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ON THE SINGULAR CAUCHY PROBLEM FOR A GENERALIZATION OF THE EULER POISSON DARBOUX EQUATION

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Abstract

In this paper, a solution is given for the following singúlar Cauchy problem:

$$\Delta u = u_{tt} + (at + \frac{b}{t})u_t$$

$$u(x,0) = f(x), u_t(x,0) = 0.$$

The solution is an uniformly and absolutely convergent power series. Where $a, b \in R, f(x)$ is a continuously differentiable function.

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1 Introduction

The singular Cauchy problem (abbreviated CP) for the Euler Poisson Darboux (EPD) equation can be formulated as follows: Let $f(x) = f(x_1, x_2, \ldots, x_n)$ be an arbitrary function which is differentiable continuously. It is required to find a function $u(x)$ which satisfies the following conditions:

$$\Delta u = u_{tt} + \frac{k}{t}u_t \tag{1}$$

$$u(x,0) = f(x), u_t(x,0) = 0 \tag{2}$$

where in the EPD equation (1) it is understood that Δ is n-dimensional Laplace operator, $x = (x_1, x_2, \ldots, x_n)$ is a point in R^n, k is a real parameter and t is time variable. The well-known case of the EPD equation is $k = 0$ for which (1) reduces to the wave equation. The EPD equation for special values of k and n has occurred in many classiccal problems for over two centuries. Euler first considered equation (1) for $n = 2$. Later Poisson treated the case $n = 2$ and the singular Cauchy problem for the case $n = 4, k = 2$. Darboux again considered (1) for $n = 2, 0 < k < 2$. Asgeirsson gave a solution of the singular Cauchy problem for all positive integers n and $k = n - 2$. Equation (1), for $n = 1, k = -1, -2, \ldots$

appears in the work of Diaz and Martin [9]. Kapılevic [11] has given solutions of $(1),(2)$, for $n = 1,2$ and $0 < k < 1$. A complete solution of the singular Cauchy problem covering all values of k and n has been given recently by Weinstein [13], [14], Diaz and Weinberger [10] and Blum [2]. For analytic initial functions $f(x)$, $(1),(2)$ singular CP was solved by Walter [11] and Dernek [8]. In this articles the solution is given by absolutely and uniformly convergent power series. Another initial value problem for EPD Equation is the regular CP. In the general mean the regular CP was solved by Davis [4], [5]. Copson [3] gave another solution of this problem in any space of even number of dimensions. A solution of the series form is given by Asral [1]. When we would like to generalize the CP $(1),(2)$ we can chose the parameter k as a function $\psi(t)$ which is regular at neighborhood of $t = 0$ or in all R-space. Then $(1),(2)$ becomes as follows:

$$\Delta u = u_{tt} + \frac{\psi(t)}{t} u_t \tag{3}$$

$$u(x,0) = f(x), u_t(x,0) = 0. \tag{4}$$

Two generalizations for the Cauchy problem of the EPD equation is given by Dernek in [6],[7].

This paper is concerned with a solution of the series form of the following Cauchy problem $(5),(6)$. In this problem the function $\psi(t)$ is chosen by $\psi(t) = at^2 + b, a > 0, b + 1 > 0$;

$$\Delta u = u_{tt} + (at + \frac{b}{t}) u_t \tag{5}$$

$$u(x,0) = f(x), u_t(x,0) = 0. \tag{6}$$

where a, b, are real parameters and $f(x)$ is an initial function. $f(x)$ must be infinitely differentiable and the sequence $(|\Delta^n f|)$ must be majorized by a suitably chosen sequence which has positive terms.

Let us consider the following series

$$u(f,t,a,b) = \sum_{k=0}^{\infty} u_k(t,a,b) \Delta^k f(x) \tag{7}$$

where, $u_0(t,a,b) = 1$ and $\Delta^0 f = f, \Delta^k = \Delta(\Delta^{k-1} f), k = 1,2,\ldots$ (see [7]). We would like to see which conditions are necessary for (7) is a special solution of the CP $(5),(6)$. We can consider (7) as a power series with respect to Δf. When (7) is derived term by term with respect to t and these values is written in (5) we obtain the following recurrence relations which are ordinary differential equations

$$\frac{d^2}{dt^2} u_n(t,a,b) + (at + \frac{b}{t}) \frac{d}{dt} u_n(t,a,b) = u_{n-1}(t,a,b) \quad (n \geq 1) \tag{8}$$

