Economic Load Dispatch Problem with Ant Lion Optimization Using Practical Constraints

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Abstract
This paper presents Ant Lion Optimization (ALO) algorithm for solving Economic Load Dispatch (ELD) problem with practical constraints. ALO is a newly developed optimization algorithm, which draws inspiration from mimics, the hunting mechanism of antlions in nature. The antlions have a unique hunting mechanism and exhibit high capability of reaching global optima, exploring the search space to find the optimal solution within a low computational time. For practical ELD problem needs to take care about the characteristics of generators, and their operational constraints, such as ramp rate limits, prohibited operating zones, generation operating limits, transmission loss, valve-point loading and non-linear emission functions. In order to validate the potency of the proposed method, four case studies are investigated on different 6-unit systems and correlated with recently published ELD solution methods. The results of the present work shows that the proposed ALO is dominant than other methods to finding out optimal results. Stastical analysis of the results among 30 trails has been carried out to validate the ALO as a highly potent method. This algorithm is considered to be a promising best alternative algorithm for solving the ELD problem in power systems.

1. INTRODUCTION

ELD problem is a major issue in the electrical generation system. The require power demand optimally allocating the available generating units known as ELD. For a given load demand and operational constraints, this optimal allocation will reduce the total fuel cost. In general, the cost function of generator model by a quadratic function and later was solving the quadratic form by different methods. The quadratic form defined for the generator can be solved by different method like lambda iteration method, gradient based method, dynamic programming etc. [1]. Usually the above-mentioned methods offer only the local optimum point and also require evaluation of derivatives of the quadratic form defined for the cost function of the generator.

To overcome these shortcomings, a lot of nature based optimization methods have been applied of which the famous technique is Particle Swarm Optimization (PSO) [2]. However other approaches like Firefly algorithm (FFA) [3], Cuckoo Search Algorithm (CSA) [4] and Grey wolf optimization (GWO) [5] are also used to solve ELD problem.

Though ELD decreases the operating cost significantly but the environmental impact is still not addressed. By considering emission constraint along with the ELD problem, redefying the problem as Economic Emission Dispatch (EED) Problem. Differential evolution (DE) [6], Glowworm Swarm Optimization (GSO) [7], Harmony Search [8], Multi Objective EA (MOEA) [9] and Summation based Multi Objective DE (SMODE) [10] methods are used for the EED problem with/without transmission losses. Multi-objective Backtracking Search Algorithm (BSA) [11] and Hybrid Ant Optimization [12] approaches used for combining fuel cost with emission as a particular objective problem.

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In customary ELD approaches, cost utility will be approximated as a single quadratic equation. In general, it can be linearized by a piecewise linear approach [1]. However, we can observe the discontinuities in the turbine-generator set performance characteristics, those owe to valve-point (non-convex) loading in plants [1]. Hybrid approaches as modified Sub-Gradient (MSG) and and Harmony Search Algorithm MSG-HS [13], hybrid GA-NSO [14], modified hybrid PSOGSA [15] and BSA [16] methods are used for solving ELD with valve point effect (ELDVPPE) problem.

Besides, instabilities occurring in the generation at some particular levels of unit loading may be caused by physical limitations or faults. This problem can be resolved by using the model known as prohibited operating zones (POZ) [1] and changes in the unit’s generation level between any two simultaneous periods has not to be exceeded its ramp rate limits [1]. Backtracking search algorithm (BSA) [17], PSO [18], Modified Cuckoo Search (MCS) [19], Exchange Market Algorithm (EMA) [20] and Improved Random Drift PSO (RDPSO) [21] methods are used for solving ELD with ramp rate limits and POZs.

So, Non-convex loading, emission, the ramp rate limits and POZs should be considered to solve a practical ELD problem, it’s extremely hard to finding optimum solution. In this paper, ALO method used to solve the ELD problems with many practical operating constraints such as valve-point effect, non-linear emission, ramp-up/down and POZs. The quality of the proposed ALO method is implemented to four case studies for solving practical ELD problems.

This paper follows the below partitioned procedure: Section 2 brings out the mathematical modeling of ELD problem done by considering valve-point loading effect, emission, POZs constraints and ramp-up/down limits. Section 3 describes the ALO Algorithm. Section 4 contains the simulation results those determine the quality of the proposed algorithm. At the end the present work with discussions will be concluded in section 5.

2. PROBLEM FORMULATION

In order to reduce the production cost, ELD problem is formulated and allocating the optimal combination of generator outputs, while satisfying load demand, system operating constraints. Normally, cost function of the generation is represented as a quadratic function. The problem can be mathematically modeled as equation (1).

\[
F_1 = \min f = \sum_{k=1}^{n} F_k(P_{Gk}) \quad (\$/h) \quad (1)
\]

where \(F_1\) is total cost of generation, \(F_k\) is the \(k^{th}\) generator unit cost function given by equation (2).

\[
F_k(P_{Gk}) = a_k P_{Gk}^2 + b_k P_{Gk} + c_k \quad (2)
\]

where coefficients of \(k^{th}\) generator represent by \(a_k\), \(b_k\) and \(c_k\). To consider discontinuities for turbine-generator set take the valve-point effects, added sinusoidal terms to the convex cost utilities as follows equation (3).

\[
F_2 = F_C(P_{Gk}) = \sum_{k=1}^{NG} \left( a_k P_{Gk}^2 + b_k P_{Gk} + c_k \right) + e_k \sin(f_k \times (P_{Gk} - P_{Gk}^{min})) \quad (\$/h) \quad (3)
\]

where \(e_k\) and \(f_k\) are constants of the unit-\(k\) with valve-point effect.

