ON \((\lambda, A)\)–STATISTICAL CONVERGENCE OF ORDER \(\alpha\)

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Abstract. In the paper [B. de Malafosse and V. Rakočević, Linear Algebra Appl. 420, no. 2-3, (2007), 377–387], authors defined the concept of \((\lambda, A)\)–statistical convergence. In this paper, the concept of \((\lambda, A)\)–statistical convergence is generalized to \((\lambda, A)\)–statistical convergence of order \(\alpha\). Also, we introduce the concept of strong \((V, \lambda, A)\)–convergence of order \(\alpha\) and give some inclusion relations between the concepts of \((\lambda, A)\)–statistical convergence of order \(\alpha\) and strong \((V, \lambda, A)\)–convergence of order \(\alpha\).

1. Introduction

In 1951, Steinhaus [34] and Fast [22] introduced the concept of statistical convergence and later in 1959, Schoenberg [32] reintroduced independently. Some arguments related to statistical convergence and its applications may be found in ([2], [5], [6], [7], [8], [9], [10], [18], [19], [20], [23], [35], [25], [20], [31], [10], [15], [38], [30], [33], [1], [17], [24]).

Let \(\lambda = (\lambda_n)\) be a non-decreasing sequence of positive real numbers tending to \(\infty\) such that \(\lambda_{n+1} \leq \lambda_n + 1\), \(\lambda_1 = 1\). The generalized de la Vallée-Poussin mean is defined by

\[
 t_n(x) = \frac{1}{\lambda_n} \sum_{k \in I_n} x_k,
\]

where \(I_n = [n - \lambda_n + 1, n]\) for \(n = 1, 2, \ldots\). A sequence \(x = (x_k)\) is said to be \((V, \lambda)\)–summable to a number \(L\) if \(t_n(x) \to L\) as \(n \to \infty\). If \(\lambda_n = n\), then \((V, \lambda)\)–summability is reduced to Cesàro summability. By \(A\) we denote the class of all non-decreasing sequence of positive real numbers tending to \(\infty\) such that \(\lambda_{n+1} \leq \lambda_n + 1\), \(\lambda_1 = 1\).

\(\lambda = (\lambda_n)\) sequence spaces were studied in ([11], [12], [21], [27], [28], [13], [14], [29], [36]) and \(A\)–statistical convergence for \(A = (a_{ik})\) an infinite matrix of complex numbers were studied in ([15], [14], [37], [3], [4]).

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Recently, the concept of $S(\lambda, A)$–convergence was defined by de Malafosse and Rakočević [14] as below:

Let $A = (a_{km})$ be an infinite matrix of complex numbers and $[AX]_k = A_k(X) = \sum_{m=1}^{\infty} a_{km}x_m$ for $k \geq 0$. A sequence $X = (x_n)_{n \geq 1}$ is said to be $(\lambda, A)$–statistically convergent to $L$ (or $S(\lambda, A)$–convergent to $L$) if for every $\varepsilon > 0$,

$$\lim_{n \to \infty} \frac{1}{\lambda_n} |\{k \in I_n : |[AX]_k - L| \geq \varepsilon\}| = 0$$

where $I_n = [n - \lambda_n + 1, n]$. In this case we write $x_k \to L(S(\lambda, A))$. The set of all $\lambda$–statistically convergent sequences will be denoted by $S(\lambda, A)$. If $\lambda_n = n$, we write $x_k \to L(S(A))$ and in the special case $A = I$, we write $x_k \to L(S(I))$ means that $x_k \to L(S)$.

2. Main Results

In this section, we will give the definition of $S^\alpha(\lambda, A)$–convergence and strong $W_p^\alpha(\lambda, A)$–convergence for $0 < p < \infty$ where $A = (a_{km})$ is an infinite matrix of complex numbers and $0 < \alpha \leq 1$ and give some results related to these concepts.

