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Analytical Design of PD Controllers for Time Delay Systems in the Second Order

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ABSTRACT

This paper intends to present the design scheme of a Proportional Derivative controller for stability and performance of time delay systems in the second order. Inspired from Bode's ideal system characteristics, phase margin and gain crossover frequency specifications are considered for the system. Then, these specifications are used to obtain the parameters of the controller. The method proposed in the study achieves general computation equations for mentioned systems. Analytically derived formulas by the proposed method are tested with existing plants in the literature and the results are illustrated graphically. It is shown that the tuning method satisfies desired gain crossover frequency and phase margin specifications.

Anahtar kelimeler: SOPTD Plant, PD Controller, Gain Crossover Frequency, Phase Margin.

İkinci Derece Zaman Gecikmeli Sistemler için Analitik PD Denetleyici Tasarımı

ÖZET

Bu yayında ikinci derece zaman gecikmeli sistemlerin kararlılığı ve performansı için oransal türev denetleyicinin analitik tasarımı şeması gösterilmiştir. Bu sistem için Bode'nin ideal transfer fonksiyonunun karakteristiğinden esinlenerek kazanç kesim frekansı ve faz payı gereksinimleri ele alınmıştır. Daha sonra bu gereksinimler denetleyicinin parametrelerini elde etmek için kullanılmıştır. Bu çalışmada önerilen yöntem sözü edilen sistemler için genelleştirilmiş denklemleri vermektedir. Önerilen yöntemle analitik olarak türetilen formüller literatürde var olan sistemler üzerinde denenmiş ve sonuçlar grafiksel olarak

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gösterilmiştir. Bu ayarlama yönteminin istenen kazanç kesim frekansı ve faz payı özelliklerini sağladığı gösterilmiştir.

Keywords: SOPTD Sistem, PD Denetleyici, Kazanç Kesim Frekansı, Faz Payı.

1. INTRODUCTION

In the last decades, second order plus time delay (SOPTD) transfer functions are widely used in approximate modeling of a large number of industrial processes. Some examples of SOPTS plants can be found in modeling of chemical (Madhuranthakam, et al., 2008), electronics (Ramakrishnan and Chidambarami 2003), and control processes (Rajapandiyan and Chidambaram, 2012) etc. The list of studies can be extended thus, control of such plants draw interest of much researchers. This thought motivated researchers on better design methods or different controller ideas (Lee, et al., 2018; Liu, et al., 2016; Tajaddodianfar, et al., 2017; Wang, et al., 2014).

As being the dominating controller for the industrial processes, proportional integral derivative (PID) controllers are utilized in so many areas of research. For instance, optimal tuning of PID controllers is presented in (Madhuranthakam, et al., 2008). Implementation of PID controllers for FOPTD plants can also be found in (Rashid, et al., 2017; Cvejn, 2016). The method in this paper utilizes proportional derivative (PD) controllers which is a type of PID controllers. There can be found numerous studies on the tuning of PD controllers in the literature as (Padhy and Sidhartha, 2017; Padhy, et al., 2017).

This paper intends to present a tuning method of PD controllers for the stability and performance of plants described by SOPTD transfer functions. Parameters are tuned to satisfy gain and phase specifications based on Bode’s ideal transfer function. General components of a Bode diagram are also reminded. The method gives generalized parameters of the PD controller for SOPTD plants. Efficiency of the proposed equations are tested with existing plants in the literature and the results are illustratively given.

After a short literature survey, this paper is organized as follows. Section 2 gives remindful information about PD controllers and SOPTD plans. Section 3 has the equations to obtain the PD controller. Illustrative examples clarify the process in section 4 and concluding remarks are given in section 5.

2. PD CONTROLLER DESIGN FOR SOPTD PLANTS

This section gives brief information about transfer functions of a PD controller and a SOPTD plant. General propoerties of a Bode diagram is also briefly reminded. Following equation denotes the general representation of a SOPTD plant.

$$P(s)=\frac{K}{(T_1 s+1)(T_2 s+1)}\mathrm{e}^{-Ls} \tag{1}$$

where, K is the gain, T is the time constant and L is the delay. Similarly, transfer function of a PD controller is given as follows.

$$C(s)=k_p+k_d s \tag{2}$$

Figure 1 shows the closed loop scheme of the system implemeted in this paper.