The thermal power plants produce pollutants as NO\(_X\), CO\(_2\), and SO\(_2\) which are commonly denoted by separate emission convex functions. Though, by combining the all pollutants as single emission introducing exponential function to the quadratic emission function as given in equation (4) for total emission level of the pollutants.

\[
F_{E2} = F_{CE}(P_{Gk}) = \sum_{k=1}^{NG} \left( a_k P_{Gk}^2 + b_k P_{Gk} + c_k \right) + e_k \times \exp(f_k \times (P_{Gk} - P_{Gk}^{min})) \quad (\$/h) \quad (4)
\]
\[ F_3 = E(P_G) = \sum_{k=1}^{N_G} (\alpha_k P_{G_k}^2 + \beta_k P_{G_k} + \gamma_k) + \xi_k \exp(P_{G_k}^\lambda_k) \quad (t/h) \]  

(4)

The fuel cost and emission objective problem is converted into single objective CEED problem given by equation (5) by assuming weighting factor proportion to the importance of the objective.

\[
\text{Minimize } F = W \times F_2 + h \times (1 - W) \times F_3
\]

(5)

Price penalty factor \((h)\) in (S/Kg) is computed by taking the ratio between the maximum value of fuel cost in (S/h) to the maximum value of emission in (ton/Kg). The objective function with price penalty factor is given in equation (6).

\[
h = \frac{F_2(P_{G_k}^{\max})}{F_3(P_{G_k}^{\max})}, \quad k = 1, 2, 3, \ldots \ldots N_G
\]

(6)

### 2.1. Equality Constraint

Any power system must follow the equality constraint as equation (7).

\[
\sum_{k=1}^{N_G} P_{G_k} = P_D + P_L
\]

(7)

where \(P_D\) denotes the load demand and \(P_L\) denotes the losses of the transmission system. The loss equation in the B-coefficient method conveyed as a convex function shown in equation (8).

\[
P_L = \sum_{j=1}^{n} \sum_{k=1}^{n} P_{G_j} B_{jk} P_{G_k} + \sum_{j=1}^{n} B_{0j} P_{G_j} + B_{00} \quad (MW)
\]

(8)

### 2.2. Power Limit Constraint

Any generator real power output must satisfy the following constraint as follows equation (9).

\[
P_{G_k}^{\min} \leq P_{G_k} \leq P_{G_k}^{\max}
\]

(9)

### 2.3. Ramp Rate Limits

The online unit’s generation level not exceed its ramp rate limitation between two successive periods.

When power increases, we have

\[
P_k - P_k^0 \leq UR_k
\]

(10)

When power decreases, we have

\[
P_k^0 - P_k \leq DR_k
\]

(11)

where

\(P_k^0\): The preceding power generation of unit \(k\),

\(UR_k\): Ramp-up boundary of the \(k^{th}\) unit,

\(DR_k\): Ramp-down boundary of the \(k^{th}\) unit.
The presence of ramp-up/down boundaries changes the generator setup limits equation (9) as follows:

\[
\max(p^\text{max}_k \cdot UR_k - P_k) \leq P_k \leq \min(p^\text{max}_k \cdot DR_k)
\]

(12)

2.4. Prohibited Operating Zones

Due to POZs a generator whole operative region will be distributed into several isolated sub-regions. The conception of POZs is consisted of the following restriction in the ELD:

\[
\begin{align*}
&\left\{ \begin{array}{l}
P^\text{min}_{Gj} \leq P_{Gj} \leq P^\text{LB}_{Gj} \\
\vdots \\
&P^\text{UB}_{Gj,k-1} \leq P_{Gj} \leq P^\text{LB}_{Gj,k} \quad k = 2,3,4\ldots, NP_j \\
P^\text{UB}_{Gj,k} \leq P_{Gj} \leq P^\text{max}_{Gj} \\
\end{array} \right.
\end{align*}
\]

(13)

Where 
\(P^\text{LB}_{j,k}\): Lower limit of POZ k of generator j,
\(P^\text{UB}_{j,k}\): Upper limit of POZ k of generator j,
NP_j: The number of POZs of generator j.

3. ANT LION OPTIMIZATION ALGORITHM

Based on ant lions hunting process Mirjalli modeled Ant lion optimization (ALO) [22] algorithm. This proposed algorithm depends on the interaction from ant lion and its prey (ant). The ant lion makes a small pit by concave in the sand, after that it is stay at the end of the pit, once the ant passes over the pit, then the ant lion catch the prey as shown in Figure 1. After it was rebuilding traps and so on. Five main steps of catching a prey as mathematically model:

3.1. Random Walk of Ants

Random walks of ants are given by equation (14).

\[
X(t) = [0, c.\text{sum}(2ra(t_1) - 1), c.\text{sum}(2ra(t_1) - 1), \ldots, c.\text{sum}(2ra(t_n) - 1)]
\]

(14)

\[
ra(t) = \begin{cases} 
1 & \text{if } \text{rand} \geq 0.5 \\
0 & \text{if } \text{rand} < 0.5 
\end{cases}
\]

(15)