Definition 1. Let $\alpha \in (0, 1]$ and $A = (a_{km})$ be an infinite matrix of complex numbers. A sequence $X = (x_k)$ is said to be $(\lambda, A)$–statistically convergent of order $\alpha$ to $L$ (or $S^\alpha(\lambda, A)$–convergent to $L$) if for every $\varepsilon > 0$,

$$\lim_{n \to \infty} \frac{1}{\lambda_n^\alpha} |\{k \in I_n : |[AX]_k - L| \geq \varepsilon\}| = 0$$

where $I_n = [n - \lambda_n + 1, n]$ and $\lambda_n^\alpha$ denotes the $\alpha$th power $(\lambda_n^\alpha)$ of $\lambda_n$, that is $\lambda_n^\alpha = (\lambda_n^\alpha) = (\lambda_n^\alpha_1, \lambda_n^\alpha_2, ..., \lambda_n^\alpha_n, ...).$ In this case we write $S^\alpha(\lambda, A) \sim \lim x_k = L$ or $x_k \to L(S^\alpha(\lambda, A))$. The set of all $(\lambda, A)$–statistically convergent sequences of order $\alpha$ will be denoted by $S^\alpha(\lambda, A)$. For $\lambda_n = n$, we shall write $S^\alpha(A)$ instead of $S^\alpha(\lambda, A)$ and in the special case $A = I$, $\alpha = 1$ and $\lambda_n = n$ we shall write $S$ instead of $S^\alpha(\lambda, A)$.

The $(\lambda, A)$–statistical convergence of order $\alpha$ is well defined for $\alpha \in (0, 1]$, but it is not well defined for $\alpha > 1$ in general. $X = (x_m)$ and $A = (a_{km})$ are defined as follows: For $A = (a_{km})$ row matrix and $i = 1, 2, ...$

$$x_m = \begin{cases} 3, & \text{if } m = 3i \\ 0, & \text{if } m \neq 3i. \end{cases}$$

and

$$a_{km} = \begin{cases} 2, & \text{if } m = 3i \\ 0, & \text{if } m \neq 3i. \end{cases}$$

Both for every $\varepsilon > 0$

$$\lim_{n \to \infty} \frac{1}{\lambda_n^\alpha} |\{k \in I_n : |[AX]_k - 0| \geq \varepsilon\}| \leq \lim_{n \to \infty} \frac{[\lambda_n^\alpha] + 1}{3\lambda_n^\alpha} = 0$$
and
\[
\lim_{n \to \infty} \frac{1}{\lambda_n^\alpha} \{ k \in I_n : |(AX)_k - 0| \geq \varepsilon \} \leq \lim_{n \to \infty} \frac{2|\lambda_n| + 1}{3\lambda_n^\alpha} = 0
\]
for \( \alpha > 1 \). So \( S^\alpha(\lambda, A) - \lim x_k = 6 \) and \( S^\alpha(\lambda, A) - \lim x_k = 0 \), but this is impossible.

**Theorem 2.** Let \( \alpha \in (0, 1] \) be positive real number. If \( S^\alpha(\lambda, A) - \lim x_k = L_1 \) and \( S^\alpha(\lambda, A) - \lim x_k = L_2 \), then \( L_1 = L_2 \).

**Proof.** Since \( S^\alpha(\lambda, A) - \lim x_k = L_1 \) and \( S^\alpha(\lambda, A) - \lim x_k = L_2 \), we can write
\[
\lim_{n \to \infty} \frac{1}{\lambda_n^\alpha} \{ k \in I_n : |(AX)_k - L_1| \geq \varepsilon \} = 0
\]
and
\[
\lim_{n \to \infty} \frac{1}{\lambda_n^\alpha} \{ k \in I_n : |(AX)_k - L_2| \geq \varepsilon \} = 0.
\]
We have
\[
|L_1 - L_2| = |L_1 - L_2 + (AX)_k - (AX)_k| \\
\leq |(AX)_k - L_1| + |(AX)_k - L_2|
\]
for \( I_n = [n - \lambda_n + 1, n] \). We get
\[
\frac{1}{\lambda_n^\alpha} \{ k \in I_n : |L_1 - L_2| \geq \varepsilon \} \leq \frac{1}{\lambda_n^\alpha} \{ k \in I_n : |(AX)_k - L_1| \geq \varepsilon \} \\
+ \frac{1}{\lambda_n^\alpha} \{ k \in I_n : |(AX)_k - L_2| \geq \varepsilon \}.
\]
This is possible with \( L_1 = L_2 \). \( \square \)

**Theorem 3.** Let \( \alpha \in (0, 1] \) be positive real number, \( A = (a_{km}) \) be an infinite matrix of complex numbers and \( X = (x_k) \), \( Y = (y_k) \) be sequences of real numbers, then
(i) If \( S^\alpha(\lambda, A) - \lim x_k = x_0 \) and \( S^\alpha(\lambda, A) - \lim y_k = y_0 \), then \( S^\alpha(\lambda, A) - \lim (x_k + y_k) = (x_0 + y_0) \).
(ii) If \( S^\alpha(\lambda, A) - \lim x_k = x_0 \) and \( c \in \mathbb{C} \), then \( S^\alpha(\lambda, A) - \lim (cx_k) = cx_0 \).