where $u_0(t,a,b) = 1$. Let us $u_n(t,a,b), (n \geq 1)$ are the solutions of the equations system (8). Thus (7) is a formal solution of CP $(5) - (6)$. Here the functions u_n

have the following initial conditions:

$$u_n(0,a,b)=0, \frac{d}{dt}u_n(0,a,b)=0 \quad (n\in N). \tag{9}$$

Let us consider the Cauchy problem $CP(1)$. A formal solution of $CP(1)$ is

$$u_1(t,a,b)=\sum_{r=0}^{\infty} N_{1,r}t^{r+2}.$$

If we substitute $u_1, \frac{d}{dt}u_1, \frac{d^2}{dt^2}u_1$ in $CP(1)$ we obtain the following recurrence relations:

$$N_{1,0}=\frac{1}{2(b+1)}, N_{1,1}=N_{1,3}=\ldots=N_{1,2r+1}=0$$

$$(2r+2)(2r+1+b)N_{1,2r}+2arN_{1,2r-2}=0 \quad (r=1,2,\ldots). \tag{10}$$

Let us write the relations (10) for $r=1,2,\ldots,n$ and product them, we obtain:

$$N_{1,2r}=(-1)^r a^r \frac{(b)_{2r}\Gamma(r+1)}{(2r+2)\Gamma(b+2r+2)}$$

where $(b)_{2r}=(b+2)(b+4)\ldots(b+2r)$. Hence a solution of $CP(1)$ is

$$u_1(t,a,b)=t^2\sum_{r=0}^{\infty}(-1)^r\frac{(b)_{2r}\Gamma(r+1)}{(2r+2)\Gamma(b+2r+2)}(t\sqrt{a})^{2r} \quad (a>0).$$

Let us seek a formal solution of $CP(2)$ as follows

$$u_2(t,a,b)=\sum_{r=0}^{\infty}N_{2,2r}t^{2r+4}.$$

Substituting $u_2, \frac{d}{dt}u_2, \frac{d^2}{dt^2}u_2$ in $CP(2)$, we obtain

$$N_{2,0}=\frac{1}{2.4(b+1)(b+3)}, N_{2,1}=N_{2,3}=\ldots=N_{2,0r+1}=0$$

$$(2r+4)(2r+3+b)N_{2,2r}+a(2r+2)N_{2,2r-2}=N_{1,2r} \quad (r=1,2,\ldots). \tag{11}$$

Now assuming that

$$N_{2,2r}=(-1)^r a^r\frac{\Gamma(b+1)(b)_{2r+2}}{(2r+4)\Gamma(b+2r+4)}\varphi_{2,2r} \quad (r=1,2,\ldots)$$

we have the following difference equations from (11):

$$\varphi_{2,2r}-\varphi_{2,2r-2}=\frac{1}{(2r+2)} \quad (r=1,2,\ldots). \tag{12}$$

Let us $\varphi_{2,0} = \frac{1}{2}$ and write the equations (12) for $r = 1, 2, \ldots, n$ and sum of them. We have $\varphi_{2,2r} = \sum_{k=0}^{r} \frac{1}{(2k+2)}$. Thus a solution of $CP(2)$ is

$$u_2(t,a,b) = t^4 \sum_{r=0}^{\infty} (-1)^r [\frac{\Gamma(b+1)(b)_{2r+2}}{(2r+4)\Gamma(b+2r+4)} \sum_{k=0}^{r} \frac{1}{(2k+2)}](t\sqrt{a})^{2r} \quad (a > 0).$$

Let us assume

$$N_{n-1,2r} = (-1)^r a^r \frac{\Gamma(b+1)(b)_{2r+2n-4}}{(2r+2n-2)\Gamma(b+2r+2n-2)} \varphi_{n-1,2r} \quad (r = 1, 2, \ldots)$$

and suppose

$$u_{n-1}(t,a,b) = \sum_{r=0}^{\infty} N_{n-1,2r} t^{2n+2r-2}$$

is a formal solution of the $CP(n-1)$. Now let us consider $CP(n)$. We can see

$$u_n(t,a,b) = \sum_{r=0}^{n} N_{n,2r} t^{2n+2r}$$

is a solution of this problem, where

$$N_{n,2r} = (-1)^r a^r \frac{\Gamma(b+1)(b)_{2r+2n-2}}{(2r+2n)\Gamma(b+2r+2n)} \varphi_{n,2r} \quad (r = 1, 2, \ldots)$$

