Testing exponential posterior distribution with scale parameter for NBUL class

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Received: 01.02.2015; Accepted: 05.05.2015

Abstract. Recently non-parametric testing using goodness of fit approach for testing standard exponential with scale parameter against the new better than used in the Laplace transform order class (NBUL) has been introduced. In this paper we represent non-parametric testing for NBUL class using goodness of fit approach with \( \lambda \) parameter that \( \lambda \) parameter has prior distribution \( E(1) \). The testing Bayesian statistic based on \( u \)-statistic, It’s provide by departure of nun hypothesis. And also we represent \( \hat{q}(x, \lambda) \) statistic.

Keywords: Bayesian estimation, NBUL aging class, Posterior distribution, U-statistic, Monte Carlo method.

1. INTRODUCTION

Recently non-parametric testing using goodness of fit approach for testing standard exponential with scale parameter against the new better than used in the Laplace transform order class (NBUL) has been introduced by P. Nasiri, R. Lotfi, F. Shojaat (2014). During the past decades, various classes of life distributions have been proposed in order to model different aspects of ageing. The best known these classes are Increase failure rate (IFR), Increase average failure rate (IFRA), New better than used (NBU), New better than used in failure rate (NBUFR), New better than used in average failure rate (NBUFRA), New better than used in convex order (NBUC), Decrease mean residual life (DMRL), New better than used in expectation (NBUE), Harmonically New better than used in expectation (HNBUUE) and Laplace class (\( \mathcal{L} \)).

The following are the relation between these classes:

\[
\text{IFR} \subset \text{IFRA} \subset \text{NBU} \subset \text{NBUFR} \subset \text{NBUFRA}
\]

\[
\text{NBU} \subset \text{NBUC} \subset \text{NBUE} \subset \text{HNBUUE} \subset \mathcal{L}, \quad \text{IFR} \subset \text{DMRL} \subset \text{NBUE}
\]

Properties and application of these ageing notation can be found, for instance, in Bryson and Siddiqui [2], Barlow and Proschan [1], Rolksi [6], Kelefsjo [4]. And a new class of life distribution New better than used in the Laplace order (NBUL), introduced by Wang [7], L.S. Diab[3] and testing exponential distribution with scale parameter for NBUL class offer by P. Nasiri, R. Lotfi, F. Shojaat (2014)[5].

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Special Issue: The Second National Conference on Applied Research in Science and Technology

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In this paper a discussion of testing exponential distribution with \( \lambda \) parameter that \( \lambda \) parameter has prior distribution of standard exponential for \( \delta(s, \lambda) \) statistic of non-censored data and Bayesian estimation of \( \lambda \) parameter is considered in section 2. In section 3 Monte Carlo null distribution critical points are simulated for sizes 15(1)80. The power estimate for two distributions are also calculated.

2. TESTING FOR NBUL CLASS WITH NON-CENSORED DATA

In the section we test the null hypothesis \( H_0: F \) is exponential with \( \lambda \) parameter, that \( \lambda \) parameter has prior distribution of standard exponential distribution against \( H_1: F \) is NBUL.

We first find the posterior distribution for \( \lambda \) parameter. And we achieve the Bayesian estimation for \( \lambda \) parameter.

We take

\[
F(x) = 1 - e^{-\lambda x} \quad \text{and} \quad \lambda \in e^{-\lambda}
\]

the posterior distribution for \( \lambda \) parameter will be

\[
\Pi(\lambda|x) \sim \text{Gama}(2, \bar{x} + 1)
\]

And Bayesian estimation with error square loss function for \( \lambda \) parameter will be

\[
\hat{\lambda} = \frac{2}{\bar{x} + 1}
\]

According to the Eq.(1) we use a measure of departure from \( H_0 \) for finding \( u \)-statistic.

\[
\delta(s, \lambda) = E \left( \bar{F}(t|x) \int_0^\infty e^{-s\lambda} \bar{F}(\lambda|x)d\lambda - \int_0^\infty e^{-s\lambda} \bar{F}(\lambda|x + t)d\lambda \right) \geq 0 \tag{2}
\]

\[
\delta(s, \lambda) = \int_0^\infty \left[ \bar{F}(t|x) \int_0^\infty e^{-s\lambda} \bar{F}(\lambda|x)d\lambda - \int_0^\infty e^{-s\lambda} \bar{F}(\lambda|x + t)d\lambda \right] dF_0(t|x) \tag{3}
\]

That

\[
F_0(t|x) \sim \text{Gama}(2, t + 1)
\]

Lemma 2.1 Let \( X \) be random variable with posterior distribution \( F(\lambda|x) \) and let

\[
\Phi(s) = \int_0^\infty e^{-s\lambda} dF(\lambda|x) \tag{4}
\]

\[
\delta(s, \lambda) = \frac{1}{s} \left[ \Phi \left( \frac{2}{\lambda} \right) \left( \frac{2}{\lambda} - s + s\Phi(s) \right) + \Phi(s) - 1 \right] \tag{5}
\]