**Proof.** Omitted. \( \square \)

**Definition 4.** Let \( \alpha \in (0, 1] \), \( 0 < p < \infty \) and \( A = (a_{km}) \) be an infinite matrix of complex numbers. We say that the sequence \( X = (x_k) \) is strong \((V, \lambda, A) - \)convergent of order \( \alpha \) to a number \( L \) (or \( W^\alpha_p(\lambda, A) - \)convergent to \( L \)) if
\[
\lim_{n \to \infty} \frac{1}{\lambda_n^\alpha} \sum_{k \in I_n} |(AX)_k - L|^p = 0.
\]
In this case, we write \( W^\alpha_p(\lambda, A) - \lim x_k = L \) or \( x_k \to L(W^\alpha_p(\lambda, A)) \).

**Theorem 5.** Let \( \alpha \in (0, 1] \) be positive real numbers and \( A = (a_{km}) \) be an infinite matrix of complex numbers, then \( W^\alpha_p(\lambda, A) \subseteq S^\alpha(\lambda, A) \) and the inclusion is strict.
Proof. \( \varepsilon > 0 \) and \( x_k \to L(W^\alpha_p(\lambda, A)) \). In this case, we have
\[
\sum_{k \in I_n} |[AX]_k - L|^p \geq \varepsilon^p \{ k \in I_n : |[AX]_k - L| \geq \varepsilon \}
\]
and
\[
\frac{1}{\lambda_n^p} |\{ k \in I_n : |[AX]_k - L| \geq \varepsilon \}| \leq \frac{1}{\lambda_n^p} \sum_{k \in I_n} |[AX]_k - L|^p.
\]
So \( x_k \to L(S^\alpha(\lambda, A)) \).
To show that the inclusion is strict define a sequence \( X = (x_m) \) and a row matrix \( A = (a_{km}) \) such that for \( i = 1, 2, \ldots \)
\[
x_m = \begin{cases} 4, & \text{if } m = i^2 \\ 0, & \text{if } m \neq i^2 \end{cases}
\]
and
\[
a_{km} = \begin{cases} 1, & \text{if } m = i^2 \\ 0, & \text{if } m \neq i^2 \end{cases}
\]
Let \( \lambda_n = n, p = 1 \) and \( L = 0 \). For \( \frac{1}{2} < \alpha \leq 1 \)
\[
\frac{1}{\lambda_n} \{ k \in I_n : |[AX]_k - L| \geq \varepsilon \} \leq \frac{\sqrt{n}}{n^\alpha} \to 0.
\]
i.e. \( x_k \to 0(S^\alpha(\lambda, A)) \). For \( 0 < \alpha < \frac{1}{2} \)
\[
\frac{1}{\lambda_n^p} \sum_{k \in I_n} |[AX]_k - L|^p = \frac{1}{n^\alpha} \sum_{k=1}^n |[AX]_k| \leq \frac{4\sqrt{n}}{n^\alpha} \to \infty
\]
and for \( \alpha = \frac{1}{2} \)
\[
\frac{1}{n^\alpha} \sum_{k=1}^n |[AX]_k| \leq \frac{4\sqrt{n}}{n^\alpha} \to 4
\]
i.e. \( x_k \to 0(W^\alpha_p(\lambda, A)) \).

Theorem 6. Let \( \alpha, \beta \in (0, 1] \) be positive real numbers such that \( \alpha \leq \beta \), then
\( S^\alpha(\lambda, A) \subseteq S^\beta(\lambda, A) \).