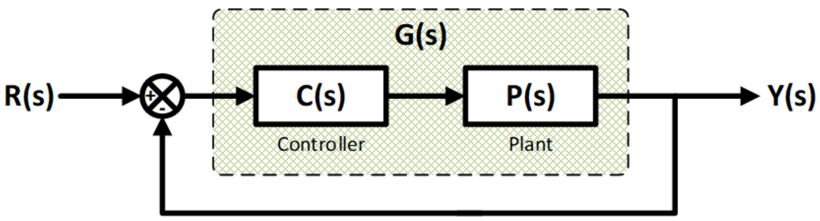

Figure 1. Block diagram of the closed loop system

Considering Figure 1, the system can be shown with the following equation.

$$G(s)=C(s)P(s) \tag{3}$$

where, $R(s)$ is the input signal and $Y(s)$ is the output signal. $P(s)$ is the transfer function of the SOPTD plant in Eq. 1 and $C(s)$ is the PD controller in Eq. 2. Figure 2 shows an example of Bode diagram of an open loop system $G(s)$.

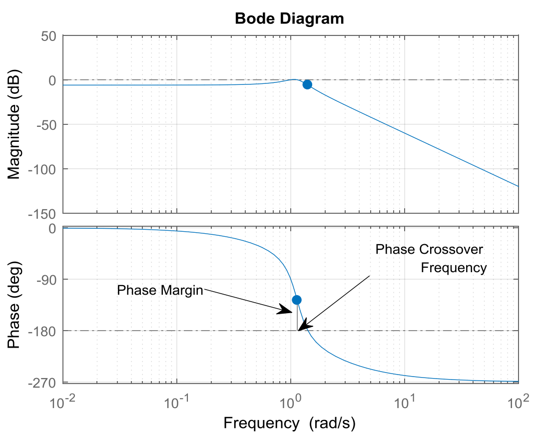

Figure 2. An example of Bode diagram of the system $G(s)$

It would be useful to describe the components of a Bode diagram. The frequency value that the gain curve crosses the $0dB$ line is called as the gain crossover frequency and denoted as ω_c is this paper. Difference of the phase value with the -180 degrees line at the gain crossover frequency is the phase margin and denoted as ϕ_m.

Now, desired gain and phase specifications can be given.

3. DESIGN SPECIFICATIONS OF PD CONTROLLER FOR SOPTD PLANT

In order to analyse the system in the frequency domain, Laplace operator s should be replaced with $j\omega$ in Eq. 3 as,

$$G(j\omega) = C(j\omega)P(j\omega). \tag{4}$$

Frequency response of the SOPTD plant in Eq. 1 can be written as,

$$\begin{aligned} P(j\omega) &= \frac{K}{\left(T_1(j\omega)+1\right)\left(T_2(j\omega)+1\right)}\mathrm{e}^{-L(j\omega)} = \frac{K}{(1+jT_1\omega)(1+jT_2\omega)}\mathrm{e}^{-jL\omega} \\ &= \left(\frac{K-KT_1T_2\omega^2}{\left(1+T_1^2\omega^2\right)\left(1+T_2^2\omega^2\right)} + j\left(-\frac{K\left(T_1+T_2\right)\omega}{\left(1+T_1^2\omega^2\right)\left(1+T_2^2\omega^2\right)}\right)\right)\mathrm{e}^{-jL\omega} \quad . \\ &= \left|P(j\omega)\right|e^{j\angle P(j\omega)} = \sqrt{\frac{K^2}{\left(1+T_1^2\omega^2\right)\left(1+T_2^2\omega^2\right)}}e^{-j(\arctan\left(\frac{K(T_1+T_2)\omega}{K-KT_1T_2\omega^2}\right)+L\omega)} \end{aligned} \tag{5}$$