Proof. According to the Eq. (3) we have

\[
\delta(s, \lambda) = \int_0^\infty \left[ \bar{F}(t|x) \int_0^\infty e^{-s\lambda} \bar{F}(\lambda|x)d\lambda - \int_0^\infty e^{-s\lambda} \bar{F}(\lambda|x + t)d\lambda \right] \cdot t e^{-t(\frac{2}{\lambda})} dt
\]

\[
= \int_0^\infty \bar{F}(t|x) e^{-t(\frac{2}{\lambda})} dt \int_0^\infty e^{-s\lambda} \bar{F}(\lambda|x)d\lambda - \int_0^\infty \int_0^\infty t e^{-s\lambda} \cdot e^{-t(\frac{2}{\lambda})} \bar{F}(\lambda|x + t)d\lambda dt
\]

Then

\[
\int_0^\infty \bar{F}(t|x) e^{-t(\frac{2}{\lambda})} dt \int_0^\infty e^{-s\lambda} \bar{F}(\lambda|x)d\lambda \quad (\ast)
\]

506
Testing exponential posterior distribution with scale parameter for NBUL class

\[ \int_0^\infty \int_0^\infty t \cdot e^{-s \lambda} \cdot e^{-t\left(\frac{s}{\lambda}\right)} F(\lambda|x + t) d\lambda dt \quad (**) \]

We let of Eq.(4) that

\[ u = e^{-s \lambda} \quad dv = dF(\lambda|x) dx \]

And we have

\[ \phi(s) = \int_0^\infty e^{-s \lambda} dF(\lambda|x) = \int_0^\infty e^{-s \lambda} F(\lambda|x) + \int_0^\infty sF(\lambda|x)e^{-s \lambda} d\lambda \]

\[ = s \int_0^\infty e^{-s \lambda}(1 - F(\lambda|x))d\lambda \]

Then we have

\[ \int_0^\infty e^{-s \lambda} F(\lambda|x)d\lambda = \frac{1}{s}(1 - \phi(s)) \]

Hence part (*) will equivalent to

\[ \int_0^\infty t \cdot F(t|x)e^{-t\left(\frac{s}{\lambda}\right)} dt \int_0^\infty e^{-s \lambda} F(\lambda|x)d\lambda = \frac{1}{s} \left( \phi\left(\frac{s}{\lambda}\right) - 1 \right)(1 - \phi(s)) \]

For part (**) consider

\[ \int_0^\infty \int_0^\infty t \cdot e^{-s \lambda} e^{-t\left(\frac{s}{\lambda}\right)} F(\lambda|x + t)d\lambda dt \overset{\lambda|x + t = \nu}{=} \int_0^\infty \int_0^\infty t \cdot e^{-s \lambda} e^{-t\left(\frac{s}{\lambda}\right)} F(\lambda)d\lambda d\nu \]

\[ = \int_0^\infty e^{-s \lambda} \Phi(u) du \int_0^\infty t \cdot e^{-t\left(\frac{s}{\lambda}\right)} dt = \frac{1}{s} \left( \frac{s}{\lambda} - s \right)^2(1 - \phi(s)) \]

Hence, u-statistic for NBUL class with Bayesian estimation for \( \lambda \) parameter will be

\[ \delta(s, \lambda) = \frac{1}{s} \left( \frac{s}{\lambda} - s \right)^2 \left[ \phi\left(\frac{2}{\lambda}\right) \left( \frac{2}{\lambda} - s + s \phi(s) - \phi(s) \right) + \phi(s) - 1 \right] \quad (6) \]

Then the result follows.

To estimate \( \delta(s, \lambda) \), let \( X_1, ..., X_n \) be a random sample from \( F(\lambda|x) \). So the empirical from of \( \delta(s, \lambda) \) in Eq.(6) is as follow:

\[ \hat{\delta}(s, \lambda) = \frac{1}{s} \left( \frac{s}{\lambda} - s \right)^2 \left[ \sum_{i=1}^n \sum_{j=1}^n e^{-x_i} \left( \frac{2}{\lambda} - s + s e^{-sX_j} - e^{-sX_j} \right) + e^{-sX_j} - 1 \right] \quad (7) \]

To find the limiting distribution of \( \hat{\delta}(s, \lambda) \), we make from two observations,

\[ \Phi_{s,\lambda}(X_1, X_2) = e^{-x_1} \left( \frac{2}{\lambda} - s + s e^{-sX_2} - e^{-sX_2} \right) + e^{-sX_2} \quad (8) \]

