a set of similar (or modular) structures (each member being a properly constructed series of bifurcations) and then checks *a posteriori* which member of the family attains the best performance under a pre-assigned set of constraints. The success of such an approach strongly and unavoidably depends on the initial choice of the geometrical features the "family" must possess: this implies that *the final (optimal or pseudo-optimal) configuration essentially depends on the ingenuity and insight of the designer*. Furthermore, once the "optimal" structure has been identified, every subsequent modification to its features may require a global iteration, i.e., a

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construction of a new family of structures each one of them possessing this "new" feature, and a new optimization. In other words, the optimum is not guaranteed to be global, when considered in the "attribute space" of the geometric or physical features of the set.

The starting point of the present analysis is essentially pragmatic: when designing a bifurcation, can the designer (be it a human or nature itself) *take entropy as the objective function of the optimisation*? And, when a tube must deliver a specified flow rate not only to its endpoint but also - through wall suction- to a domain surrounding it, can again the designer *take entropy as the objective function of the optimisation*? Limiting our discussion for the moment to anthropic structures, let us observe that for (laminar or turbulent) liquid flow in pipes, all current design manuals suggest to "minimize the pressure drop for a specified mass flowrate", which is attained by limiting for instance sharp curves, sudden restrictions or expansions, narrow diaphragms and valves, etc. But if the design goal is to carry a given mass flow rate m_0 from one "source" O to two delivery "sinks" A and B (Figure 1), can an "optimal" geometry be identified *a priori*? Equivalently, if the design goal is to carry a given mass flow rate m_0 from one "source" O to a delivery sink A and through wall suction, to the immediate surroundings of the tube itself (Figure 2), does an "optimal" geometry exist?

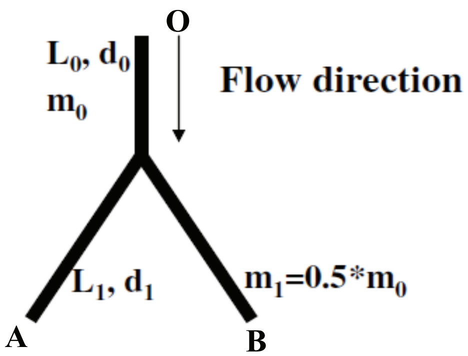


Figure 1. The bifurcation geometry.

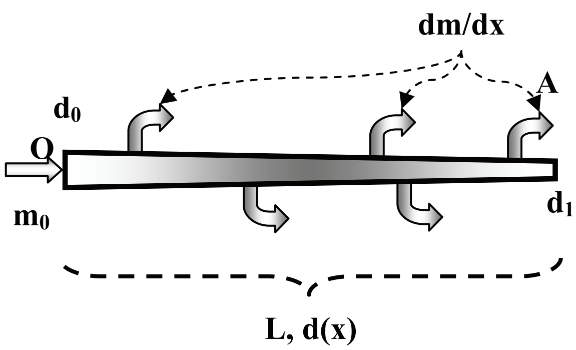


Figure 2. The tube with wall suction

The answer to all of the above questions can be found if we denote as the "optimal structure" the flow path that, under the given design constraints, delivers the specified mass flow rate with the minimum possible irreversibility. This statement can be formulated in a much broader sense, namely that *in the real world the optimisation criterion is the minimum entropy generation rate compatible with the available exergy input and with the prescribed constraints*.

Bifurcated flows are important *per se*, in piping for instance, but they are also interesting because they may be used as "building stones" to construct a porous matrix, or to reproduce, by successive splits and downstream recombinations, "fractal" geometries. A first goal of this study is to decide whether an optimal configuration exists, and if it exists whether the optimum is sharp and unique. The method falls under the class of "Entropy Generation Minimization" (formulated in its modern form by Bejan (1995)). In the laminar, isothermal and incompressible case analysed here, the only contribution to the entropy generation comes from the viscous dissipation, which is commonly measured by the pressure drop per unit length. The two "measures" (the pressure drop and the viscous entropy generation rate) may be numerically equivalent, in the sense that the minimum of the former coincides with the minimum of the latter, but the insight provided by an entropic analysis is much deeper, if only for the fact that it can be assessed locally (Robbe, 2007) and immediately applied to design modifications. Furthermore, whenever an additional objective function is added (like for instance minimum surface temperature or maximum heat transfer rate), EGM provides a unique measure of "optimality" absent in other methods.

2. The entropy generation rate in a simple bifurcation

2.1 Analytical formulation

Consider a rectangular element of a given aspect ratio a_r=H/L (Figure 3). In this simple model, discussed in detail in (Sciubba 2010), the fluid proceeds from left to right, and the "goal" of the device is to deliver the mass flow rate $m_0/2$ to the end points A and B. The tube has a circular section, so that the usual formulae for steady, laminar, fully developed flow in a round pipe apply.

The relevant variables are: m_0, m_1, L_0, L_1, d_0, d_1, and the (constant) fluid properties. The mass flow rate m_0 is prescribed, and the bifurcation is flow-wise symmetrical, so that $m_1=0.5\ m_0$. The flow is isothermal.