This is a consequence of the Mathematical Induction Principle. If we substitute the values of u_n, $\frac{d}{dt}u_n$, $\frac{d^2}{dt^2}u_n$ in $CP(n)$, we obtain the general recurrence relations and the following general difference equations:

$$(2r+2n)(2r+2n-1+b)N_{n,2r} + a(2r+2n-2)N_{n,2r-2} = N_{n-1,2r}$$

$$\varphi_{n,2r} - \varphi_{n,2r-2} = \frac{1}{2r+2n-2} \varphi_{n-1,2r} \quad (r = 1, 2, \ldots) \tag{13}$$

On the other hand, if we take $a = 0$ the equation (5) becomes EPD equation. The functions $u_n(t,a,b)$ are continuous with respect to a (this will prove the next section). Hence we can write $a = 0$ in $u_n(t,a,b)$,thus

$$u_n(t,0,b) = \frac{\Gamma(b+1)(b)_{2r+2n-2}}{2n\Gamma(b+2n)} \varphi_{n,0} t^{2n}.$$

But under the condition $b+1 > 0$, a solution of the series form of Cauchy problem (1), (2) is (see [7])

$$u(f,t,b) = \sum_{n=0}^{\infty} u_n(t,b) \Delta^n f$$

where

$$u_n(t,b) = \frac{t^{2n}}{2^n n!(b+1)(b+3)\ldots(b+2n-1)} \quad (n=1,2,\ldots).$$

From $u_n(t,0,b) = u_n(t,b)$ we obtain $\varphi_{n,0} = 1/(2^{n-1}(n-1)!) \quad (n \geq 2)$.

Now let's consider the difference equation (13) for $r = 1,2,\ldots,s$ and let's sum of them. Then we have, $(n = 2,3,\ldots; s = 0,1,2,\ldots)$

$$\varphi_{n,2s} = \sum_{k_{n-1}=0}^{s} \frac{1}{2k_{n-1}+(2n-2)} \sum_{k_{n-2}=0}^{k_{n-1}} \frac{1}{2k_{n-2}+(2n-4)} \cdots \sum_{k_1=0}^{k_2} \frac{1}{2k_1+2} \tag{14}$$

Thus the series which gives a solution of $CP(n)$ can be written as follows:

$$u_n(t,a,b) = t^{2n} \sum_{r=0}^{\infty} (-1)^r \frac{\Gamma(b+1)(b)_{2r+2n-2}}{(2r+2n)\Gamma(b+2r+2n)} \varphi_{n,2r} (t\sqrt{a})^{2r}. \tag{15}$$

Where, $a > 0$ and $\varphi_{n,2r} > 0$ for $b+1 > 0$ which are given by (14), $n \geq 2, r \geq 0$. When we fix the indices n then the coefficient $\varphi_{n,2r}$ increases with respect to r.

2 Convergence of The Series $u_n(t,a,b)$

In this section it will be shown that the series $u_n(t,a,b)$ $(n = 1,2,\ldots)$ are uniformly convergent. Thus the series which are obtained with derivated u_n term by term are well defined. Furthermore we will show that all of our hypothesis turn out to be true. For this purpose first we calculate the radius of convergence of the series $u_1(t,a,b)$. It is clear that $|N_{1,2r+2}|/|N_{1,2r}| = O(r^{-1})$. Hence the series u_1 convergent for every real values of t. Under the condition $b+1 > 0$, the series u_1 is uniformly convergent for every t in R.It is also shown that the series u_1 is differantiable infinitely and it is a continuous function of the variables a and t for $a > 0$. Let us consider the series $u_2(t,a,b)$ and the following numbers

$$\varphi_{2,2r} = \sum_{r=0}^{n} \frac{1}{2k+2} = \frac{1}{2}(1 + \sum_{k=1}^{n} \frac{1}{k+1}).$$

