

SOME NEW IDENTITIES CONCERNING GENERALIZED FIBONACCI AND LUCAS NUMBERS

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Abstract

In this paper we obtain some identities containing generalized Fibonacci and Lucas numbers. Some of them are new and some are well known. By using some of these identities we give some congruences concerning generalized Fibonacci and Lucas numbers such as

$$V_{2mn+r} \equiv (-(-t)^m)^n V_r \pmod{V_m},$$

$$U_{2mn+r} \equiv (-(-t)^m)^n U_r \pmod{V_m},$$

and

$$V_{2mn+r} \equiv (-t)^{mn} V_r \pmod{U_m},$$

$$U_{2mn+r} \equiv (-t)^{mn} U_r \pmod{U_m}.$$

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1. Introduction

Let k and t be nonzero real numbers. Generalized Fibonacci sequence $\{U_n\}$ is defined by $U_0 = 0$, $U_1 = 1$, and $U_{n+1} = kU_n + tU_{n-1}$ for $n \geq 1$ and generalized Lucas sequence $\{V_n\}$ is defined by $V_0 = 2$, $V_1 = k$, and $V_{n+1} = kV_n + tV_{n-1}$ for $n \geq 1$. U_n and V_n are called generalized Fibonacci numbers and generalized Lucas numbers respectively.

For $k = t = 1$, we have classical Fibonacci and Lucas sequences $\{F_n\}$ and $\{L_n\}$. For $k = 2$ and $t = 1$, we have Pell and Pell-Lucas sequences $\{P_n\}$ and $\{Q_n\}$. For more

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