Research Article GU J Sci 32(2): 667-672 (2019)

Gazi University
Journal of Science
http://dergipark.gov.tr/gujs

Packing Chromatic Number of Bismuth Tri-iodide and First Type Nanostar Dendrimers

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Article Info

Received: 23/06/2017
Accepted: 01/02/2018

Keywords

Packing coloring
Bismuth tri-iodide
Lead chloride

Abstract

The packing chromatic number $\chi_\rho(G)$ of a connected graph G is the smallest number m for which a function $g:V(G)\to\{1,2,...,m\}$ exists, such that if $g(a)=g(b)=j$, then $d(a,b)>j$. Here, we determine the packing chromatic numbers of bismuth tri-iodide and first type nanostar dendrimers.

1. INTRODUCTION

A packing m-coloring of a connected graph G is a function $g:V(G)\to\{1,2,...,m\}$ such that if $g(a)=g(b)=j$, then $d(a,b)>j$. The packing chromatic number $\chi_\rho(G)$ of G is the smallest number m for which G has packing m-coloring. The idea of packing coloring comes from the zone of frequency assignment in wireless networks and was proposed by Goddard et al. [1] underneath the title broadcast coloring. The name packing chromatic number was coined by Bresar et al. [2].

Dendrimers are novel artificial polymeric structures having extended bodily and chemical properties due to their special three dimensional architecture and they also have a nicely defined shape, size and molecular weight. These are adaptable with drug moieties as properly as bioactive molecules like DNA, Liquaemin and different polyanions [3,4].

A crystal structure is made out of a unit cell, a set of atoms arranged particularly; repeated on a lattice periodically in three dimensions. The crystal structure plays a crucial role in deciding several physical properties, like electronic band structure, cleavage, and optical transparency [5]. Here, we determine the packing coloring of first type nanostar dendrimers and bismuth tri-iodide.

2. SOME FAMILIES OF NANOSTAR DENDRIMERS

The first type of nanostar dendrimers $D_1(n)$ are shown in Figures 1, 2 and 3, where n is the stage of growth. The order and size of $D_1(n)$ nanostar dendrimers are $24+36(n-1)$ and $27+42(n-1)$ respectively. In the following Theorems, by $c(v)$, $d(u,v)$ and $d(u,\{v,w\})$ we mean the color of a vertex v, the distance between vertices u and v, and the distances between u and v and u and w.

Proposition 2.1. [1] Let H be a subgraph of G. Then $\chi_\rho(H)\le\chi_\rho(G)$

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Proposition 2.2. [1] For a cycle graph C_n of length n, we have

$$\chi_\rho(C_n) = \begin{cases} 3 & \text{when } n \text{ is divisible by 4} \\ 4 & \text{otherwise} \end{cases}$$

Theorem 2.3. For $D_1(1)$, $\chi_\rho(D_1(1)) = 5$.

Proof: Assume for logical inconsistency this is not valid. Then $\chi_\rho(D_1(1)) = 4$. Obviously there must be a vertex to which color 1 or 2 or 3 or 4 is assigned. Label $D_1(1)$ as it is shown in Figure 1(a). Let x be a vertex in $D_1(1)$ with $c(x)=2$ or $c(x)=1$ or $c(x)=3$ or $c(x)=4$. Now we will discuss these four cases in detail.

Case 1: For $c(x)=1$, note that each of the neighbors of x have pairwise various colors and let $c(u_1)=2$, $c(v_1)=3$ and $c(w_1)=4$. Now vertices u_2 and u_6 can't be colored except if $c(u_2)=c(u_6)=1$. Since $d(w_1,\{u_3,u_5\})=4$ and $d(v_1,\{u_3,u_5\})=4$, either u_3 or u_5 can be colored 3. But if $c(u_3)=3$, u_5 can't be colored except if a color more than 4 is used. Similarly, if $c(u_5)=3$, u_3 can't be colored except if a color more than four is used.

Case 2: For $c(x)=2$, let $c(u_1)=c(v_1)=c(w_1)=1$. Consider the vertices u_2, u_6 and v_2. Since $c(x)=2$ and $d(v_2,\{u_2,u_6\})=4$, vertices u_2 and v_2 can be colored 3, which also implies that $c(w_2)=3$ because $d(w_2,\{u_2,u_6\})=4$. Now for vertices u_6, v_6 and w_6, the only choice is to get color 4. Without loss of all inclusive statement, let $c(u_6)=4$. Then v_6 and w_6 can't be colored except if a color more than four is used.

Case 2.1: For $c(x)=2$, let $c(v_1)=3$ and $c(w_1)=4$ or vice versa. This forces $c(u_1)=1$. Since $d(v_1,\{u_2,u_6\})=3$, $d(w_1,\{u_2,u_6\})=3$ and $d(x,\{u_2,u_6\})=2$, vertices u_2 and u_6 can't be colored except if a color more than 4 is used.