Where \(\text{rand}\) produces random number in the range of [0, 1]. In the local space random walks of ant can be given by the following equation,

\[
X^t_k = \frac{(X^t_k - a_k) \ast (d^t_k - c^t_k)}{(b^t_k - a^t_k)} + c^t_k
\]

(16)

Where \(a_k, b_k\) is the minimum and the maximum random walk of k-th variable, \(d^t_k, c^t_k\) is the maximum and minimum at t-th iteration of k-th variable.
The location of the ants is formulated as in [23] and the corresponding fitness function matrix. If ants and ant lions are hidden in the search area, the appropriate location and fitness matrices are provided by

\[
M_{\text{antlion}} = \begin{bmatrix}
A_{l,1} & A_{l,2} & \cdots & A_{l,d} \\
A_{2,1} & A_{2,2} & \cdots & A_{2,d} \\
\vdots & \vdots & \ddots & \vdots \\
A_{n,1} & A_{n,2} & \cdots & A_{n,d}
\end{bmatrix}
\]

(17)

\[
M_{\text{OAL}} = \begin{bmatrix}
f([A_{l,1} & A_{l,2} & \cdots & A_{l,d}]) \\
f([A_{2,1} & A_{2,2} & \cdots & A_{2,d}]) \\
\vdots & \vdots & \ddots & \vdots \\
f([A_{n,1} & A_{n,2} & \cdots & A_{n,d}])
\end{bmatrix}
\]

(18)

3.2. Building trap

To get greater chance for catching fittest ant lions roulette wheel mechanism is used.

3.3. Trapping in ant lion pits

The accurate equations for trapping are set by equation (19) and equation (20).

\[
c_k^l = \text{Antlion}_k^l + c^l
\]

(19)

\[
d_k^l = \text{Antlion}_k^l + d^l
\]

(20)
where \( c^t_i, d^t_i \) is the minimum and maximum at \( t \)-th iteration of all variables. \( d^t_k, c^t_k \) is the maximum and minimum for \( k \)-th ant of all variables, and \( \text{Antlion}^t_k \) shows the position at the \( t \)-th iteration of the selected \( k \)-th antlion.

### 3.4. Sliding ants towards ant lions

The mathematical model to move ants near to ant lions can be modeled as equation (21).

\[
\begin{cases} 
    c^t_i = \frac{c^t_i}{i}, & \quad d^t_i = \frac{d^t_i}{i} \\
\end{cases} 
\]

(21)

Where \( i = 10^w \frac{t}{T} \);

t is the present iteration, \( T \) is the maximum iterations, and constant \( W \) denoted as follows based on the current iteration,

\[
W = \begin{cases} 
    2 & \text{when } t > 0.1T, \\
    3 & \text{when } t > 0.5T, \\
    4 & \text{when } t > 0.75T, \\
    5 & \text{when } t > 0.9T, \text{ and} \\
    6 & \text{when } t > 0.95T. \\
\end{cases} 
\]

The accuracy level of exploitation adjusts by the constant \( W \).

### 3.5. Catching prey and rebuilding the pit

The antlion feeding an ant completed when prey reaches at the bottommost of the pit and further ant lion need to appraise its place to the newest place for catching the prey by equation (22).

\[
\text{Antlion}_j^t = \text{Ant}^t_k \quad \text{if } f(\text{Ant}^t_k) > f(\text{Antlion}^t_k) 
\]

(22)

### 3.6. Elitism

Elitism is essential in proposed method to maintain finest solution. This can be molded as equation (23),

\[
\text{Ant}^t_k = \frac{R^t_e + R^t_a}{2} 
\]

(23)

where

\( R^t_a \): At \( t \)-th iteration rand walk nearby the antlion nominated by the roulette Wheel,

\( R^t_e \): At \( t \)-th iteration rand walk nearby the elite,

\( \text{Ant}^t_k \): indicates the position of \( k \)-th ant at \( t \)-th iteration.

The Flowchart of the ALO algorithm is depicted in Figure 2.
Figure 2. Flowchart of ALO for ELD problem
3.7. Implementation of the ALO for ELD Problem

The steps of the ALO, as implemented in the solution of the ELD problem of this work, are shown below.

Step 1: Initialization
(a) Read cost curve coefficients and B matrix.
(b) Set number of search agents and maxiter.
(c) Set output power limits of each generator.

Step 2: Read ALO parameters, Set upper and lower boundaries.

Step 3: Check the boundaries of all the search agents.

Step 4: Initialize variables to save the antlions fitness, and ant’s fitness, position of elite, sorted antlions.

Step 5: Set current_iteration=2, because first iteration was used for antlions fitness calculation.

Step 6: Based on random walks selects the fitness of antlions and also find the elite antlion fitness.

Step 7: Ant position calculate by using equation (23).

Step 8: If any antlions, ant’s positions exceed the boundaries then bring back into the search space. The ants position update by using equation (24).

\[ \text{ant}_\text{pos}(k) = \text{ant}_\text{pos}(k) + \left( \left( \text{flag4ub} + \text{flag4lb} \right) \right) + \text{ub} \cdot \text{flag4ub} + \text{lb} \cdot \text{flag4lb} \]  

(24)

Step 9: Update the historical best position of antlion and fitness based on the ants, if an ant was caught by the antlion and the antlion update its position to rebuild the trap.

Step 10: Run the program upto met the tolerance (0.00001). Display the best score, which gives the optimum result with respect to best ant position.