Since the "goal" of the (natural or artificial) device is to deliver fluid to endpoints A and B, a bifurcation must occur somewhere in L, at a point P identified by an additional parameter $\lambda=L_0/L$, and the general expression for the entropy generation rate is (Sciubba 2010):

$$\dot{S}_{bifurcated-flow} = \dot{S}_{L_0,r_0} + 2\dot{S}_{L_1,r_1} = K_f\,\frac{\dot{m}_0^2 L}{r_0^4}\left[\lambda + \frac{\sqrt{(1-\lambda^2)+0.25a_r^2}}{2\delta^4}\right] \tag{1}$$

with $K_f = \dfrac{8\nu}{\pi\rho T}$.

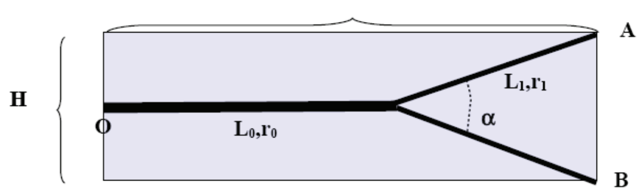


Figure 3. Definition of the bifurcation geometry.

2.2 Results and discussion for the laminar bifurcation

Equation (1) was solved analytically for different aspect ratios a_r=H/L and different bifurcation lengths (identified by the ratio $\lambda=L_0/L$), under three physically meaningful situations:

1) The Reynolds number remains constant over the entire fluid path: $Re_0=Re_1$. This defines a diameter ratio $\delta=r_1/r_0=0.5$;
2) The velocity remains constant over the entire fluid path: $U_0=U_1$. This defines a diameter ratio $\delta=r_1/r_0=0.707$;
3) The volume occupied by the fluid in the unsplit portion is equal to that occupied by the two bifurcated branches. This defines a diameter ratio $\delta=r_1/r_0=\sqrt{\frac{L_0}{2L_1}}$.

Notice that none of the above situations is *per se* "optimal" in any sense: they were selected because they provide a good spectrum for the actually possible (i.e., naturally occurring) configurations.

In Section 2.2.1 the results are presented for an ideal case, in which the entropy generation rate is given by Equation (1). In Section 2.2.2, the losses due to the bifurcation shall be accounted for on the basis of an equivalent hydraulic length.

2.2.1 No bifurcation losses

The results for this case are shown in Figures 4a, b and c. The values of the entropy generation rates are made dimensionless by dividing them by the value

$$\dot{S}_{ref} = K_f \frac{\dot{m}_0^2 L}{r_0^4} \tag{2}$$

(with the constant K_f defined in Section 2.1 above), that represents the viscous dissipation in a straight non-bifurcated tube of length L and radius r_0.

The following features are apparent:

a) For all cases, *the entropy generation rate strongly depends on the aspect ratio a_r of the domain in which the bifurcation occurs*. With the geometry selected here, a higher a_r leads -for the same splitting ratio λ- to a longer bifurcated stretch, of smaller diameter and therefore with higher losses;
b) *The constant velocity case displays a lower entropy generation rate than the constant Re case for any bifurcation length*. This is due to the fact that the mass conservation constraint imposes a higher diameter ratio on the split portions of the tubes that are therefore affected by a lower dissipation rate;
c) *The constant fluid volume configurations* display extremely high entropy generation rates for low splitting ratios λ, but *are the least dissipative structure for high λ*. However, the minimum dissipation is attained with diameter ratios near unity: the short bifurcations have a <u>larger</u> diameter than the initial portion of the channel, and the velocities are -under the specified mass flowrate constraint- correspondingly lower.
d) For each physical situation (constant Re, constant U, constant $\mathcal{V}_{fluid}$) there is indeed a rather well-identifiable "optimal" configuration for each aspect ratio, that displays a minimum value of the entropy generation rate.

The above results are compatible with those obtained by using allometric or arithmetic/geometrical laws only if some additional features are introduced in those models:

1. Use of an "optimal" diameter ratio (suggested for example by Constructal Theory (Bejan 2000)) is avoided, since this parameter is uniquely specified once the physical flow type has been assigned (constant Re, constant U, constant fluid volume);
2. A suitable correlation is introduced in the above mentioned models between the "optimal" splitting ratio λ -for each diameter ratio δ- and the minimum entropy generation, which is the most reasonable indicator of "optimal performance";
3. An additional correction is also introduced in the models to allow for the higher-than-unity diameter ratio δ found for the "constant volume" configurations, which is a case not contemplated either by allometric (Bejan & Lorente 2004; Cano-Andrade et al. 2010; Rubio-Jimenez et al. 2009) nor by arithmetic/geometrical (including fractal) (Escher et al. 2008; Rubio-Jimenez et al. 2009) paradigms.

2.2.2 Including the bifurcation losses

In real flows, an additional viscous dissipation is generated at the bifurcation, due to the stagnation flow at the cusp. These losses were demonstrated to be non-negligible in a previous numerical study (Robbe & Sciubba 2008), and have been accounted for here, following (Sciubba 2010), by introducing an "equivalent length" of tube on the bifurcated portion of the domain. Adopting this approximate method, the length L_1 in Equation (9) is multiplied by a factor $(1+n_D)$: the constant n_D can be derived from one of the many semi-empirical correlations for pressure losses in sudden restrictions, and the values assumed here are displayed in Figure 5 (α is the bifurcation angle defined in Figure 3). The results are shown in Figure 6. The values of the entropy generation rates are made dimensionless like in the previous case.

