ON ONE SIDED STRONGLY PRIME IDEALS

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Abstract
The notion of strongly prime right ideal is analogous to that of completely prime ideal in a commutative ring. We prove that the intersection of all strongly prime right ideals of a ring $R$ coincides with the Levitzki radical of this ring. We also give various conditions on a noncommutative ring $R$ so that $R$ is 2-primal.

Keywords: Prime right ideal, Strongly prime right ideal, 2-primal ring, AC-ring, Regular ring.

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1. Introduction
Throughout this article $R$ denotes an associative ring and $I \neq R$ a right ideal of $R$. In [6], a right ideal $I$ in $R$ is called a prime right ideal if $AB \subseteq I$ implies that either $A \subseteq I$ or $B \subseteq I$ for any right ideals $A,B$ of $R$. In [3], the right ideal $I$ was defined to be strongly prime if for each $x$ and $y$ in $R$, $xIy \subseteq I$ and $xy \in I$ imply that either $x \in I$ or $y \in I$. Let $m(R)$ (resp., $p(R)$, $sp(R)$) be the set of maximal right ideals (resp., prime right ideals, strongly prime right ideals) of $R$. Clearly, any strongly prime right ideal is prime. But the converse need not be true. For example, the zero ideal in the ring of all $n \times n$ matrices over a division ring is a prime right ideal but not strongly prime.

Recall that a two-sided ideal $P$ of $R$ is completely prime (completely semiprime) if $ab \in P$ implies $a \in P$ or $b \in P$ (if $a^2 \in P$ implies $a \in P$) for $a,b \in R$. Note that any ideal (two-sided) of a ring is strongly prime if and only if it is completely prime.

The goal of this paper is to prove that the intersection rad$_R(R)$ of all strongly prime right ideals coincides with the largest locally nilpotent ideal of the ring $R$. Also, we give some characterizations of rings through strongly prime right ideals.

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