A Time Series Approach for Precipitation in Turkey

Mesut BALIBEY¹,*, Serpil TÜRKYILMAZ²

¹ Tunceli University, Faculty of Economics and Administrative Sciences, Department of Business Administration
² Bilecik Şeyh Edebali University, Faculty of Science & Art, Department of Mathematics,

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ABSTRACT

The climate that has a dynamic structure varies continuously. Basic climatological features of any region depend on many meteorological variables, such as precipitation and temperature records. The changes in these two variables reveal important clues about climate variations. Turkey is one of the sensitive areas for climate variation around the world. In this study, statistical analysis is applied to historical data set consisted of monthly precipitation totals (mm.) which are provided by the Turkish State Meteorological Service (TSMS) for 1999-2010 period in Turkey. Furthermore, a sinusoidal model for each region obtained by using a time series method is proposed in order to obtain a sinusoidal form for the precipitation to each region. Besides a basic form of Frequency Domain Techniques, usual Box-Jenkins ARIMA approach is applied in this study. In conclusion, it is observed that the sinusoidal model predictions are more appropriate than ARIMA techniques and usual generalized linear model (GLM) techniques.

Keywords: Precipitation; Periodicity; Time Series Model; Periodogram; Frequency Domain Technique.

1. INTRODUCTION

The information on temperature and precipitation has a great importance in determining the characters of global climate changes. The fundamental climatological characteristics of any region depend on meteorological variables such as the precipitation and temperature. Oscillations in these two variables reveal important clues to understand the overall structure of the climate. With increase in the human population, the increased need for water and energy requires sensitive socio-economic awareness about the precipitation changes. The most changing element for climate is the precipitation which varies in respect to time and space. In recent years, the studies about climate changes have focused on trend analysis of these two variables. Turkey is one of the countries which has sensitive regions to climate changes in the World [1]. Particularly, some regions of Turkey are more sensitive against the climate changes such as the Mediterranean and the Southeastern Anatolia due to their geographical locations. In these regions, vegetation and other natural sources could be damaged because of climate changes. Furthermore, climate oscillations could cause drought. As a result of this, some important problems could be occurred such as decrease in agricultural production. The unpredictable climate changes in these regions create a great pressure on water resources. Because of the effects of global climate changes in the last years, water resources are limited. Therefore, the statistical analysis of the precipitations has become more important in recent years.

*Corresponding author, e-mail: mstblby@hotmail.com
In this study, precipitation data is investigated from region to region during 1999-2010 period in Turkey. Many studies have contributed to the analysis of the precipitation variability for various time periods for different countries. [2] investigate the monthly rainfall series by SARIMA and spectrum models from January 1989 to December 2005 in China. [3] examine the seasonal precipitation distributions and seasonal trend changes of Trabzon and Rize which are located on the northeast coast of Turkey by using Matrix and Marginal analysis methods and ARIMA model. [4] examines north south directional temperature and precipitation change forecasts of Turkey. In the study, temperature and precipitation trends are realized by analyzing selected 6(six) stations characterizing the three main climate types along a north-south directional line in the central part of Turkey by the ARIMA models. [5] examine the relationships between rainfall, temperature, minimum temperature and maximum temperature as well as over time by using time series methods in Queensland. In addition, they investigate autocorrelation and seasonality by using ARIMA models. This study showed that the rainfall data was variable while the temperature data was stationary in 2009-2010 period. [6] investigate spatiotemporal variability of precipitation by the principal component analysis for 1930-2002 period. [7] propose a new approach for determining the water potential of Turkey for 1971-2000 period. [8] determine the characteristics of the precipitation pattern for Thessaly region in Central Greece by using the Principal Component Analysis. [1] examine the trends in the precipitation at annual, seasonal and monthly time scales by using non-parametric tests, linear regression for 1975-2005 periods for Kahramanmaras which is mostly located in the mediterranean region of Turkey. [9] propose an approach based on a statistical model for daily rainfall that achieves a high level of real-time control by the inclusion of both spatial and temporal components for Australia. [10] present a statistical analysis of the daily maximum and monthly mean precipitation measured at two meteorological stations in Serbia during 1948-2007 period. [11] analyze long-term changes and trends in the series of seasonal and total annual precipitation and precipitation intensity of 111 stations in Turkey by the methods of Kruskal-Wallis homogeneity test and Mann-Kendall rank correlation coefficient test. [12] investigate spectral characteristics of the Annual Mean Rainfall in Ghana by using the Maximum Entropy Spectral Analysis technique for 1961-1998 period. [13] present a trend analysis in Turkish precipitation data for 1929-1993 period. [14] analyze changes in daily precipitation frequency and distribution in Italy over the last 120 years by a Principal Component Analysis. [15] examines some Indian monthly meteorological data by means of fractional integration. The results of this study show that long memory is present in the monthly structure of various rainfall data. [16] analyze the relationships between the variability of the North Atlantic oscillation indices and the normalized precipitation at 78 stations in Turkey. [17] analyzes monthly and annual precipitation data of Ankara Station in Turkey statistically by using a trend analysis for a 75 year period between 1926-2000. [18] investigate persistence and periodicity in normalized precipitation anomaly series of 91 stations over Turkey using serial correlation coefficients and power spectra. [19] indicate an analysis of precipitation climatology in Jordan. [20] present changes in total precipitation, rainy days and extreme events in northeastern Italy. This study shows a negative trend in wet days numbers associated with the increase in the contribution of heavy rainfall events to total precipitation. [21] develop a validate optimum interpolation method for the spatial analysis of monthly precipitation for 30 years in Turkey. [22] consider only precipitation records for depicting spatial periodic features over the whole of Turkey by using harmonic analysis. [23] investigates climatic factors in Turkey by the analysis of the spatial and temporal variations of the precipitation and aridity series for 1930-1993 period. [24] tries to reveal daily rainfall intensity in the area of Dikili and Bergama in Turkey during 1950-1995 period.