Proof. For \( \varepsilon > 0 \), we can write
\[
\frac{1}{\lambda_n^\beta} \{ k \in I_n : |[AX]_k - L| \geq \varepsilon \} \leq \frac{1}{\lambda_n^\alpha} \{ k \in I_n : |[AX]_k - L| \geq \varepsilon \}.
\]
So \( S^\alpha(\lambda, A) \subseteq S^\beta(\lambda, A) \) for \( 0 < \alpha \leq \beta \leq 1 \).
To show that the inclusion is strict define a sequence \( X = (x_k) \) by
\[
x_k = \begin{cases} 3, & \text{if } k \text{ is square} \\ 0, & \text{otherwise} \end{cases}
\]
Let \( A = I \). Then \( X \in S^\beta(\lambda, A) \) for \( \beta \in \left(\frac{1}{2}, 1\right] \), but \( X \notin S^\alpha(\lambda, A) \) for \( \alpha \in (0, \frac{1}{2}] \). \( \square \)
Theorem 7. Let \( \alpha \in (0, 1] \), \( S^{\alpha} (\lambda, A) = \lim x_k = x_0 \) and \( S^{\alpha} (\lambda, A) = \lim y_k = y_0 \).

If \( ||AX|| = |A_k (X)| = \sum_{m=1}^{\infty} a_{km} x_m | < M, (M > 0) \), then
\[
\lim_{n \to \infty} \frac{1}{\lambda_n^n} |\{ k \in I_n : ||(AX)_{[k]}[AY]_{[k]} - (x_0 y_0) || \geq \varepsilon \}| = 0.
\]

Proof. For \( \varepsilon > 0 \), we can write
\[
\frac{1}{\lambda_n^n} \left| \{ k \in I_n : ||(AX)_{[k]}[AY]_{[k]} - (x_0 y_0) || \geq \varepsilon \} \right|
\]
\[
= \frac{1}{\lambda_n^n} \left| \{ k \in I_n : ||AX||_{[k]}[AY]_{[k]} - (x_0 y_0) + ||AX||_{[k]} y_0 - ||AX||_{[k]} x_0 || \geq \varepsilon \} \right|
\]
\[
= \frac{1}{\lambda_n^n} \left| \{ k \in I_n : ||AX||_{[k]} ([AY]_{[k]} - y_0) + y_0 ([AX]_{[k]} - x_0) || \geq \varepsilon \} \right|
\]
\[
\leq \frac{1}{\lambda_n^n} \left| \left\{ k \in I_n : ||AX||_{[k]} ([AY]_{[k]} - y_0) \geq \frac{\varepsilon}{2} \right\} \right|
\]
\[
+ \frac{1}{\lambda_n^n} \left| \left\{ k \in I_n : |y_0 (||AX||_{[k]} - x_0) \geq \frac{\varepsilon}{2} \right\} \right|
\]
\[
= \frac{1}{\lambda_n^n} \left| \left\{ k \in I_n : ||[AY]_{[k]} - y_0) \geq \frac{\varepsilon}{2 |AX||_{[k]} y_0} > \frac{\varepsilon}{2 M} \right\} \right|
\]
\[
+ \frac{1}{\lambda_n^n} \left| \left\{ k \in I_n : ||AX||_{[k]} - x_0) \geq \frac{\varepsilon}{2 |y_0} \right\} \right|
\]

Therefore, \( \lim_{n \to \infty} \frac{1}{\lambda_n^n} |\{ k \in I_n : ||(AX)_{[k]}[AY]_{[k]} - (x_0 y_0) || \geq \varepsilon \}| = 0. \)

Theorem 8. Let \( \alpha \in (0, 1] \), \( S(A) \subseteq S^{\alpha} (\lambda, A) \) if and only if
\[
\lim_{n \to \infty} \inf \frac{\lambda_n^n}{n} > 0.
\]

Proof. For a given \( \varepsilon > 0 \), since
\[
\{ k \leq n : ||AX||_{[k]} - L || \geq \varepsilon \} \supset \{ k \in I_n : ||AX||_{[k]} - L || \geq \varepsilon \},
\]
we can write
\[
\frac{1}{n} \left| \{ k \leq n : ||AX||_{[k]} - L || \geq \varepsilon \} \right| \geq \frac{1}{n} \left| \{ k \in I_n : ||AX||_{[k]} - L || \geq \varepsilon \} \right|
\]
\[
= \frac{\lambda_n^n}{n} \cdot \frac{1}{\lambda_n^n} \left| \{ k \in I_n : ||AX||_{[k]} - L || \geq \varepsilon \} \right|.
\]