With respect to the previous "ideal" case, the following additional features emerge:

a) For all cases, the entropy generation rate displays a marked increase: for the same aspect ratio a_r, same splitting length λ and same diameter ratio δ, the dissipation is increased by a factor between 1.5 and 4. This confirms *the importance of real-flow effects on the optimal configuration*;
b) For each physical situation (constant Re, constant U, constant $\mathcal{V}_{fluid}$) there is still an "optimal" configuration that displays a minimum value of the entropy generation rate, but the minimum is markedly shifted towards lower splitting ratios (earlier bifurcation), except for the constant volume case, in which it is very near the "T-shaped" configuration ($\lambda\approx1$).

As in the previous case, and for the same reasons, the above results are incompatible with allometric and geometric/fractal paradigms.

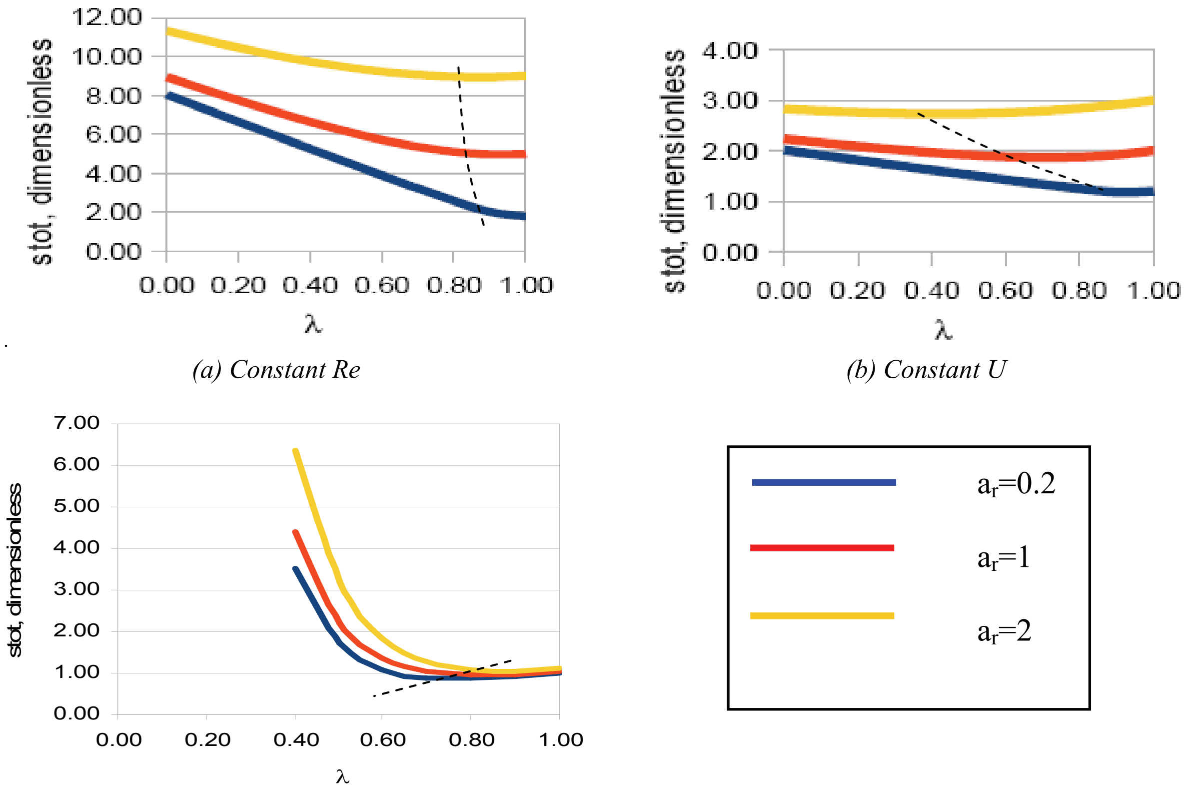


Figure 4. Dimensionless entropy generation in the bifurcation, without splitting losses. The dashed lines indicate the loci of the optimal λ. (figure is in color in the on-line version of the paper)(Adapted from [22]).

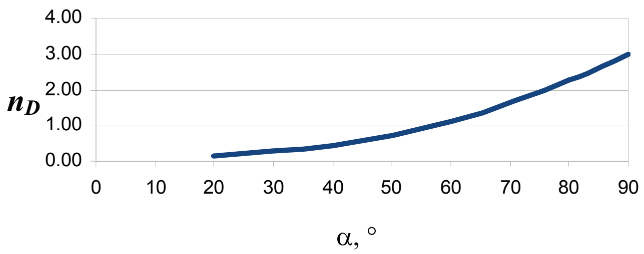


Figure 5. The additional loss factor used to assess the viscous losses localized at the cusp

3. Straight tube with wall suction

Consider again a rectangular element of a given aspect ratio $a_r=H/L$ (Figure 2). The fluid proceeds from left to right, and the "goal" of the device is to deliver the inlet mass flow rate m_0 to a portion of the solid domain that surrounds the tube, by means of a small amount of wall suction: we stipulate that the wall of the tube is porous and that a mass flow rate dm/dx permeates through this wall for each length dx. The channel is of circular section, and the formulae for steady, laminar, isothermal fully developed flow in a round pipe apply. The relevant variables are: m_0, dm/dx, $d(x)$ and the (constant) fluid properties. The inlet mass flow rate m_0 is prescribed, and it is convenient to assume that the mass flow rate leaked through the wall is proportional to the local wetted area:

$$\frac{dm}{dx} = \sigma r(x) \tag{3}$$

where σ is an area-weighted wall permeability. To facilitate the calculation of a closed-form expression, it is convenient to assume that the entire m_0 is delivered to the surrounding domain, so that the mass flow rate at the endpoint A is equal to zero. This implies the possibility that the tube diameter varies along x, to adjust to the additionally imposed stipulation of constant Re or constant fluid velocity.