Then

(i) \( \Phi_{s,\lambda}(X_1) = E\left[ \phi_{s,\lambda}(X_1, X_2) \right] \)

\[ = \int_0^\infty \left[ e^{-x_1} \left( \frac{2}{\lambda} - s + s e^{-sX_2} - e^{-sX_2} \right) + e^{-sX_2} \right] \cdot x_2 e^{-x_2} (\frac{2}{\lambda}) dx_2 \]

\[ = e^{-x_1} \left( \frac{\lambda^3 s^3 (\lambda s - 1) + 4 \lambda^2 s (\lambda - s) - 12 \lambda}{4(2 + \lambda s)^2} \right) - \frac{\lambda^2}{(2 + \lambda s)^2} \quad (9) \]
\begin{align*}
(ii) \Phi_{2,3, \lambda}(X_1) &= E \left( \Phi_{s, \lambda}(X_2, X_1 | X_1) \right) \\
&= \int_0^{\infty} e^{-x_2} \left( \frac{2}{\lambda} - s + se^{-sx_1} - e^{-sx_1} \right) e^{-sx_1} \, dx_2 \\
&= \left( \frac{2}{\lambda} - s + se^{-sx_1} - e^{-sx_1} \right) \left( \frac{-\lambda^2}{(2 + \lambda)^2} \right) - \frac{\lambda^2}{4} e^{-sx_1}
\end{align*}

We have
\[ \psi_{s, \lambda}(X_1) = \Phi_{1, s, \lambda}(X_1) + \Phi_{2, 3, \lambda}(X_1) \quad \forall (s \neq 1, \lambda \neq -2) \]

**Theorem 2.1**

As \( n \to \infty \), \( \sqrt{n} \left( \delta(s, \lambda) - \delta(s, \bar{x}) \right) \), is asymptotically normal with mean \( \mu_{s, \lambda} \) and variance \( \sigma^2_{s, \lambda} \).

That
\[ \mu_{s, \lambda} = E \left( \psi_{s, \lambda}(X_1) \right) \]
\[ \sigma^2_{s, \lambda} = \text{var} \left( \psi_{s, \lambda}(X_1) \right) \]

3. **MONTE CARLO NULL DISTRIBUTION CRITICAL POINTS**

In this section we simulate the Monte Carlo null distribution critical points for \( \hat{\delta}(s, \lambda) \) statistic in Eq.(7). Based on \( N = 5000 \) simulated sample sizes \( n = 15 \) (1) 80 from the exponential distribution with scale parameter \( \lambda \) that \( \lambda \) distribution is standard exponential. Table 1 presents the upper percentage points of the \( \hat{\delta}(s, \lambda) \) statistic and fig presents the relation between critical values, sample size, and confidence levels. Also we perform and estimation of power \( \hat{\delta}(s, \lambda) \) statistic with significant level \( \alpha = 0.05 \), based on \( N = 1000 \) simulated sample sizes \( n = 15, 20, 25, 30 \) from posterior distribution with the survival function \( 1 - e^{-\lambda(1+\frac{x^2}{\lambda})} \) and posterior distribution with the survival function \( 1 - e^{-\lambda-x} \). Table 2 presents an estimation of the power for test statistic \( \delta(s, \bar{x}) \).

| Table 1: the upper percentages of \( \hat{\delta}(s, \lambda) \) with 5000 simulated values of posterior distribution \[ \Phi(\lambda|x) = \Gamma a(n, 2, \lambda + 1) \] with \( \lambda = 0.01 \) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| n  | p=90 | p=95 | p=98 | p=99 | n  | p=90 | p=95 | p=98 | p=99 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 15 | -0.0107 | -0.0084 | -0.0066 | -0.0056 | 48 | -0.1492 | -0.1329 | -0.1187 | -0.1037 |
| 16 | -0.0124 | -0.0101 | -0.0080 | -0.0073 | 49 | -0.1623 | -0.1444 | -0.1229 | -0.1123 |
| 17 | -0.0152 | -0.0120 | -0.0094 | -0.0082 | 50 | -0.1686 | -0.1524 | -0.1339 | -0.1228 |
| 18 | -0.0169 | -0.0142 | -0.0111 | -0.0092 | 51 | -0.1735 | -0.1598 | -0.1423 | -0.1277 |
Testing exponential posterior distribution with scale parameter for NBUL class

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It is noted from table 1 and fig that the critical values are increasing as the confidence level increasing and decreasing as the sample size increasing.

**Table 2.** power estimate for posterior distributions:

\[
(1) \Pr = (\lambda|x) = 1 - e^{-\lambda(1+\frac{x^2}{2})^x}
\]

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\[
(2) \Pr = (\lambda|x) = 1 - e^{-\lambda-x^4}
\]

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It is clear from the above table that our test has good power.
Testing exponential posterior distribution with scale parameter for NBUL class

4. CONCLUSIONS

In this paper we have introduce u-statistic based on a goodness of fit approach for testing exponentially with scale parameter $\lambda$ that prior distribution is standard exponential. Against the new better than used in the Laplace transform (NBUL) order. We provide critical values and power for posterior distributions with non-censored data.

REFERENCES