The paper is organized as follows. In Section 2, data and methodology are presented. Section 3 gives the empirical results. Conclusions are provided in Section 4.

2. DATA AND METHODOLOGY

The precipitation data set used in this study consists of monthly average precipitation from region to region calculated by monthly total precipitation values recorded at the 270 stations of the Turkish State Meteorological Service (TSMS) during 1999-2010 period. In addition, these data at station level is grouped from region to region and monthly averages are calculated.

As shown in the previous chapter, many of the studies have focused on the trend analysis. However, the precipitation data consists of seasonal effects. The aim of this study is to obtain a sinusoidal model related to each region for monthly average precipitation values from region to region by using time series techniques by considering this situation. Furthermore, the application presents that the validity of these sinusoidal models are checked statistically. The predictions for precipitation values are obtained by means of ARIMA and frequency domain techniques. In the study, different ARIMA and SARIMA models for all the regions are estimated.

For instance,
\[(X_t - \mu) = \alpha(X_{t-1} - \mu) + e_t , \quad t=1,2,...,n .\]
\[(X_t - \mu) = \alpha(X_{t-n} - \mu) + e_t , \quad t=1,2,...,n .\]
\[(X_t - \mu) = \alpha(X_{t-12} - \mu) + e_t , \quad t=1,2,...,n . \ldots \text{etc}^1.\]

All these models have been considered and the model which has the least AIC selection criteria has been used.

The simplest periodic functions are the sinusoids, given by either [26], or

\[^1\text{For more information see [25].}\]
The function has a maximum at the time

\[ t_0 = \phi / 2 \pi f \]  

[26-29]. In this form, we see that

the function has a maximum at the time \( t_0 \), which is a
time shift. Any periodic data can be expressed as,

\[ y_i = R \cos(2\pi ft + \phi) + \varepsilon_i \]  

[4] or

\[ y_i = R \cos(2\pi ft) \cos(\phi) - R \sin(\phi) \sin(2\pi ft) + \varepsilon_i. \]  

[5]

The purpose is to estimate model parameters, \( \mu, R \) and \( \phi \) which are called mean, amplitude, and phase respectively. If we set \( A = R \cos(\phi) \) and \( B = R \sin(\phi) \), the model in (1) can be written as a regression equation in (6) [25, 28].

\[ y_i = \mu + A \cos(2\pi ft) + B \sin(2\pi ft) + \varepsilon_i; \quad t = 1,2,n. \]  

[6]

Then the OLS estimators of the parameter according to

(2) are,

\[ \hat{\mu} = \bar{y}, \quad \hat{\lambda}_a = \frac{2}{n} \sum_{i=0}^{n-1} y_i \cos(2\pi ft). \]  

[7]

\[ \hat{\lambda}_b = \frac{2}{n} \sum_{i=0}^{n-1} y_i \sin(2\pi ft). \]  