4. SIMULATION RESULTS

The quality of the proposed ALO algorithm has been validating over four case studies on different 6-unit systems for solving ELD problems. The valve point loading and emission are addressed in the generator’s cost function. In addition to the power limits and power balance constraints ramp rate limits and POZs constraints also included. For programming Matlab R2014a software is used on personal computer. The effectiveness of the proposed method has been evaluated by performing 30 runs for each case study. To achieve the highest quality results, control parameter values are tuned. The convergence criterion for the proposed method is checked by setting the tolerance level to 0.00001.

4.1. Case 1: Conventional ELD

Conventional ELD (CELD) performs without valve-point loading and emission. For this case study consider 6-unit system with three different power demands (600, 700 and 800) MW [4]. The characteristics of the units and loss coefficients are presented in appendix. The generators cost are calculated using quadratic functions shown in equation (2), and losses are calculated from equation (6). There are 12 boundary power limit constraints in addition to power balance constraint. Table 1 shows optimal solution for the case-1 at various power demands.

Table 1. Best solution of the ELD in Case 1

<table>
<thead>
<tr>
<th>Unit</th>
<th>( P_d = 600 \text{ MW} )</th>
<th>( P_d = 700 \text{ MW} )</th>
<th>( P_d = 800 \text{ MW} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_1 (MW)</td>
<td>24.7779</td>
<td>29.4029</td>
<td>33.9118</td>
</tr>
<tr>
<td>P_2 (MW)</td>
<td>10.0000</td>
<td>10.0000</td>
<td>14.4026</td>
</tr>
<tr>
<td>P_3 (MW)</td>
<td>95.3216</td>
<td>118.7180</td>
<td>141.2739</td>
</tr>
<tr>
<td>P_4 (MW)</td>
<td>100.1918</td>
<td>118.3324</td>
<td>135.6484</td>
</tr>
<tr>
<td>P_5 (MW)</td>
<td>202.1601</td>
<td>230.4543</td>
<td>257.3125</td>
</tr>
<tr>
<td>P_6 (MW)</td>
<td>181.7099</td>
<td>212.4059</td>
<td>242.6253</td>
</tr>
<tr>
<td>P_L (MW)</td>
<td>14.1613</td>
<td>19.3135</td>
<td>25.1745</td>
</tr>
<tr>
<td>P_T (MW)</td>
<td>614.1613</td>
<td>719.3135</td>
<td>825.1745</td>
</tr>
<tr>
<td>F_1 ($/h)</td>
<td>32091.6309</td>
<td>36907.6939</td>
<td>41890.5076</td>
</tr>
</tbody>
</table>
Figure 3 shown convergence characteristics of ALO in CELD problem for various load demands. From the curve it can be concluded that an optimal result of ALO for 600 MW, 700 MW and 800 MW power demands are 32091.6309 ($/h), 36907.6939 ($/h) and 41890.5076 ($/h) respectively. In this case, search agents are set as 30 and maximum iterations are set as 1500, convergence criteria met nearly at 1350th iteration and Matlab program terminated before reaching the maximum iterations.

In this case study, proposed ALO is compared with FFA [3], CSA [4] and GWO [5] in the units of total fuel cost. The result in the Table 2 provides the comparison for power demand 600 MW between all the methods, which confirms that ALO converged to best fuel cost among all the other methods.

![Figure 3. Convergence characteristics of ALO in CELD](image)

**Table 2. Comparison and Statistical Result of the ELD in Case-1**

<table>
<thead>
<tr>
<th>Unit (MW)</th>
<th>POWER DEMAND = 600 MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_1</td>
<td>23.860</td>
</tr>
<tr>
<td>P_2</td>
<td>10.000</td>
</tr>
<tr>
<td>P_3</td>
<td>95.638</td>
</tr>
<tr>
<td>P_4</td>
<td>100.708</td>
</tr>
<tr>
<td>P_5</td>
<td>202.832</td>
</tr>
<tr>
<td>P_6</td>
<td>181.198</td>
</tr>
<tr>
<td>P_T</td>
<td>614.237</td>
</tr>
<tr>
<td>Min.cost($/h)</td>
<td>32094.7</td>
</tr>
<tr>
<td>Avg.cost($/h)</td>
<td>---</td>
</tr>
<tr>
<td>Max.cost($/h)</td>
<td>---</td>
</tr>
</tbody>
</table>

For the given test system 30 independent runs are performed for 600 MW demand and variations in optimal results obtained by ALO method are presented in Table 2. The optimal results with negligible standard deviation (0.007172 ($/h)) validate the potential of ALO for solving CELD problem. The total cost validate for 30 runs is between 32091.6309 ($/h) to 32091.6342 ($/h). Figure 4 confirms the robustness of ALO as it shows the similar results in 30 independent runs.
4.2. Case 2: ELD with Valve Point Effect

ELD with Valve Point Effect (ELDVPE) is considered as case-2 study. In this case-2 study we considered IEEE 30-bus 6-unit systems with power demand 283.4 MW and data is taken from [16-17] or presented in appendix. The generators cost functions are combination of quadratic functions and sinusoidal terms which are shown in equation (3) and losses are calculated from equation (6). The best solution in this case study is shown in Table 3.

In this case study, ALO is compared with hybrid MSG-HS [13], hybrid GA-NSO [14], modified hybrid PSOGSA [15] and BSA [16] methods in terms of total fuel cost. Table 3 provides the comparison, it can be confirmed that ALO converged to best fuel cost among other methods.