Similarly, frequency response of the PD controller is,

$$C(j\omega) = k_p + k_d\left(j\omega\right) = k_p + jk_d\omega. \tag{6}$$

Magnitude and phase of the SOPTD plant are obtain in the following way

$$\left|P(j\omega)\right| = \sqrt{\frac{K^2}{\left(1+T_1^2\omega^2\right)\left(1+T_2^2\omega^2\right)}}, \tag{7}$$

$$\angle P(j\omega) = -\arctan\left(\frac{K\left(T_1+T_2\right)\omega}{K-KT_1T_2\omega^2}\right) - L\omega. \tag{8}$$

Likewise, magnitude and phase of the PD controller are,

$$|C(j\omega)| = \sqrt{k_p^{\,2} + (k_d\omega)^2} = \sqrt{k_p^{\,2} + k_d^{\,2}\omega^2}\,, \tag{9}$$

$$\angle C(j\omega) = \arctan\left(\frac{k_d\omega}{k_p}\right). \tag{10}$$

Therefore, magnitude and phase of the system can be written as follows.

$$|G(j\omega)| = |C(j\omega)P(j\omega)| = |C(j\omega)||P(j\omega)| \tag{11}$$

$$\angle G(j\omega) = \angle C(j\omega)P(j\omega) = \angle C(j\omega) + \angle P(j\omega) \tag{12}$$

Assuming that the gain crossover frequency is ω_c and the phase margin is ϕ_m, following gain and phase specifications are desired to be satisfied.

$$|G(j\omega_c)| = 1 \tag{13}$$

$$\angle G(j\omega_c) = \phi_m - \pi \tag{14}$$

Considering Eq. 11 and Eq. 13, gain specification of the system can be rewritten as,

$$|G(j\omega_c)| = |C(j\omega_c)||P(j\omega_c)| = \sqrt{k_p^{\,2} + k_d^{\,2}\omega_c^{\,2}}\sqrt{\frac{K^2}{\left(1+T_1^2\omega_c^{\,2}\right)\left(1+T_2^{\,2}\omega_c^{\,2}\right)}} = 1. \tag{15}$$

Similarly, considering Eq. 12 and Eq. 14 phase margin specification of the system is,

$$\begin{aligned} \angle G(j\omega_c) &= \angle C(j\omega_c) + \angle P(j\omega_c) \\ &= \arctan\left(\frac{k_d\omega_c}{k_p}\right) - \arctan\left(\frac{K(T_1+T_2)\omega_c}{K - KT_1T_2\omega_c^{\,2}}\right) - L\omega_c = \phi_m - \pi \end{aligned} \tag{16}$$

Together solution of Eq. 15 and Eq. 16 leads to the following parameters of the PD controller.

$$k_p = \pm\frac{\sqrt{1+T_1^2\omega_c^{\,2}}\sqrt{1+T_2^{\,2}\omega_c^{\,2}}}{K\sqrt{1+\tan\left(\phi_m + L\omega_c + \arctan\left(\frac{K(T_1+T_2)\omega_c}{K - KT_1T_2\omega_c^{\,2}}\right)\right)^2}}\,, \tag{17}$$

$$k_d = \pm \frac{\sqrt{1+T_1^2\omega_c^2}\sqrt{1+T_2^2\omega_c^2}\tan\left(\phi_m + L\omega_c + \arctan\left(\frac{K\left(T_1+T_2\right)\omega_c}{K-KT_1T_2\omega_c^2}\right)\right)}{K\omega_c\sqrt{1+\tan\left(\phi_m + L\omega_c + \arctan\left(\frac{K\left(T_1+T_2\right)\omega_c}{K-KT_1T_2\omega_c^2}\right)\right)^2}}. \tag{18}$$

Parameters of the PD controller in Eq. 17 and Eq. 18 can be used to obtain the PD controller to satisfy given gain and phase margin specifications in this paper. It would be clarifying to explain the results on illustrative examples.

4. ILLUSTRATIVE EXAMPLES

This section gives two examples to clarify the given procedure.