Case 2.2: For $c(x)=2$, let $c(u_1)=c(v_1)=1$. Then $c(w_1)\in\{3,4\}$. Since $d(w_1,\{v_2,v_6\})=3$, if $c(w_1)=3$, vertex v_6 can be colored 4 but v_2 needs a color greater than 4. Similarly, if $c(w_1)=4$ instead of $c(w_1)=3$, vertex v_6 can be colored 3 but v_2 needs a color more than 4.

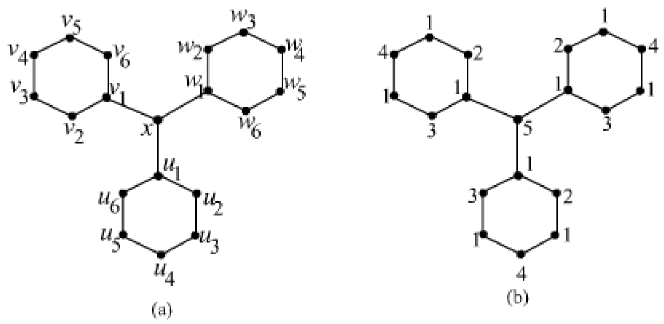

Figure 1*. a) Labeling of* $D_1(1)$ *b) Packing 5-coloring of* $D_1(1)$

Case 3: For $c(x)=3$, let $c(u_1)=c(v_1)=c(w_1)=1$. Consider the vertices u_6, v_6 and w_6. Since $d(u_6,\{v_6,w_6\})=4$ and $d(v_6,w_6)=4$, u_6, v_6 and w_6 can be colored 2. Now for vertex v_2, the only choice is to get color 4. But if $c(v_2)=4$, vertices u_2 and w_2 can't be colored except if a color more than four is used.

Case 3.1: For $c(x)=3$, let $c(v_1)=2$ and $c(w_1)=4$. This forces $c(u_1)=1$. Since $d(v_1,\{u_2,u_6\})=3$, if $c(u_2)=2$, u_6 can't be colored except if a color greater than 4 is used. Similarly, if $c(u_6)=2$ instead of $c(u_2)=2$, u_2 can't be colored except if a color more than four is used.

Case 3.2: For $c(x)=3$, let $c(v_1)=c(w_1)=1$. Then $c(u_1)\in\{2,4\}$. Since $d(u_1,\{u_6,w_6\})=3$, u_1, u_6 and w_6 can be colored 2. This forces $c(w_2)=4$. But if $c(w_2)=4$, v_2 can't be colored except if a color greater than four is used and suppose if $c(u_1)=4$ instead of $c(u_1)=2$, vertex v_2 should receive a color more than four.

Case 4: For $c(x)=4$, let $c(u_1)=1$. Then $c(u_2)$ and $c(u_6)\in\{2,3\}$. Without loss of all inclusive statement, let $c(u_2)=2$ and $c(u_6)=3$. This forces $c(u_3)=c(u_5)=1$. But for vertex u_4, the only choice is to get a color more than four.

Case 4.1: For $c(x)=4$, let $c(u_1)\in\{2,3\}$. To minimize the packing coloring, vertices u_2 and u_6 can be colored 1. When $c(u_1)=2$, either $c(u_3)=3$ or $c(u_5)=3$. Without loss of all inclusive statement let $c(u_3)=3$. Then u_5 needs a color more than four. Similarly, when $c(u_1)=3$ instead of $c(u_1)=2$, either $c(u_3)=2$ or $c(u_5)=2$. Without loss of generality let $c(u_3)=2$. Then u_5 needs a color more than four. Hence from above four Cases, $\chi_\rho(D_1(1))\geq 5$. From Figure 1(b), we conclude that $\chi_\rho(D_1(1))=5$.

Theorem 2.4. For $D_1(n), n=2,3$, $\chi_\rho(D_1(n))=5$.

Proof: Since $D_1(1)$ is a subgraph of $D_1(n), n=2,3$, by Propositions 2.1 and 2.2, $\chi_\rho(D_1(n))\geq 5$. From Figures 2 and 3, the upper bound is $\chi_\rho(D_1(n))\leq 5$. Hence for $D_1(n), n=2,3$, $\chi_\rho(D_1(n))=5$.