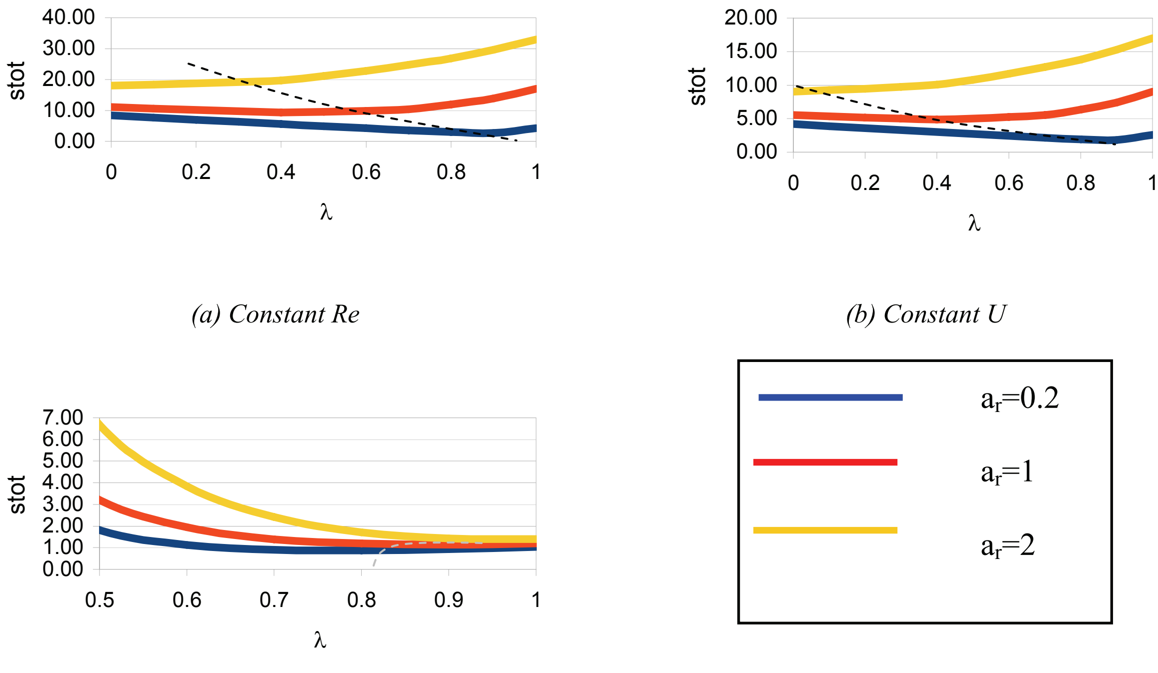
Under the above mentioned assumptions, the entropy generation rate in a slice of tube length dx is given by two terms: the first one being the viscous generation rate in the pipe and the second the entropy generation through the porous wall:

$$\frac{ds}{dx} = K_f \frac{m^2(x)}{r^4(x)} + \Theta\left(\frac{dm}{dx}\right)^2 \tag{4}$$

In Equation (4), the entropy generation rate through the porous wall has been expressed in terms of the local radius of the tube and of the (dimensional) constant Θ. The mass flow rate through the tube varies as:

$$m(x) = m_0 - \int_0^x \left(\frac{dm}{d\xi}\right) d\xi \tag{5}$$

With the assumption of $Re(x)=Re_0$, the radius of the tube varies downstream according to:



(a) Constant Re

(b) Constant U

(c) Constant Volume

Figure 6. Dimensionless entropy generation in the bifurcation with splitting losses. The dashed lines indicate the loci of the optimal λ. (figure is in color in the on-line version of the paper).

$$\frac{r(x)}{r_0} = \delta(x) = e^{-\gamma x} \tag{6}$$

With $\gamma=(\pi r_0 \sigma L)/m_0$. Equation (1) can then be integrated over the length L to obtain the total entropy generation rate in the flow:

$$\frac{S_{tot}}{S_{ref}} = \frac{(e^{2\gamma} - \Theta e^{-2\gamma}) - (1-\Theta)}{2\gamma} \tag{7}$$

Where $S_{ref} = \frac{8\upsilon}{\pi\rho T}\frac{m_0^2 L}{r_0^4}$ is the entropy generation rate in a tube of length L and radius r_0 in the absence of wall suction.

3.1 Results and discussion

Equation (6) was solved analytically for different values of Θ and γ: the case $Re_0=Re(x)$, which defines a diameter ratio $\delta=e^{-\gamma x}$, is compared here with a constant-diameter configuration.