Conversely, suppose that \( \lim_{n \to \infty} \inf \frac{\lambda_n^n}{n} = 0 \). We can choose a subsequence \( \{ n(j) \}_{j=1}^{\infty} \) such that \( \frac{\lambda_n^n}{n} < \frac{1}{7} \). Define a sequence \( X = (x_k) \) by for \( j = 1, 2, \ldots \)
\[
x_k = \begin{cases} 1, & \text{if } k \in I_{n(j)} \\ 0, & \text{otherwise} \end{cases}
\]
Let $A = I$. Then $X \in S(A)$, but $X \notin S(\lambda, A)$. From Theorem 6, since $S^\alpha(\lambda, A) \subseteq S(\lambda, A)$, we have $X \notin S^\alpha(\lambda, A)$. Hence (1) is necessary.

\textbf{Theorem 9.} Let $\alpha, \beta \in (0, 1]$ be positive real numbers such that $\alpha \leq \beta$, then $W^\alpha_p(\lambda, A) \subseteq W^\beta_p(\lambda, A)$.

\textbf{Proof.} For $\varepsilon > 0$, we can write $$\frac{1}{\lambda_n^\beta} \sum_{k \in I_n} ||AX||^p_k - L||^p \leq \frac{1}{\lambda_n^\alpha} \sum_{k \in I_n} ||AX||^p_k - L||^p.$$ So $W^\alpha_p(\lambda, A) \subseteq W^\beta_p(\lambda, A)$ for $0 < \alpha \leq \beta \leq 1$.

To show that the inclusion is strict define a sequence $X = (x_k)$ by

$$x_k = \begin{cases} 2, & \text{if } k \text{ is square} \\ 0, & \text{otherwise} \end{cases}.$$ 

Let $A = I$. Then $X \in W^\beta_p(\lambda, A)$ for $\beta \in \left(\frac{1}{2}, 1\right]$, but $X \notin W^\alpha_p(\lambda, A)$ for $\alpha \in \left(0, \frac{1}{2}\right]$.

\textbf{Theorem 10.} Let $\lambda = (\lambda_n), \mu = (\mu_n) \in \Lambda$ such that $\lambda_n \leq \mu_n$ for all $n \in N$ and $\alpha, \beta \in (0, 1]$ be positive real numbers such that $0 < \alpha \leq \beta \leq 1$.

(i) If

$$\lim_{n \to \infty} \inf \frac{\lambda_n^\alpha}{\mu_n^\beta} > 0$$

then $S^\beta(\mu, A) \subseteq S^\alpha(\lambda, A)$.

(ii) If

$$\lim_{n \to \infty} \frac{\mu_n}{\lambda_n^\beta} = 1$$

then $S^\alpha(\lambda, A) \subseteq S^\beta(\mu, A)$, where $I_n = [n - \lambda_n + 1, n], J_n = [n - \mu_n + 1, n]$.

\textbf{Proof.} (i) Suppose that $\lambda_n \leq \mu_n$ for all $n \in N$ and let (2) be satisfied. For given $\varepsilon > 0$ we have

$$\left\{ k \in J_n : ||AX||_k - L \geq \varepsilon \right\} \supseteq \left\{ k \in I_n : ||AX||_k - L \geq \varepsilon \right\}$$

and so

$$\frac{1}{\mu_n} \left| \left\{ k \in J_n : ||AX||_k - L \geq \varepsilon \right\} \right| \geq \frac{\lambda_n^\alpha}{\mu_n} \frac{1}{\lambda_n^\beta} \left| \left\{ k \in I_n : ||AX||_k - L \geq \varepsilon \right\} \right|$$

for all $n \in N$.

(ii) Let $X = (x_k) \in S^\alpha(\lambda, A)$ and (3) be satisfied. We have

$$\lim_{n \to \infty} \frac{1}{\lambda_n^\alpha} \left| \left\{ k \in I_n : ||AX||_k - L \geq \varepsilon \right\} \right| = 0.$$ 

Since $I_n \subset J_n$, for $\varepsilon > 0$ we may write
for all \( n \in \mathbb{N} \). This implies that \( S^\alpha(\lambda, A) \subseteq S^\beta(\mu, A) \). \( \square \)

**Corollary 11.** Let \( \lambda = (\lambda_n) \), \( \mu = (\mu_n) \) \( \in \Lambda \) such that \( \lambda_n \leq \mu_n \) for all \( n \in \mathbb{N} \) and \( 0 < \alpha \leq \beta \leq 1 \).