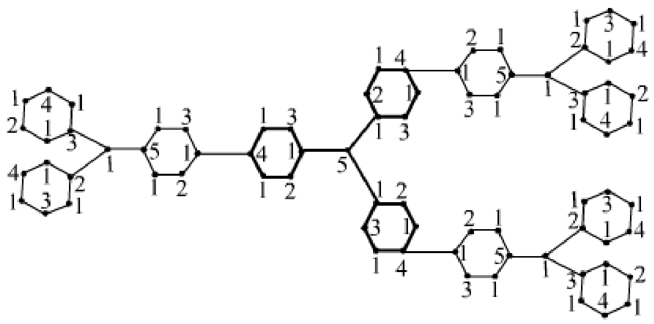

***Figure 2**. Packing 5-coloring of* $D_1(2)$

Open Problem 2.5. By a comparable, yet considerably more monotonous investigation as in the above verification one can set up $\chi_\rho(D_1(n))=5$, for $n\geq 4$.

3. THE STRUCTURE OF BISMUTH TRI-IODIDE

Bismuth tri-iodide (BiI3) is one of the inorganic compounds. The result of the response of bismuth and iodine is used to be of attractive for subjective inorganic investigation. Bismuth tri-iodide is additionally shaped by the activity of ethyl iodide on bismuth trichloride within the sight of ethyl chloride. Layered BiI3 crystal is viewed as 3-layered stacking structure. The occasional stacking of 3 layers frames rhombohedral BiI3 crystal with R-3 symmetry. The progressive stacking of one I-Bi-I layer frames hexagonal structure with symmetry [6]. Figure 4(a) demonstrates one unit of bismuth tri-iodide. An m - bismuth chain is obtained by linearly arranging m unit cells.

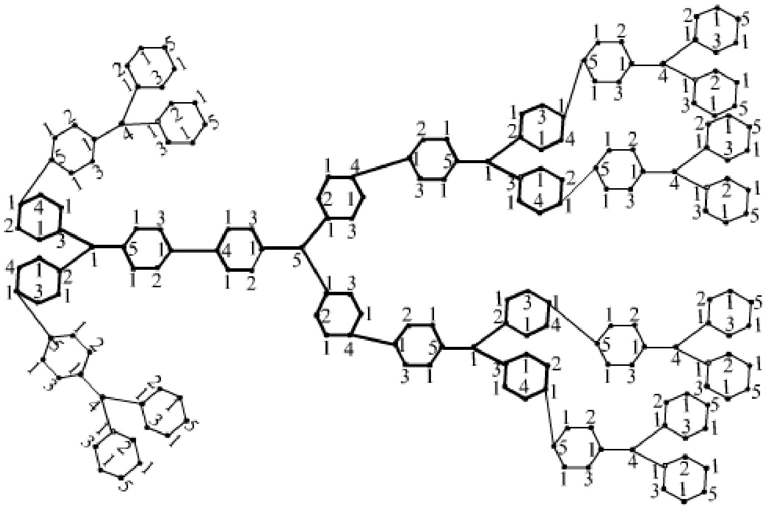

***Figure 3**. Packing 5-coloring of* $D_1(3)$

Theorem 3.1. Let G be an m-bismuth chain. Then $\chi_\rho(G) = 3$.

Proof: Since C_4 is a subgraph of G, by Propositions 2.1 and 2.2, $\chi_\rho(G) \geq 3$. The upper bound pursues from the coloring of an m-bismuth chain with 3 colors, whose pattern is shown in Figure 4(b). Thus, from Figure 4(b), $\chi_\rho(G) \leq 3$. Hence $\chi_\rho(G) = 3$.

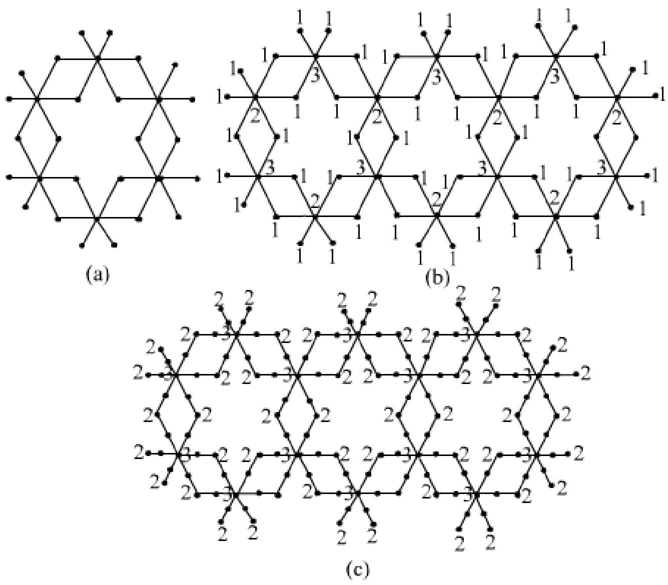

***Figure 4**. a) One unit of bismuth b) Packing 3-coloring of 3-bismuth chain c) Packing 3-coloring of subdivided 3-bismuth chain*

By subdividing each edge of a graph G, *subdivision graph* $S(G)$ is acquired.

Theorem 3.2. Let $S(G)$ be a subdivided m-bismuth chain. Then $\chi_\rho(S(G)) = 3$.