[8]

the properties of trigonometric function implies that

\[ \sum_{i=0}^{n-1} \cos(2\pi ft) = \sum_{i=0}^{n-1} \sin(2\pi ft) = 0, \]  

[9] for \( f = k/n \) and therefore, the estimators can be written as,

\[ \hat{\lambda}_a = \frac{2}{n} \sum_{i=0}^{n-1} (Y_i - \bar{Y}) \cos(2\pi ft) \]  

[10] and

\[ \hat{\lambda}_b = \frac{2}{n} \sum_{i=0}^{n-1} (Y_i - \bar{Y}) \sin(2\pi ft). \]  

These properties implies that the OLS estimators are

invariant to the mean which is an important advantage

over the usual OLS method.

3. EMPIRICAL RESULTS

In this section, the prediction models are obtained for

monthly average precipitation values related to each

region. Furthermore, the results of all statistical analysis

are presented only for the Mediterranean Region.

Moreover, the results of similar analysis for the other

regions are indicated briefly.

Due to Turkey’s geographical structure, the

precipitation amount varies according to the

regions. The area which has the highest average precipitation in

a year in Turkey is the Black Sea Region. The

maximum precipitation amount in this region reaches

the highest values in all regions. In the empirical

analysis of the study, it has been observed that there is a

significant difference between regions statistically. Let

\( y_{ijk} \) denote the kth amount of monthly average

precipitation in the jth region for ith month. In that case,

the model is given by,

\[ y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha \beta)_{ij} + e_{ijk}, \quad i = 1,2,\ldots,12; \]  

[11] \( j = 1,2,\ldots,7; \) \( k = 1,2,\ldots,n \)  

According to the model, \( \text{H}_0: \) “There isn’t significant
difference between regions”, the null hypothesis is

rejected at the significance level \( \alpha = 0.05. \) (The value

of \( F = 2.60 \) is greater than the critical value \( F_{0.05, 6,77}=2.23, \) and \( p \) value is 0.0240). Table 1 indicates the

values of mean and standart deviations according to

regions and months.
Table 1. The Precipitation Means According to Regions and Months

<table>
<thead>
<tr>
<th></th>
<th>Regions</th>
<th>M</th>
<th>Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agean</td>
<td>12</td>
<td>12</td>
<td>54.5965833</td>
<td>38.0646250</td>
</tr>
<tr>
<td>Blacksea</td>
<td>12</td>
<td>12</td>
<td>76.9176667</td>
<td>20.1469727</td>
</tr>
<tr>
<td>Central Anatolia</td>
<td>12</td>
<td>12</td>
<td>34.3405833</td>
<td>13.2232683</td>
</tr>
<tr>
<td>Eastern Anatolia</td>
<td>12</td>
<td>12</td>
<td>46.4996667</td>
<td>21.4809929</td>
</tr>
<tr>
<td>Marmara</td>
<td>12</td>
<td>12</td>
<td>60.5996667</td>
<td>27.3794241</td>
</tr>
<tr>
<td>Mediterranean</td>
<td>12</td>
<td>12</td>
<td>60.8393333</td>
<td>42.2371361</td>
</tr>
<tr>
<td>Southeastern</td>
<td>12</td>
<td>12</td>
<td>43.2719167</td>
<td>34.1820259</td>
</tr>
</tbody>
</table>

Table 2. The General Linear Model Procedure Results

<table>
<thead>
<tr>
<th>Class Level Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class</td>
</tr>
<tr>
<td>MONTH</td>
</tr>
<tr>
<td>Number of observations</td>
</tr>
</tbody>
</table>

The detailed statistical analysis and results are presented only for the Mediterranean Region. Furthermore, Table 2 presents the results of general linear model procedure for the Mediterranean Region.

Table 2. General Linear Model Procedure Results

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
</table>

* M is the number of months. R is the number of regions.

According to Table 1, the Black Sea region has the highest average precipitation in a year in Turkey. Moreover, maximum precipitation rate in Turkey is occurred in December, January and February which are the winter months respectively.