Table 3. Comparison and Statistical Result of the ELD in Case-2

<table>
<thead>
<tr>
<th>Unit (MW)</th>
<th>POWER DEMAND = 283.4 MW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GA-NSO</td>
</tr>
<tr>
<td>P₁</td>
<td>182.47</td>
</tr>
<tr>
<td>P₂</td>
<td>48.3525</td>
</tr>
<tr>
<td>P₃</td>
<td>19.8553</td>
</tr>
<tr>
<td>P₄</td>
<td>17.1370</td>
</tr>
<tr>
<td>P₅</td>
<td>13.6677</td>
</tr>
<tr>
<td>P₆</td>
<td>12.3487</td>
</tr>
<tr>
<td>P₇</td>
<td>10.4395</td>
</tr>
<tr>
<td>P₈</td>
<td>293.839</td>
</tr>
<tr>
<td>Min.cost($/h)</td>
<td>984.936</td>
</tr>
<tr>
<td>Avg.cost($/h)</td>
<td>---</td>
</tr>
<tr>
<td>Max.cost($/h)</td>
<td>992.4815</td>
</tr>
</tbody>
</table>
Figure 5. Convergence characteristics of ALO in ELDVPE

Figure 5 shown convergence characteristics of ALO in ELDVPE problem. From the curve it can be concluded that an optimal result of ALO for 283.4 MW power demand is 924.9693 ($/h). In this case search agents are set as 50 and maximum iterations are to 3000. The convergence criteria met nearly at 2850th iteration and terminated at this iteration rather than program run up to maximum iterations.

For the case 2 test system 30 independent runs are performed and optimal results obtained by ALO method are presented in Table 3. The optimal results with negligible standard deviation (0.10165 ($/h)) validate the potential of ALO for solving ELDVPE problem. From the result conclude that due to valve-point effect deviation in optimal result increases. The total cost validate for 30 runs is between 924.9693 ($/h) to 925.2884 ($/h). Figure 6. confirms the robustness of ALO as it shows the similar results in 30 independent runs.

Figure 6. Convergence of ALO for Case 2 with 30 runs
4.3. Case 3: Environmental ELD

Environmental ELD (EELD) is considered as case-3, IEEE 30-bus 6-unit system with power demand of 283.4 MW, and data is extracted from [10] or presented in appendix. The generator fuel cost functions are shown in equation (2), emission function shown in equation (4) and losses are calculated from equation (6). Table 4 represents the cost minimization, emission minimization solution for the represented test system without and with power losses. Combining cost and emission as single objective function formed CEED minimization also presented in Table 4 for given test system.

Table 4. Best solution of the EELD in Case 3

<table>
<thead>
<tr>
<th>Unit (MW)</th>
<th>POWER DEMAND = 283.4 MW</th>
<th>Cost Minimization</th>
<th>Emission Minimization</th>
<th>CEED Minimization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Power loss = 0</td>
<td>Power loss ≠ 0</td>
<td>Power loss = 0</td>
<td>Power loss ≠ 0</td>
</tr>
<tr>
<td>P1</td>
<td>10.9715</td>
<td>5.0000</td>
<td>40.6074</td>
<td>40.9150</td>
</tr>
<tr>
<td>P2</td>
<td>29.9760</td>
<td>30.6699</td>
<td>45.9069</td>
<td>45.5209</td>
</tr>
<tr>
<td>P3</td>
<td>52.4303</td>
<td>60.8552</td>
<td>53.7938</td>
<td>54.1174</td>
</tr>
<tr>
<td>P4</td>
<td>101.6199</td>
<td>111.7194</td>
<td>38.2953</td>
<td>38.2194</td>
</tr>
<tr>
<td>P5</td>
<td>52.4298</td>
<td>38.1076</td>
<td>53.7939</td>
<td>54.3954</td>
</tr>
<tr>
<td>P6</td>
<td>35.9725</td>
<td>38.8151</td>
<td>51.0027</td>
<td>53.0130</td>
</tr>
<tr>
<td>P_L</td>
<td>0</td>
<td>1.7671</td>
<td>0</td>
<td>3.2977</td>
</tr>
<tr>
<td>P_T</td>
<td>283.4</td>
<td>285.1671</td>
<td>283.4</td>
<td>286.6977</td>
</tr>
<tr>
<td>F_2 ($/h)</td>
<td>600.1114</td>
<td>604.9684</td>
<td>638.2734</td>
<td>650.3641</td>
</tr>
<tr>
<td>E(ton/h)</td>
<td>0.222145</td>
<td>0.232755</td>
<td>0.194203</td>
<td>0.194179</td>
</tr>
</tbody>
</table>

Figure 7 shows convergence curve for cost minimization to the system for both the cases. When the generation cost is objective function, ALO reaches the optimal value of 600.1114 ($/h) during without losses and 604.9684 ($/h) with loss consideration. Figure 8. shows convergence curve for emission minimization to the system for both the cases. In emission minimization, the optimal value during without loss is 0.194203 (ton/h) and optimal value with loss is 0.194179 (ton/h).

Figure 7 shown convergence characteristics of ALO in EELD for cost minimization. From the curve it can be concluded that ALO produce optimal result for the test system. In this case search agents are set as 50 and maximum iterations are to 5000. With cost minimization convergence criteria met nearly at 4800th iteration and emission minimization nearly at 955th iteration when we took system with losses.