The results, displayed in Figure 7 and 8, show that:

a) For Re(x)=constant, any value of γ, i.e., *any amount of spillage, causes a total entropy generation rate higher than in the case of an impermeable wall*;

b) As expected, *the higher Θ* (steeper pressure drop across the membrane) the *higher the irreversibility*;

c) There exists a minimum $\delta_1=r(L)/r_0$: when $\gamma=1$ (the entire inlet mass flow rate permeates through the wall), the tube has a truncated conoidal shape, with a $\delta(L)_{min}=0.368$

It is apparent that, for the configurations considered in this example, there is no advantage in increasing the wall spillage: for ANY value of the wall suction, the entropy generation is higher than that of a pipe with an impermeable wall. This holds true even in the limiting case of lossless membrane (Θ=0 in Eqn. 4), and indicates that the Re=constant assumption is not the most "efficient" one. It is instructive to compare the above configuration with a simpler one in which the tube diameter remains constant: under the same assumptions, the dimensionless entropy generation rate, shown in Figure 8, decreases with the amount of spillage γ, and is always lower than in the case Re=constant. Moreover, for the lower values of Θ and high values of γ (very permeable membrane and high mass spillage), there is an entropic advantage in extracting the fluid along the tube.

This simple comparison shows how the EGM method may be used in design to select the "less irreversible" flow configuration for a given task and boundary conditions.

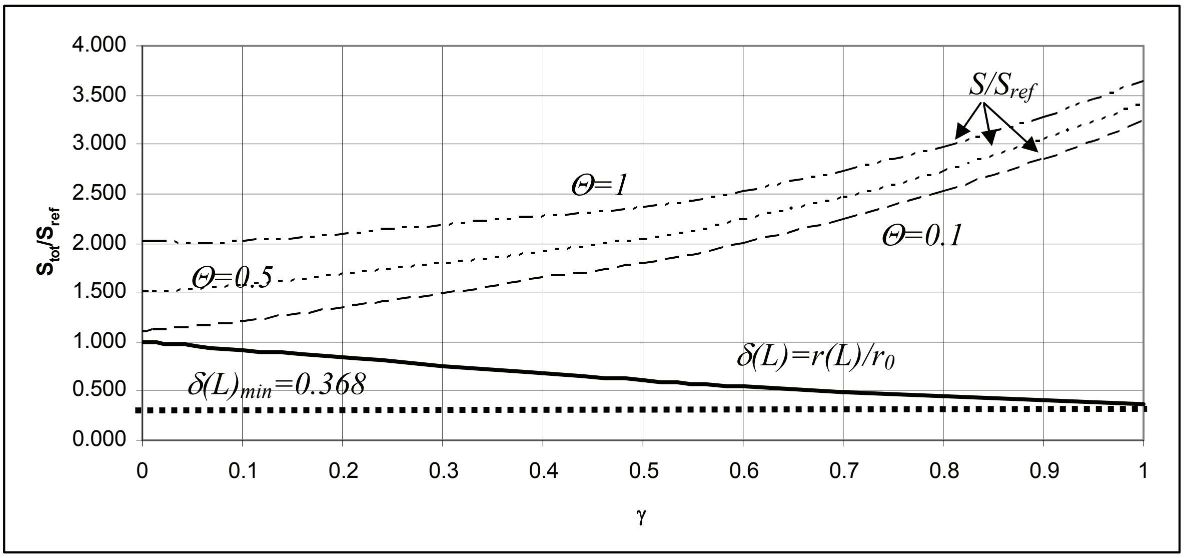


Figure 7. Dimensionless entropy generation in a circular tube with wall suction

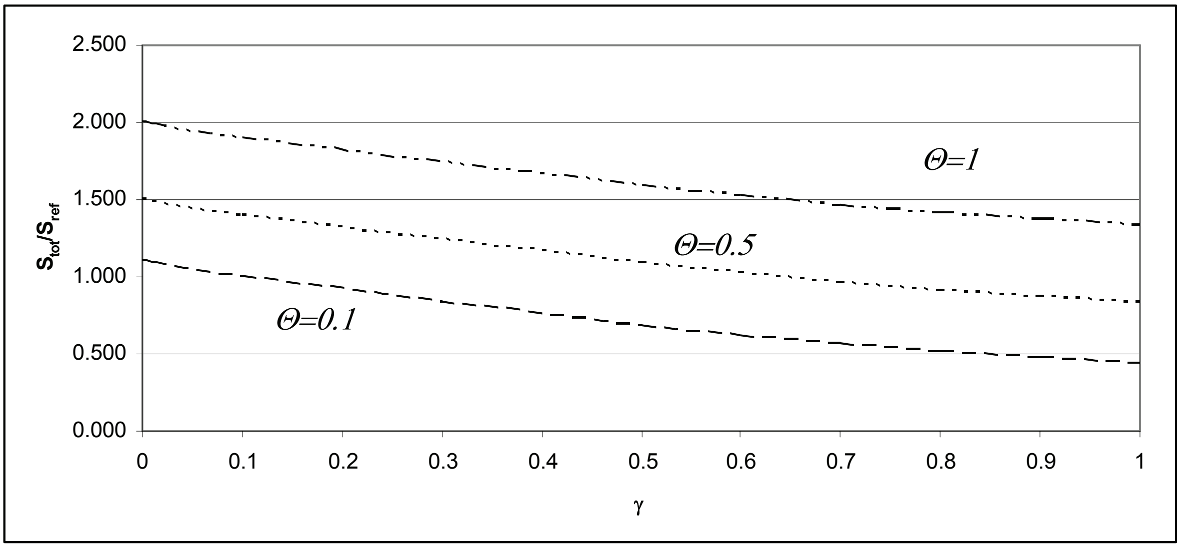


Figure 8. Dimensionless entropy generation in a constant diameter circular tube with wall suction