If (2) holds, then

(i) \( S^\alpha(\mu, A) \subseteq S^\alpha(\lambda, A) \),

(ii) \( S(\mu, A) \subseteq S^\alpha(\lambda, A) \),

(iii) \( S(\mu, A) \subseteq S(\lambda, A) \).

If (3) holds, then

(i) \( S^\alpha(\lambda, A) \subseteq S^\alpha(\mu, A) \),

(ii) \( S^\alpha(\lambda, A) \subseteq S(\mu, A) \),

(iii) \( S(\lambda, A) \subseteq S(\mu, A) \).

**Theorem 12.** Let \( \lambda = (\lambda_n) \), \( \mu = (\mu_n) \) \( \in \Lambda \) such that \( \lambda_n \leq \mu_n \) for all \( n \in \mathbb{N} \) and \( 0 < \alpha \leq \beta \leq 1 \). Then

(i) If (2) holds, then \( W^p_\beta(\mu, A) \subseteq W^\alpha_p(\lambda, A) \),

(ii) If (3) holds and \( \sup_k |A_k(x)| < \infty \) then \( W^\alpha_p(\lambda, A) \subseteq W^\beta_p(\mu, A) \).

**Proof.** (i) Suppose that \( \lambda_n \leq \mu_n \) for all \( n \in \mathbb{N} \) and let (2) be satisfied. For given \( \varepsilon > 0 \) we have

\[
\frac{1}{\mu_n^\beta} \sum_{k \in J_n} ||AX||_k - L||^p > \frac{\lambda_n^\alpha}{\mu_n^\beta \lambda_n^\alpha} \sum_{k \in I_n} ||AX||_k - L||^p.
\]

This implies that \( W^\alpha_p(\lambda, A) \subseteq W^\beta_p(\mu, A) \).

(ii) Let \( X = (x_k) \in W^\alpha_p(\lambda, A) \) and suppose that (3) holds. Then

\[
\lim_{n \to \infty} \frac{1}{\lambda_n^\alpha} \sum_{k \in I_n} ||AX||_k - L||^p = 0.
\]
Since \( \sup_k |A_k(x)| < \infty \) then there exists some \( M > 0 \) such that \( ||AX_k - L|| \leq M \) for all \( k \). Now, since \( I_n \subseteq J_n \) and \( \lambda_n \leq \mu_n \) for all \( n \in \mathbb{N} \), we may write

\[
\frac{1}{\mu_n^\alpha} \sum_{k \in J_n} ||AX_k - L|^p = \frac{1}{\mu_n^\beta} \sum_{k \in J_n - I_n} ||AX_k - L|^p + \frac{1}{\mu_n^\beta} \sum_{k \in I_n} ||AX_k - L|^p
\]

\[
\leq \frac{\mu_n - \lambda_n}{\mu_n^\beta} M^\alpha + \frac{1}{\mu_n^\beta} \sum_{k \in I_n} ||AX_k - L|^p
\]

\[
\leq \left( \frac{\mu_n}{\lambda_n^\beta} - 1 \right) M^\alpha + \frac{1}{\lambda_n^\beta} \sum_{k \in I_n} ||AX_k - L|^p
\]

for every \( n \in \mathbb{N} \). Therefore \( W_p^\alpha (\lambda, A) \subset W_p^\beta (\mu, A) \).

Corollary 13. Let \( \lambda = (\lambda_n) \), \( \mu = (\mu_n) \in \Lambda \) such that \( \lambda_n \leq \mu_n \) for all \( n \in \mathbb{N} \) and \( 0 < \alpha \leq \beta \leq 1 \).

If (2) holds, then

(i) \( W_p^\alpha (\mu, A) \subset W_p^\alpha (\lambda, A) \),

(ii) \( W_p^\beta (\mu, A) \subset W_p^\alpha (\lambda, A) \),

(iii) \( W_p^\beta (\mu, A) \subset W_p^\beta (\lambda, A) \).

If (3) holds and \( \sup_k |A_k(x)| < \infty \), then

(i) \( W_p^\alpha (\lambda, A) \subset W_p^\alpha (\mu, A) \),

(ii) \( W_p^\beta (\lambda, A) \subset W_p^\beta (\mu, A) \),

(iii) \( W_p^\lambda (\lambda, A) \subset W_p^\beta (\lambda, A) \).

References


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