Figure 7. Convergence characteristics of ALO in EELD for Cost Minimization
Figure 8. Convergence characteristics of ALO in EELD for Emission Minimization
When combining both cost and emission functions CEED problem framed and its convergence characteristics for 30 independent runs are shown in Figure 9. It shows the potential of ALO for solving CEED problem and optimal results have negligible standard deviation (0.007261 ($/h)). The total cost validate for 30 runs is between 598.9678 ($/h) to 599.0041 ($/h) when considering the weighting factor as 0.6. Figure 9, confirms the robustness of ALO as it shows the similar results in 30 independent runs.

![Figure 9. Convergence of ALO for CEED with 30 runs](image)

In this test system, ALO has been compared with MOEA [9], SMODE [10] and BSA [11] methods in terms of total generation fuel cost and emission minimization. Table 5 provides the cost minimization and emission minimization comparison, it can be confirmed that ALO converged to best fuel cost among other methods.

<table>
<thead>
<tr>
<th>Table 5. Comparison Result of the ELD in Case-3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cost Minimization ($/h)</strong></td>
</tr>
<tr>
<td>Without Loss</td>
</tr>
<tr>
<td>MOEA</td>
</tr>
<tr>
<td>SMODE</td>
</tr>
<tr>
<td>BSA</td>
</tr>
<tr>
<td>ALO</td>
</tr>
</tbody>
</table>

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>With Loss</td>
</tr>
<tr>
<td>MOEA</td>
</tr>
<tr>
<td>SMODE</td>
</tr>
<tr>
<td>BSA</td>
</tr>
<tr>
<td>ALO</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Emission Minimization (ton/h)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Without Loss</td>
</tr>
<tr>
<td>MOEA</td>
</tr>
<tr>
<td>SMODE</td>
</tr>
<tr>
<td>BSA</td>
</tr>
<tr>
<td>ALO</td>
</tr>
</tbody>
</table>

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>With Loss</td>
</tr>
<tr>
<td>MOEA</td>
</tr>
<tr>
<td>SMODE</td>
</tr>
<tr>
<td>BSA</td>
</tr>
<tr>
<td>ALO</td>
</tr>
</tbody>
</table>
4.4. Case 4: ELD with POZs and Ramp rate limits.

ELD with POZs and ramp rate limits (ELDRPOZ) is considered as case-4. For this case study considers 6-unit system with power demand 1263 MW, data is taken from [18] or presented in appendix. All units have two POZs with ramp-up/down limit constraints. There are 12 boundary constraints, 12 POZs, and 12 ramp limit constraints emerging in 36 inequity constraints summing together one equality constraint. Table 6 shown the optimal solution for the case 4 test system with power loss.

Table 6. Comparison and Statistical Result of the ELD in Case-4

<table>
<thead>
<tr>
<th>Unit</th>
<th>POWER DEMAND =1263 MW</th>
<th>BSA</th>
<th>PSO</th>
<th>MCS</th>
<th>EMA</th>
<th>ALO</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_1</td>
<td>447.490</td>
<td>447.497</td>
<td>447.5038</td>
<td>447.3872</td>
<td>446.8292</td>
<td></td>
</tr>
<tr>
<td>P_2</td>
<td>173.330</td>
<td>173.322</td>
<td>173.3182</td>
<td>173.2524</td>
<td>173.1392</td>
<td></td>
</tr>
<tr>
<td>P_3</td>
<td>263.455</td>
<td>263.474</td>
<td>263.4628</td>
<td>263.3721</td>
<td>264.3302</td>
<td></td>
</tr>
<tr>
<td>P_4</td>
<td>139.060</td>
<td>139.059</td>
<td>139.0653</td>
<td>138.9894</td>
<td>139.1030</td>
<td></td>
</tr>
<tr>
<td>P_5</td>
<td>165.480</td>
<td>165.476</td>
<td>165.4734</td>
<td>165.3650</td>
<td>165.7180</td>
<td></td>
</tr>
<tr>
<td>P_6</td>
<td>87.1409</td>
<td>87.1280</td>
<td>87.1347</td>
<td>87.0781</td>
<td>86.2976</td>
<td></td>
</tr>
<tr>
<td>P_T</td>
<td>1275.95</td>
<td>1276.01</td>
<td>1276.9582</td>
<td>1275.443</td>
<td>1275.4171</td>
<td></td>
</tr>
<tr>
<td>Min.cost($/h)</td>
<td>15449.8995</td>
<td>15450.0</td>
<td>15449.8995</td>
<td>15443.0749</td>
<td>15442.8410</td>
<td></td>
</tr>
<tr>
<td>Avg.cost($/h)</td>
<td>15449.9001</td>
<td>15454.0</td>
<td>15449.8995</td>
<td>15443.075</td>
<td>15442.8410</td>
<td></td>
</tr>
<tr>
<td>Max.cost($/h)</td>
<td>15449.9056</td>
<td>15492.0</td>
<td>15449.8995</td>
<td>---</td>
<td>15442.8410</td>
<td></td>
</tr>
</tbody>
</table>

In this case, ALO is paralleled with BSA [18], PSO [19], MCS [20] and EMA [21] methods in terms of fuel cost. Table 6 provides the comparison which confirming that ALO has been converged to best fuel cost among other methods.

From Figure 10, it can be concluded that optimal valve for ELDRPOZ is 15442.8410 ($/h). In this case search agents are set to 50 and maximum iterations are to 1500. With cost minimization convergence criteria met nearly at 1130th iteration, the program terminated at this iteration.