The detailed statistical analysis and results are presented only for the Mediterranean Region. Furthermore, Table 2 presents the results of general linear model procedure for the Mediterranean Region.
The results of the general linear model show that there is a significant difference among months, and the monthly means tell the story (high rainfall in December, January and February, low in June, July and August, and so on). For the Mediterranean region, the monthly precipitation can be the usual linear model for the monthly precipitation data and can be written as:

\[ y_{ij} = \mu + \alpha_i + e_{ij}, \quad i=1,\ldots,12; \quad j=1,\ldots,n \] (11)

Where \( Y_{ij} \) indicates jth monthly precipitation data in ith month. It is needed to estimate 12 parameters to describe the variation among months. However, a better model for each region is obtained by estimating a few parameters by using the sinusoidal forms. Moreover, there are more meaningful models to catch the periodicities in the data. For the Mediterranean region, firstly, the Model I is considered.

Model I:

\[ y_i = \mu + A \cos(2\pi t/12) + B \sin(2\pi t/12) + \epsilon_i \] (12)

and then Model II is evaluated.

Model II:

\[ y_i = \mu + A_1 \cos(2\pi t/12) + B_1 \sin(2\pi t/12) + A_2 \cos(2\pi t/6) + B_2 \sin(2\pi t/6) + \epsilon_i \] (13)

Therefore, the null hypothesis (\( H_0 \): Model I is true) cannot be rejected against the alternative hypothesis (\( H_1 \): Model II is true). Several alternative models have been considered and it has been obtained that the Model I cannot be rejected. By using iterative trigonometric calculations, the Model I turns out to be

\[ Y_i = R \cos(2\pi ft + \phi) + \epsilon_i \] (14)

For example, 3 parameters (\( \mu, A, B \)) should be estimated instead of f=12 parameter by using this model. In empirical analysis, the period of sinusoidal function is 12. Table 3 indicates the results for the Mediterranean Region.
Table 3. The Results of Model for the Mediterranean Region

Model: MODEL I

| Variable | DF | Estimate | Error   | t Value | Pr>|t| |
|----------|----|----------|---------|---------|------|
| Intercept| 1  | 60.83924 | 3.32174 | 18.32   | <.0001 |
| c1       | 1  | 49.32597 | 4.69764 | 10.50   | <.0001 |
| s1       | 1  | 24.86349 | 4.69764 | 5.29    | <.0001 |

Model: MODEL II

| Variable | DF | Estimate | Error   | t Value | Pr>|t| |
|----------|----|----------|---------|---------|------|
| Intercept| 1  | 60.83924 | 3.26738 | 18.62   | <.0001 |
| c1       | 1  | 49.32597 | 4.62077 | 10.67   | <.0001 |
| s1       | 1  | 24.86349 | 4.62077 | 5.38    | <.0001 |
| c2       | 1  | 11.98701 | 4.62077 | 2.59    | 0.0105 |
| s2       | 1  | -0.14301 | 4.62077 | -0.03   | 0.9754 |

(* c1=12, s1=12, c2=6, s2=6 indicate periods)

According to the results in Table 3, F statistic is calculated (c1=12, s1=12) as;

\[
F = \frac{(SSE(\text{Model}) - SSE(\text{Full model})) / 2}{MSE(\text{Full model})}
\]

\[
= \frac{(224033 - 213686) / 2}{3,3653} = 8.53
\]

and therefore, the null hypothesis (H\_0: c2=s2=0) is accepted. In other words, it can be assumed that the model I is true. The sinusoidal model and ARIMA procedure \( \hat{Y}_t = \hat{\mu} + \hat{\alpha} (Y_{t-12} - \hat{\mu}) \) for the Mediterranean Region can be obtained as follows,

Sinusoidal Model:

\[
\hat{Y}_t = 60.83924 + 49.32597 \cos\left\{ \frac{2\pi t}{12} \right\} + 24.86349 \sin\left\{ \frac{2\pi t}{12} \right\}
\]

ARIMA Procedure: \( \hat{Y}_t = 62.74370 + 0.53847 Y_{t-12} \)

According to the ARIMA procedure, the most appropriate model is observed as,

\[
\text{Std.Err. (8,49273) (0,07318)}
\]

The monthly averages and predictions according to this sinusoidal model and Box Jenkins ARIMA procedure are given in Figure 1.
According to Figure 1, the sinusoidal model predictions are more close to the observed monthly averages than the predictions obtained from ARIMA procedure. Moreover, monthly precipitation data predictions for other regions are obtained by means of sinusoidal model and ARIMA procedure. Figure 2a, Figure 2b, Figure 2c, Figure 2d, Figure 2e and Figure 2f indicate the predictions of Sinusoidal Models and ARIMA procedures for Eastern Anatolia, South Eastern Anatolia, Central Anatolia, Marmara, Black Sea and Aegean Region respectively.