Example 1: Consider the following SOPTD plant provided from (Rajapandiyan and Chidambaram, 2012).

$$P_1(s) = \frac{0.3}{(s+1)(2s+1)}e^{-0.01s} \tag{19}$$

Desired gain crossover frequency is $\omega_c = 10 rad/\sec$ and the phase margin is $\phi_m = 45^o$. Replacing the unknown variables in Eq. 17 and Eq. 18, following PD controller is obtained.

$$C_1(s) = 4.972943x10^2 + 45.023269s \tag{20}$$

Bode diagram of the system $G_1(s) = C_1(s)P_1(s)$ is illustrated in Figure 3. It is clearly seen in the figure that the gain crossover frequency is tuned to be $\omega_c = 10 rad/\sec$ and the phase margin is $\phi_m = 45^o$. Thus, the proposed method is successfully implemented.

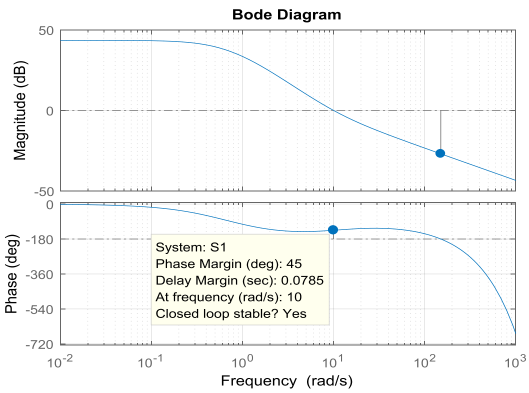

Figure 3. Bode diagram of the system $G_1(s) = C_1(s)P_1(s)$.

We can also check the stability of the system with the step response of the closed loop system given in Figure 4.

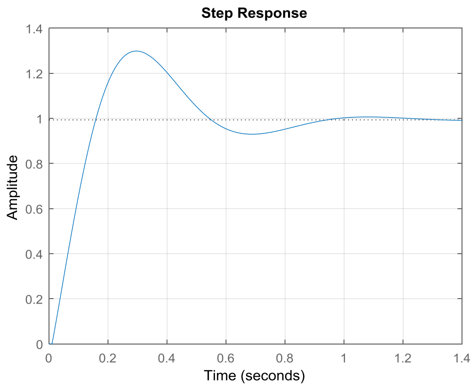

Figure 4. Step response of the closed loop system in Example 1.

The method can also be tested by varying phase margin values. Table 1 lists the parameters of the PD controller found for $\phi_m \in [30^o, 50^o]$ with increment steps of 5^o at $\omega_c = 10 rad/\sec$.

Table 1. Parameters of the PD controller found for $\phi_m \in [30^o, 50^o]$.

ϕ_m	k_p	k_i
30°	5.968782122937050 $x10^2$	30.618215297463372
35°	5.679213775348870 $x10^2$	35.703840136599815
40°	5.346423181727249 $x10^2$	40.517737193728628
45°	4.972943079434104 $x10^2$	45.023269805534540
50°	4.561615877561335 $x10^2$	49.186148148326730
55°	4.115572024481349 $x10^2$	52.974690204301915
60°	3.638206183244730 $x10^2$	56.360062880829844

Bode diagrams of the systems with $P_1(s)$ and the 7 controllers listed in Table 1 are given in Figure 5.

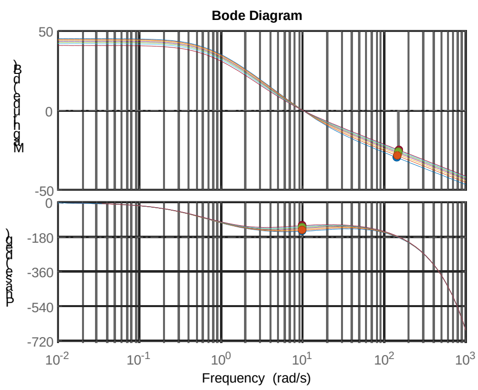

Figure 5. Bode diagrams of the systems with $P_1(s)$ and the 7 controllers listed in Table 1.