Figure 10. Convergence characteristics of ALO in ELD WITH RAMP LIMITS AND POZs

Figure 11. Convergence of ALO for in ELD WITH RAMP LIMITS AND POZs with 30 runs.

For the case 4 test system 30 independent runs are performed and optimal results obtained by ALO method are presented in Table 6. The optimal results with negligible standard deviation (0.00001 ($/h)) validate the potential of ALO for solving ELDRPOZ problem. The total cost validate for 30 runs is 15442.8410 ($/h). Figure 11. confirms the robustness of ALO as it shows the similar results in 30 independent runs.
5. CONCLUSIONS

In this paper, the proposed Ant Lion Optimization (ALO) was effectively applied to solve ELD problems considering practical constraints such as valve-point effects, emission constraints, ramp rate limits and prohibited operating zones. The proposed ALO approach is tested on four case studies on different 6-unit systems. The first case consider as conventional ELD without any practical constraints for various load demands. The ALO produce lowest optimal with similar results in 30 runs as compared with other optimization methods. In case 2 valve-point effect include to CELD problem to test the potential of ALO. The robustness of ALO again approved and produce optimal results with standard deviation (0.10165 ($/h)). To employing clean energy technology emission impact also considered as case 3, combining cost and emission as multiobjective also included. By considering weighting factor as 0.6 and perform CEED problem on ALO, it produce optimal results have negligible standard deviation (0.00726 ($/h)). The ramp rate limits and POZs constraints are included to ELD problem in case 4. In case 4 study ALO produce optimal results with standard deviation (0.00001 ($/h)) in 30 independent runs. From the results concluded that as the complexity of ELD problem increases proposed method obtain optimal point better than prescribed other top methods those are stated in literatures. The result has proved that the proposed ALO is a reliable tool to solve many optimization problems. For further works, the proposed method can be implemented to solve security constraint ELD problem and the unit commitment problem.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES


## APPENDIX

### CASE 1 (CELD)
**(Power Demand = 600/700/800 MW)**

<table>
<thead>
<tr>
<th>Unit i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{i}^{\text{min}}$ (MW)</td>
<td>10</td>
<td>10</td>
<td>35</td>
<td>35</td>
<td>130</td>
<td>125</td>
</tr>
<tr>
<td>$p_{i}^{\text{max}}$ (MW)</td>
<td>125</td>
<td>150</td>
<td>225</td>
<td>210</td>
<td>325</td>
<td>315</td>
</tr>
<tr>
<td>$a_i$ ($/$(MW)$^2$h)</td>
<td>0.15240</td>
<td>0.10587</td>
<td>0.02803</td>
<td>0.03546</td>
<td>0.02111</td>
<td>0.01799</td>
</tr>
<tr>
<td>$b_i$ ($/$/MWh)</td>
<td>38.53973</td>
<td>46.15916</td>
<td>40.39655</td>
<td>38.30553</td>
<td>36.32782</td>
<td>38.27041</td>
</tr>
<tr>
<td>$c_i$ ($/hr$)</td>
<td>756.79886</td>
<td>451.3251</td>
<td>1049.997</td>
<td>1243.531</td>
<td>1658.559</td>
<td>1356.659</td>
</tr>
</tbody>
</table>

Loss Coefficients taken from Reference [4],

\[
B = 0.000001 \begin{bmatrix} 14 & 17 & 15 & 19 & 26 & 22 \\
17 & 60 & 13 & 16 & 15 & 20 \\
15 & 13 & 65 & 17 & 24 & 19 \\
19 & 16 & 17 & 72 & 30 & 25 \\
26 & 15 & 24 & 30 & 69 & 32 \\
22 & 20 & 19 & 25 & 32 & 85 \end{bmatrix};
\]

### CASE 2 (ELDVPE)
**(Power Demand = 283.4/400/500 MW)**

<table>
<thead>
<tr>
<th>Unit i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{i}^{\text{max}}$ (MW)</td>
<td>200</td>
<td>80</td>
<td>50</td>
<td>35</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>$a_i$ ($/$(MW)$^2$h)</td>
<td>0.0016</td>
<td>0.0100</td>
<td>0.0625</td>
<td>0.00834</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>$b_i$ ($/$/MWh)</td>
<td>2</td>
<td>2.5</td>
<td>1</td>
<td>3.25</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$c_i$ ($/hr$)</td>
<td>150</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$e_i$ ($/hr$)</td>
<td>50</td>
<td>40</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$f_i(1/$MW)</td>
<td>0.063</td>
<td>0.098</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Loss Coefficients taken from Reference [16-17],

\[
B = \begin{bmatrix} 0.0224 & 0.0103 & 0.0016 & -0.0053 & 0.0009 & -0.0013 \\
0.0103 & 0.0158 & 0.0010 & -0.0074 & 0.0007 & 0.0024 \\
0.0016 & 0.0010 & 0.0474 & -0.0687 & -0.0060 & -0.0350 \\
-0.0053 & -0.0074 & -0.0687 & 0.3464 & 0.0105 & 0.0534 \\
0.0009 & 0.0007 & -0.0060 & 0.0105 & 0.0119 & 0.0007 \\
-0.0013 & 0.0024 & -0.0350 & 0.0534 & 0.0007 & 0.2353 \end{bmatrix};
\]

\[
B_0 = \begin{bmatrix} -0.0005 & 0.0016 & -0.0029 & 0.0060 & 0.0014 & 0.0015 \end{bmatrix};
\]