**Sinusoidal Model:**
\[
\hat{Y}_t = 46,49951 + 10,62687 \cos\left\{\frac{2\pi t}{12}\right\} + 22,81562 \sin\left\{\frac{2\pi t}{12}\right\}
\]

**ARIMA Procedure:**
\[
\hat{Y}_t = 46,28569 + 0,38759 Y_{t-12}
\]

Std.Err. (3,67335) (0,07737)

Figure 2a. The Monthly Precipitation Averages, Sinusoidal Function Graph and ARIMA Model Estimations for Eastern Anatolia Region.

**Sinusoidal Model:**
\[
\hat{Y}_t = 43,27188 + 35,1768\cos\left\{\frac{2\pi t}{12}\right\} + 29,6135\sin\left\{\frac{2\pi t}{12}\right\}
\]

**ARIMA Procedure:**
\[
\hat{Y}_t = 46,28569 + 0,38759 Y_{t-12}
\]

Std.Err. (3,67335) (0,07737)

Figure 2b. The Monthly Precipitation Averages, Sinusoidal Function Graph and ARIMA Model Estimations for South Eastern Anatolia Region.
Sinusoidal Model: $\hat{Y}_t = 34.34049 + 8.10317 \cos\left\{ \frac{2 \pi t}{12} \right\} + 13.23112 \sin\left\{ \frac{2 \pi t}{12} \right\}$

ARIMA Procedure: $\hat{Y}_t = 34.43642 + 0.25037 Y_{t-12}$

Std. Err. (2.40216) (0.08260)

Figure 2c. The Monthly Precipitation Averages, Sinusoidal Function Graph and ARIMA Model Estimations for Central Anatolia Region.

Sinusoidal Model: $\hat{Y}_t = 60.59958 + 34.81254 \cos\left\{ \frac{2 \pi t}{12} \right\} + 9.84516 \sin\left\{ \frac{2 \pi t}{12} \right\}$

ARIMA Procedure: $\hat{Y}_t = 60.79841 + 0.32728 Y_{t-12}$

Std. Err. (4.97149) (0.08005)

Figure 2d. The Monthly Precipitation Averages, Sinusoidal Function Graph and ARIMA Model Estimations for Marmara Region.
Sinusoidal Model: \( \hat{Y}_t = 76.91764 + 22.48255 \cos\left(\frac{2\pi t}{12}\right) - 9.09738 \sin\left(\frac{2\pi t}{12}\right) \)

ARIMA Procedure: \( \hat{Y}_t = 76.91244 + 0.8903 Y_{t-12} \)

Std.Err. (0.08903) (0.08374)

Figure 2e. The Monthly Precipitation Averages, Sinusoidal Function Graph and ARIMA Model Estimations for Black Sea Region.

Sinusoidal Model: \( \hat{Y}_t = 54.59646 + 45.03981 \cos\left(\frac{2\pi t}{12}\right) + 22.552 \sin\left(\frac{2\pi t}{12}\right) \)

ARIMA Procedure: \( \hat{Y}_t = 55.96544 + 0.50892 Y_{t-12} \)

Std.Err. (7.34400) (0.07292)

Figure 2f. The Monthly Precipitation Averages, Sinusoidal Function Graph and ARIMA Model Estimations for Aegean Region.
4. CONCLUSIONS

In recent years, the statistical analysis of the precipitations has become important in terms of agricultural politics. Therefore, in this study, precipitation data are investigated according to the regions by time series techniques during 1999-2010 period in Turkey.

Due to Turkey’s geographical structure, the precipitation amount varies from region to region. The region which has the highest average precipitation in a year in Turkey is the Black Sea region. Besides the yearly average expanding to the whole year, the maximum precipitation amount in this region reaches very high figures. In the empirical analysis of the study, it has been observed that there is a significant difference between regions statistically. Moreover, maximum precipitation rate in Turkey is occurred in December, January and February which are the winter months respectively.

In the empirical analysis of the study, two models are obtained (the sinusoidal model and ARIMA procedure) for the precipitation data according to the regions. It is observed that the sinusoidal models are more appropriate than ARIMA procedures for monthly averages. Moreover, several sinusoidal forms are considered to get the most appropriate sinusoidal model for each region. The validity of sinusoidal models are supported by statistical findings.

Consequently, the sinusoidal functions obtained for each region can contribute to decision makers about agricultural politics and environmental policies to take preventive measures against drought.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES


