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ON ONE WEIGHTED INEQUALITIES FOR CONVOLUTION TYPE OPERATOR

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Abstract

In this paper we prove the boundedness of certain convolution operator in a weighted Lebesgue space with kernel satisfying the generalized Hörmander's condition. The sufficient conditions for the pair of general weights ensuring the validity of two-weight inequalities of a strong type and of a weak type for convolution operator with kernel satisfying the generalized Hörmander's condition are found.

Keywords: Weighted Lebesgue space, Singular integral, Kernel, Generalized Hörmander's condition, Boundedness.

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1. Introduction.

Let $\mathbb{R}^n$ be n-dimensional Euclidean spaces of points $x = (x_1, \dots, x_n)$, where $n \in \mathbb{N}$ and $\mathbb{R}^n_0 = \mathbb{R}^n \setminus \{0\}$. Suppose that ω is a non-negative, Lebesgue measurable and real function defined on $\mathbb{R}^n$, i.e., ω is a weight function defined on $\mathbb{R}^n$. By $L_{p,\omega}(\mathbb{R}^n)$ we denote the weighted Lebesgue space of measurable functions f on $\mathbb{R}^n$ such that

$$\|f\|_{L_{p,\omega}(\mathbb{R}^n)} = \|f\|_{p,\omega} = \left(\int_{\mathbb{R}^n} |f(x)|^p \, \omega(x)\, dx \right)^{1/p} < \infty, \ \ 1 \le p < \infty.$$

In the case $p = \infty$, the norm on the space $L_{\infty,\omega}(\mathbb{R}^n)$ is defined as

$$\|f\|_{L_{\infty,\omega}(\mathbb{R}^n)} = \|f\|_\infty = ess \sup_{x \in \mathbb{R}^n} |f(x)|.$$

For $\omega = 1$ we obtain the nonweighted L_p spaces, i.e., $\|f\|_{L_{p,1}(\mathbb{R}^n)} = \|f\|_{L_p(\mathbb{R}^n)} = \|f\|_p$.

Our aim in this paper is to show the boundedness of certain convolution operator in a weighted Lebesgue space with kernel satisfying the generalized Hörmander's condition.

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