$B_{00} = 0.0011$;
### CASE-3 (EELD)

**Problem Parameters**

<table>
<thead>
<tr>
<th>Unit</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_i^\text{max}$ (PU)</td>
<td>0.5</td>
<td>0.6</td>
<td>1</td>
<td>1.2</td>
<td>1</td>
<td>0.6</td>
</tr>
<tr>
<td>$a_i$ ($/\text{(MW)}^2\text{h}$)</td>
<td>10</td>
<td>10</td>
<td>20</td>
<td>10</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>$b_i$ ($$/\text{MWh}$)</td>
<td>200</td>
<td>150</td>
<td>180</td>
<td>100</td>
<td>180</td>
<td>150</td>
</tr>
<tr>
<td>$c_i$ ($$/\text{hr}$)</td>
<td>100</td>
<td>120</td>
<td>40</td>
<td>60</td>
<td>40</td>
<td>100</td>
</tr>
</tbody>
</table>

**Emission Coefficients**

| $\alpha_i$ (ton/\text{MW}^2\text{h}) | 0.0409 | 0.0254 | 0.0426 | 0.0533 | 0.0426 | 0.0613 |
| $\beta_i$ (ton/\text{MWh}) | -0.0555 | -0.0605 | -0.0509 | -0.0355 | -0.0509 | -0.0555 |
| $\gamma_i$ (ton/\text{hr}) | 0.0649 | 0.0564 | 0.0459 | 0.0338 | 0.0459 | 0.0515 |
| $\eta_i$ (ton/\text{hr}) | 0.0002 | 0.0005 | 0.0000 | 0.0020 | 0.0000 | 0.0000 |
| $\delta_i$ (1/\text{MW}) | 2.8570 | 3.3330 | 8.0000 | 2.0000 | 8.0000 | 6.6670 |

Loss Coefficients taken from Reference [10].

$B = [0.0218 \ 0.0107 \ -0.00036 \ -0.0011 \ 0.00055 \ 0.0033 \ 0.0107 \ 0.01704 \ -0.0001 \ -0.00179 \ 0.00026 \ 0.0028 \ -0.0004 \ -0.0002 \ 0.02459 \ -0.01328 \ -0.0118 \ -0.0079 \ -0.0011 \ -0.00179 \ -0.01328 \ 0.0065 \ 0.0098 \ 0.0045 \ 0.00055 \ 0.00026 \ -0.00118 \ 0.0098 \ 0.0216 \ -0.0001 \ 0.0033 \ 0.0028 \ -0.00792 \ 0.0045 \ -0.00012 \ 0.02978];$

$B_0 = [0.010731 \ 1.7704 \ -4.0645 \ 3.8453 \ 1.3832 \ 5.5503] \times 10^{-3};$

$B_{00} = 0.0014;$

### CASE-4 (ELDPOZ)

**Problem Parameters**

<table>
<thead>
<tr>
<th>Unit</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>DR(MW)</td>
<td>120</td>
<td>90</td>
<td>100</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>$P_i^\text{max}$ (MW)</td>
<td>440</td>
<td>170</td>
<td>200</td>
<td>150</td>
<td>190</td>
<td>150</td>
</tr>
<tr>
<td>$P_i^\text{min}$ (MW)</td>
<td>100</td>
<td>50</td>
<td>80</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$P_i^\text{max}$ (MW)</td>
<td>500</td>
<td>200</td>
<td>300</td>
<td>150</td>
<td>200</td>
<td>120</td>
</tr>
<tr>
<td>$a_i$ ($/\text{(MW)}^2\text{h}$)</td>
<td>0.0070</td>
<td>0.0095</td>
<td>0.0090</td>
<td>0.0090</td>
<td>0.0080</td>
<td>0.0075</td>
</tr>
<tr>
<td>$b_i$ ($$/\text{MWh})</td>
<td>7.0000</td>
<td>10.0000</td>
<td>8.5000</td>
<td>11.0000</td>
<td>10.5000</td>
<td>12.0000</td>
</tr>
<tr>
<td>$c_i$ ($$/\text{hr})</td>
<td>240</td>
<td>200</td>
<td>220</td>
<td>200</td>
<td>220</td>
<td>190</td>
</tr>
</tbody>
</table>

Loss Coefficients taken from Reference [17].
\[
\begin{align*}
B &= 0.01 \times \left[ 
\begin{array}{ccccccc}
0.0017 & 0.0012 & 0.0007 & -0.0001 & -0.0005 & -0.0002 \\
0.0012 & 0.0014 & 0.0009 & 0.0001 & -0.0006 & -0.0001 \\
0.0007 & 0.0009 & 0.0031 & 0.0000 & -0.0010 & -0.0006 \\
-0.0001 & 0.0001 & 0.0000 & 0.0024 & -0.0006 & -0.0008 \\
-0.0005 & -0.0006 & -0.0010 & -0.0006 & 0.0129 & -0.0002 \\
-0.0002 & -0.0001 & -0.0006 & -0.0008 & -0.0002 & 0.0150 \\
\end{array}
\right]; \\
B_0 &= 1e-5 \times \left[ 
\begin{array}{ccccccc}
-0.3908 & -0.1279 & 0.7047 & 0.0591 & 0.2161 & -0.6635 \\
\end{array}
\right]; \\
B_{00} &= 1e-2 \times 0.0056;
\end{align*}
\]