Proof: Since C_8 is a subgraph of $S(G)$, by Propositions 2.1 and 2.2, $\chi_\rho(S(G)) \geq 3$. Then upper bound pursues from the labelling of a subdivided m-bismuth chain with 3 colors, whose pattern is shown in Figure 4(c). In Figure 4(c), it is expected that the vertices which are not colored get color 1. Thus, from

Figure 4(c), $\chi_\rho(S(G)) \le 3$. Hence $\chi_\rho(S(G)) = 3$.

4. THE STRUCTURE OF LEAD CHLORIDE

Lead chloride is a salt crystal which is utilized in the creation of infra red disseminating glass. It is additionally used as an intermediate in refinement of bismuth (Bi) ore. The structure of lead chloride is orthorhombic dipyramidal [6]. The diagram of one unit of lead chloride is gotten from that of bismuth tri-iodide by joining only one two degree vertex of every one of the 4-cycles to another vertex. As on account of bismuth tri-iodide, chains and sheets of lead chloride are characterized. See Figure 5(a).

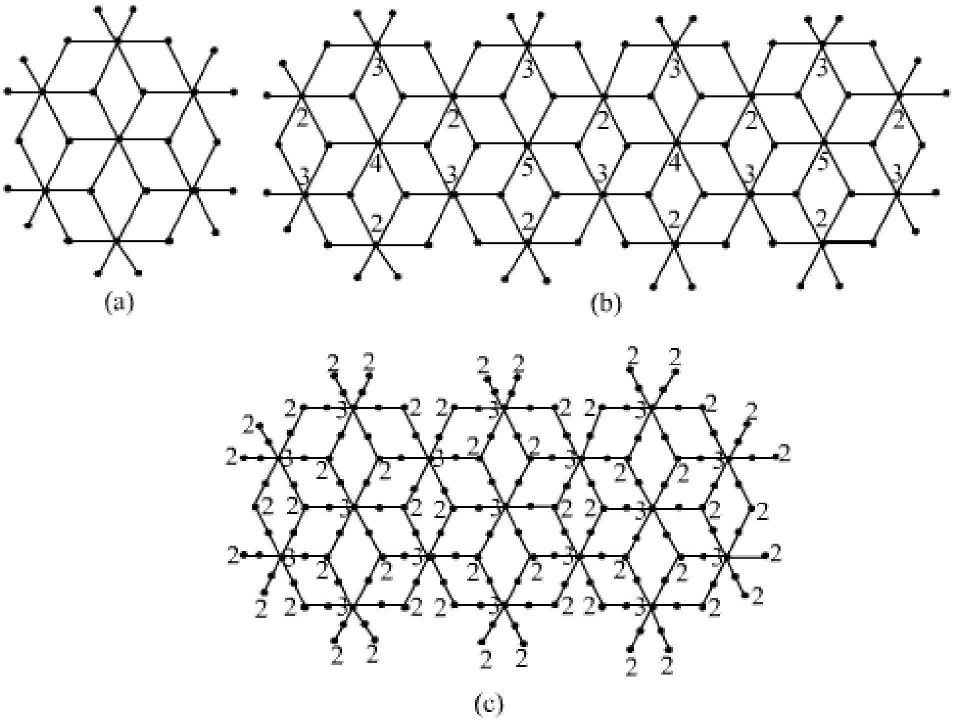

Figure 5. *a) One unit of lead chloride b) Packing 5-coloring of 4-lead chloride chain c) Packing 3-coloring of subdivided 4-lead chloride chain*

Theorem 4.1. Let G be an m -lead chloride chain. Then $4 \le \chi_\rho(G) \le 5$.

Proof: Since C_6 is a subgraph of G, by Propositions 2.1 and 2.2, $\chi_\rho(G) \ge 4$. Then upper bound pursues from the labelling of an m -lead chloride chain with 5 colors, whose pattern is shown in Figure 5(b). In Figure 5(b), it is expected that the vertices which are not colored get color 1. Thus, from Figure 5(b), $\chi_\rho(G) \le 5$. Hence $4 \le \chi_\rho(G) \le 5$.

Theorem 4.2. Let $S(G)$ be a subdivided m -lead chloride chain. Then $\chi_\rho(S(G)) = 3$.

Proof: Since C_8 is a subgraph of $S(G)$, by Propositions 2.1 and 2.2, $\chi_\rho(S(G)) \ge 3$. Then upper bound pursues from the labelling of a subdivided m -lead chloride chain with 3 colors, whose pattern is shown in Figure 5(c). In Figure 5(c), it is expected that the vertices which are not colored get color 1. Thus, from Figure 5(c), $\chi_\rho(S(G)) \le 3$. Hence $\chi_\rho(S(G)) = 3$.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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