4. Bifurcated flow with wall suction

Consider again a rectangular element of a given aspect ratio $a_r=H/L$ (Figure 1). The flow configuration is the same as in Section 2, but now the additional goal of the device is to deliver the inlet mass flow rate m_0 to a portion of the solid domain that surrounds the tube, by means of a small amount of wall suction: a mass flow rate dm/dx permeates through the wall for each length dx. The channel is of circular section, and the flow is steady, laminar, isothermal and fully developed at the inlet. The relevant variables are: m_0, dm/dx, L_0, L_1, d_0, d_1, and the (constant) fluid properties. The inlet mass flow rate m_0 is prescribed, and the bifurcation is flow-wise symmetrical, as in Section 2. The mass flow rate leaked through the wall is assumed to be proportional to the local wetted area (Eqn. 3 above). To facilitate the calculation of a closed-form result, it is also convenient to assume that the entire m_0 is delivered to the surrounding domain, so that the mass flow rate at the endpoints A and B is equal to zero. A bifurcation occurs somewhere in the fluid path between $x=0$ and $x=L$, at a point S identified by a splitting parameter $\lambda=L_0/L$.

Under the above mentioned assumptions, the entropy generation rate in a slice of tube length dx is given by Eqn. (4) above, which can be integrated over the entire flow path and, under the assumption of a constant leakage flow through the wall provides the total entropy generation rate in the bifurcated path:

$$\frac{S_{tot}}{S_{ref}} = \lambda\left(1-\gamma\lambda+\frac{\gamma^2\lambda^2}{3}+\Theta\gamma^2\right)+2\frac{\left(L_1/L\right)_{equiv}}{\delta^4}\left\{1-\gamma\delta_1\left(L_1/L\right)_{equiv}+\frac{\gamma^2\left(L_1/L\right)^2_{equiv}\delta_1^2}{3}+\Theta\gamma^2\delta_1^2\right\} \tag{8}$$

where $S_{ref} = \frac{8\upsilon}{\pi\rho T}\frac{m_0^2 L}{r_0^4}$ is the entropy generation rate in an unsplit tube of length *L* and radius r_0.

Since $L_1/L = \sqrt{H^2/(4L^2) + (1-\lambda^2)}$, the total entropy generation rate is seen to depend on the aspect ratio of the domain (a_r=*H/L*), on the splitting length (λ=L_0/L), on the diameter ratio (δ=d_1/d_0) and on the wall suction parameter Θ. In this example, an arbitrary value Θ=0.5 has been imposed, and a minimum value of S_{tot}/S_{ref} is sought in the three-dimensional solution space identified by the three configuration parameters a_r, λ and δ.

4.1 Results and discussion

Equation (8) was solved analytically for different aspect ratios a_r and different bifurcation lengths λ, including bifurcation losses as in Section 2.2, for three physically meaningful situations:

1) The Reynolds number remains constant over the entire fluid path: Re_0=Re_1. This defines a diameter ratio δ=r_1/r_0=0.5;
2) The velocity remains constant over the entire fluid path: U_0=U_1. This defines a diameter ratio δ=r_1/r_0=0.707;
3) The volume $\mathcal{V}$ occupied by the fluid in all of the studied configurations remains constant. This defines a diameter ratio $\delta = \frac{r_1}{r_0} = \sqrt{\frac{(1-\lambda)}{\left(2L_1/L\right)}}$.

The results are displayed in Figures 9a-9c. The following features are apparent:

a) For all cases, the entropy generation rate grows with the aspect ratio a_r of the domain in which the bifurcation occurs;
b) For both Re=const and U=const, the entropy generation rate displays a rather flat behaviour up to λ~0.5: after that, it decreases to a minimum for lower aspect ratios and increases more rapidly for higher aspect ratios. The optimal splitting ratios λ are in the range 0.8-0.9. This is a peculiar feature that can be checked by experiment, to validate or disprove the present approach;
d) For the constant $\mathcal{V}_{fluid}$ case, the lower aspect ratios display a rather flat behaviour, while for a_r>1 the entropic penalty to bifurcating becomes extremely high.