Similarly, stability of the systems with $P_1(s)$ and the 7 controllers can be checked with the step responses in Figure 6.

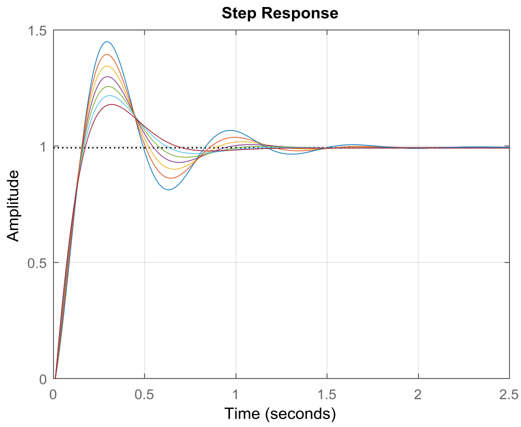

Figure 6. Step responses of the systems with $P_1(s)$ and the 7 controllers.

From this example, efficiency of the proposed method is clearly shown. It would be advantageous to apply the proposed method in another example.

Example 2: Consider the following SOPTD plant.

$$P_2(s) = \frac{1}{(0.9299s+1)(3.2004s+1)} e^{-4.3s} \tag{21}$$

Gain crossover frequency for this example is desired to be $\omega_c = 0.3 rad/\sec$. Phase margin is assumed to change in the interval $\phi_m \in [30^o, 60^o]$ with increment steps of 5^o. Table 2 shows the parameters of the PD controller obtained for this case.

Table 2. Parameters of the PD controller found for $\phi_m \in [30^o, 60^o]$.

ϕ_m	k_p	k_d
30°	1.378772602583570	-1.375910027504024
35°	1.409501494688229	-0.970114440256468
40°	1.429503229338038	-0.556935696347445
45°	1.438625581254936	-0.139518335502251
50°	1.436799123832608	0.278960844119590
55°	1.424037757514882	0.695316963276517
60°	1.400438604004961	1.106381300499049

Figure 7 shows the Bode diagrams and Figure 8 gives the step responses of the systems with $P_2(s)$ and the 7 controllers in Table 2.

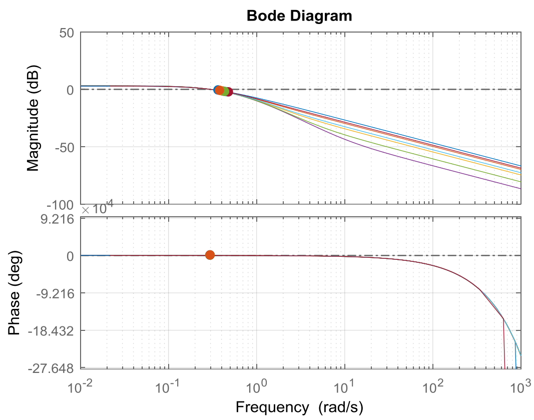

Figure 7. Bode diagrams of the system with $P_2(s)$ and the 7 controllers in Table 2.

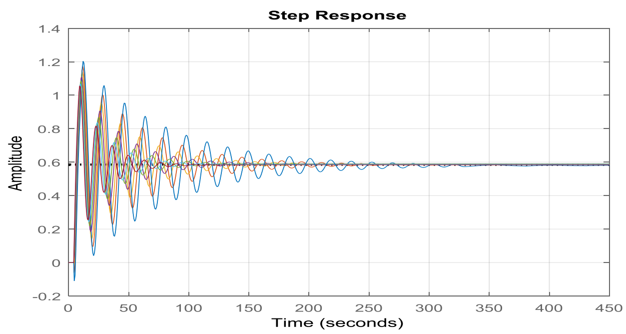

Figure 8. Step responses of the systems with $P_2(s)$ and the 7 controllers in Table 2.

Thus, the method is proved illustratively.

5. CONCLUSIONS

This paper proposes a design scheme of proportional integral controllers for performance and stability of second order plus time delay plants. Gain and phase specifications for the system are inspired from Bode's ideal loop. The method analytically obtains general computation equations for the mentioned systems.

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