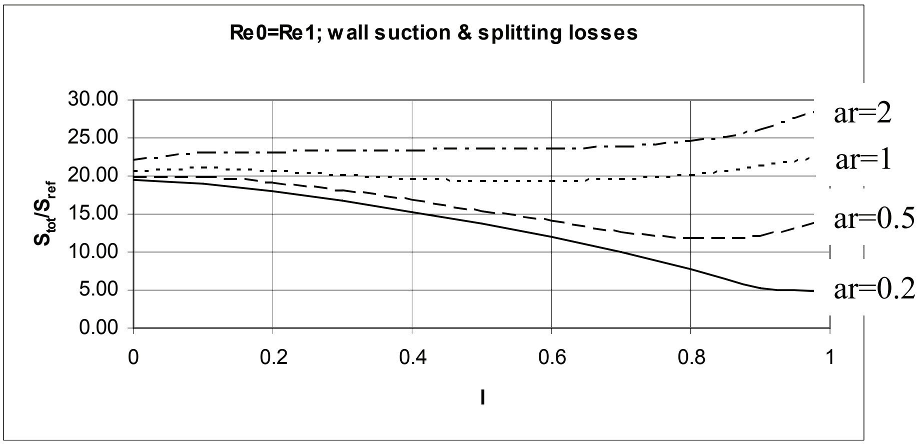


Figure 9a. Dimensionless entropy generation in a bifurcation with wall suction: Re(x)=Re_0

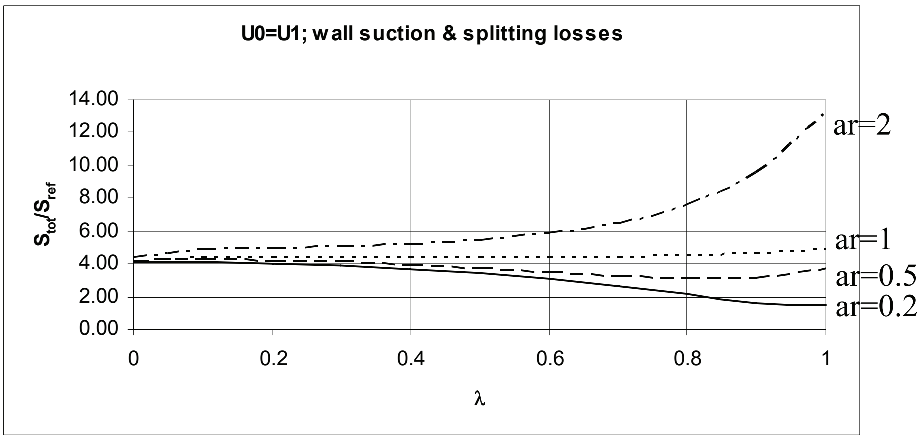


Figure 9b. Dimensionless entropy generation in a bifurcation with wall suction: U(x)=U_0

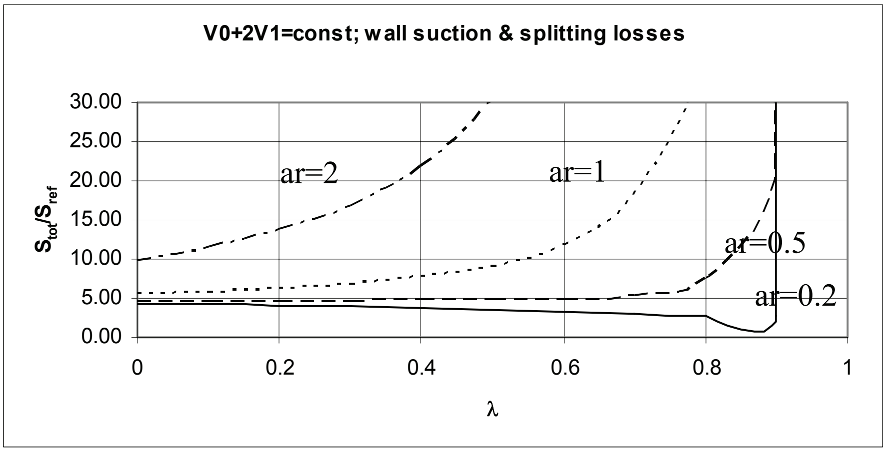


Figure 9c. Dimensionless entropy generation in a bifurcation with wall suction: $\mathcal{V}_{\lambda}+2\mathcal{V}_1=\mathcal{V}_0$

As remarked in Section 2, the diameter ratio is completely specified once the physical flow type has been assigned (constant *Re*, constant *U*, constant fluid volume), and therefore is not a relevant parameter in the optimization.

Also in this case, the minimum entropy generation indicates that the splitting ratio λ and the diameter ratio δ are correlated in a way profoundly different from those dictated by either one of the semi-empirical or geometric laws. A more detailed discussion of the physical features of such optimal configurations is presented below.

4.2 Some considerations on the physics of the outcome of the present analysis

Given the numerous simplifying assumptions made at the onset, the question arises of the physical validity of the above derived results. This Section offers some reflections on this -important- side of the issue.

With respect to the case of the absence of wall suction, discussed in Section 2, the optimal configurations display some interesting differences:

a) In the presence of wall suction there is in practice no optimal splitting ratio for aspect ratios H/L higher than 1: for all of the configurations that bifurcate between λ=0 or λ=0.8, the entropic penalty is rather high and almost constant. The marked difference between these results and those obtained in Section 2 demonstrates once more that the entropy generation rate depends strongly on the *purpose* of the device, and therefore is a significant indicator of the correlation between *shape* and *function*;
b) In general, the lower the aspect ratio, the lower the S_{min}: this is a physically meaningful result, because if the goal of the structure is that of delivering fluid through the wall of the tube to the surrounding domain, the slimmer the domain is, the less resources will be spent to perform the task. The present analysis reinforces the suggestion that emerged in Section 2: in a domain with an *H/L>>1* the flow ought to occur *in the direction perpendicular to that stipulated here*.

5. Conclusions

The study reported here is performed under a strongly idealized set of assumptions that may limit its practical applications: in fact, what has been discussed in this paper is a metaphor rather than a design paradigm. It is clear that an experimental or numerical validation of the results obtained here is in order, and this will be done in a follow-up article. However, even at the present level of an analytical and strongly simplified model, the general trends displayed by the solution are very relevant, and can be summarized as follows:

a) For the examined situations, a configuration exists that displays the lowest viscous entropy generation rate compatible with the imposed constraints;
b) In all cases, the diameter ratio δ can be derived from purely phenomenological considerations: there is no *a priori* optimal value for this parameter;
c) The entropy generation rate appears to be a consistent Lagrangian for the identification of the "optimal" configuration, and furthermore, the optima thus derived appear different from those suggested by both allometric and arithmetic/geometric models (Bejan & Lorente 2004; Ramos-Alvarado et al. 2009).

A general, very widely published theory exists (Constructal Theory, formulated by Bejan (Bejan 1997; Bejan 2000; Bejan et al. 2000; Bejan & Lorente 2004)) that attempts to explain and interpret the geometry of material and immaterial flows without recurring to any explicit "optimization method" or "objective function". It is very likely that an intimate, hitherto undiscovered, relationship exists between Constructalism and Entropy Generation Minimization (see also (Bejan A. 2000; Robbe & Sciubba 2009; Robbe et al. 2006; Sciubba 2005)): this is in fact a problem left to future studies. This preliminary investigation demonstrates -on the basis of simple analytical considerations- that the entropy generation rate is indeed a proper quantifier for the bifurcation topology, in that it identifies the shape that, for each set of assigned boundary conditions, performs the assigned task (to deliver a given mass flow rate) with the minimal exergy destruction (irreversible entropy generation rate). It is useful here to stress the fundamental similarities and differences between the EGM method and the present formulation of Constructal Theory (CT):

a) In CT and EGM alike, "the structure springs out of thermodynamic optimization" (Bejan 2003): this is an extremely important feature of both methods, in that they solve an inverse problem (given the purpose, find the most efficient geometry) rather than a direct one (given a set of geometries, find the most efficient one);

b) In all of the published applications, CT basically starts with a search for the "minimum resistance to the flow" of some quantity whose transport is driven by a gradient of a physical parameter (Bejan & Lorente 2008). In fluid dynamic applications, CT invariably minimizes the Δp between the source and the user, and then proceeds to a separate search for a secondary optimum, that can be, depending on the application, the minimum material temperature, the maximum heat transfer rate, etc. In some cases, CT even contends that this second optimization is not needed, because the first one already provides a satisfactory optimum (Bejan & Lorente 2008). By contrast, EGM minimizes the total entropy generation rate (viscous plus thermal), i.e. the total rate of irreversibility in the transfer process. In the context of resource use efficiency, EGM is thus more comprehensive than CT.

c) CT -by its very nature- produces (and justifies) branching of the (heat or fluid) transporting structure, and successfully demonstrates that it is the "function" that creates the "shape". But it does so by posing *ad hoc* and at times unclearly stated assumptions: thus, d_1/d_0 is either assumed from the Hess-Murray law (Bejan 2000; Bejan & Lorente 2008, Wechsatol et al. 2006) or derived from the assumption of a fixed volume of fluid (Bejan & Marden 2008) or minimal length (Bejan & Lorente 2007) or fixed elemental geometry (Bejan et al 2008). It ought to be considered though that the so-called Hess-Murray's law was originally derived under a completely different reasoning (Hess 1914; Murray 1926), and its tout-court application to different situations is unjustified. By contrast, a correct application of the EGM does not require additional phenomenological assumptions, and permits to treat any conceivable geometry and any physical phenomenon, solely on the basis of the Second Law of Thermodynamics, which is a conceptual and practical advantage with respect to CT.

A last remark: the EGM procedure adopted in the present study is perfectly *falsifiable*: if a minimum entropy generation rate configuration can be demonstrated to exist outside of the ranges identified by the method presented here, then the method is incorrect and needs to be revised (it has been "falsified"). This feature suggests a verification procedure: generate - numerically or experimentally- a sufficiently extensive series of bifurcated configurations, and identify the "least dissipative" ones. A multi-variable fit of the generated solutions may then be used to heuristically determine the underlying Lagrangian.

List of Symbols

a_r	=H/L, aspect ratio
d	Tube diameter
H	Domain height
L	Domain length
m	Mass flow rate
n_d	Branching loss factor
r	Tube radius
Re	=Ud/ν, Reynolds number
s	Local entropy generation
S	Total entropy generation rate
T	Absolute temperature
U	Mean flow velocity
$\mathcal{V}$	Total volume of fluid
x	Axial coordinate
α	Bifurcation semi-angle
γ	Wall suction mass ratio
δ	$=r_1/r_0$, Diameter ratio
Θ	Seeping flow loss factor
λ	$=L_0/L$, splitting length ratio
ν	Fluid viscosity
ρ	Fluid density
σ	Wall permeability
0	(suffix): main branch
1	(suffix): split branch

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