We would like to obtain convenient upper bounds for the numbers $\varphi_{2,2r}$. Let us consider the following inequalities

$$\ln(r+1) - 1 < \sum_{k=2}^{r} \frac{1}{k} < \ln r \quad (r \geq 2, r \in N).$$

it can be written

$$\ln(r+2) < 1 + \sum_{k=1}^{r} \frac{1}{k+1} < 1 + \ln(r+1).$$

Hence from, $\lim_{r\to\infty}\frac{\varphi_{2,2r}}{r} = 0$, we have for sufficiently great $r, \varphi_{2,2r} = o(r)$ and then we obtain

$$\varphi_{2,2r} = O(r) \tag{16}$$

and

$$|N_{2,2r+2}|/|N_{2,2r}| = O(r^{-1}).$$

Thus the radius of convergence of the series $u_2(t,a,b)$ is infinite and $u_2(t,a,b)$ convergence absolutely and uniformly for every $t \in R$. Now, we would like to find an available upper bound for the numbers

$$\varphi_{3,2r} = \sum_{k=0}^{r} \frac{1}{2k+4}\varphi_{2,2k}.$$

Since the coefficients $\varphi_{2,2k}$ are monotone increasing for $k = 0, 1, \ldots, r$ we can write

$$\varphi_{3,2r} = \frac{1}{4}(r+1)\varphi_{2,2r}.$$

¿From (16) $\varphi_{3,2r} = O(r^2)$. Then we have

$$|N_{3,2r+2}|/|N_{3,2r}| = O(r^{-1}).$$

In general we assume that

$$\varphi_{n-1,2r} = O(r^{n-2}) \quad (b+1>0). \tag{17}$$

For to prove $\varphi_{n,2r} = O(r^{n-1})$ we can consider the coefficients $\varphi_{n-1,2k}, (k = 0, 1, \ldots, r)$ are monotone increase and then

$$\varphi_{n,2r} = \sum_{k=0}^{r} \frac{1}{2k+(2n-2)}\varphi_{n-1,2k} \leq \frac{1}{2n-2}(r+1)\varphi_{n-1,2r}.$$

By the last inequality, (17) and the induction hypothesis we can write $\varphi_{n,2r} = O(r^{n-1})$ and then,

$$|N_{n,2r+2}|/|N_{n,2r}| = O(r^{-1}).$$

Hence for every $n \in N$ and $b+1>0, a>0$, the radius of convergence of the series $u_n(t,a,b)$ are infinite. We can give the following Lemma from above observation.

Lemma 1. 1 *The series $u_n(t,a,b), (n \in N)$ are absolutely and uniformly convergent for every t when $b+1>0$ and $a>0$. They are continuous functions of the parameter a and the variable t. Then the functions $u_n(t,a,b)$ can be differentiated infinitely.*

3 The Upper Bounds

Now we would like to find an available upper bound for the series, $u_n(t,a,b)$, $(n \in N)$ which is absolutely and uniformly convergent. For this purpose we may transform (8) to an equal integral equation system. For to define the transformation first we write the equation (8) for $t = \xi$:

$$\frac{d^2}{d\xi^2}u_n(\xi,a,b) + (a\xi + \frac{b}{\xi})\frac{d}{d\xi}u_n(\xi,a,b) = u_{n-1}(\xi,a,b) \quad (n \in N) \tag{18}$$

$$u_0(\xi,a,b) \equiv 1.$$

We product (18) with $\xi^b e^{a\xi^2/2}$ and integrate both sides with respect to ξ on $(0,\mu)$. We consider the initial conditions (9), then

$$\frac{d}{d\mu}u_n(\mu,a,b) = \mu^{-b}e^{-a\mu^2/2}\int_0^\mu \xi^b e^{a\xi^2/2}u_{n-1}(\xi,a,b)d\xi \quad (n \in N). \tag{19}$$

If we integrate both sides of (19) with respect to μ on $(0,t)$, from the initial conditions (9) we have

$$u_n(t,a,b) = \int_0^t \int_0^\mu \left(\frac{\xi}{\mu}\right)^b e^{-a(\mu^2-\xi^2)/2}u_{n-1}(\xi,a,b)d\mu d\xi \quad (n \in N). \tag{20}$$

$$u_0(t,a,b) \equiv 1$$

(20) is an integral representation for solutions of $CP(n)$ which is given by (15). In this section we shall give which conditions are necessary for the equality of (15) and (20). We consider that the integrals (20) are calculated on the domain B which is defined with the points $(0,0),(t,0),(t,t)$ and the line $\xi = \mu$. If we transform the variables as follows

$$T: \quad \rho = \mu^2 - \xi^2, \; r = \frac{\xi}{\mu}.$$

The functional determinant of this transformation is $\frac{D(\mu,\xi)}{D(\rho,r)} = \frac{1}{2(1-r^2)}$ $(r \neq 1)$. Under this transformation (20) is transformed to the following integral:

$$0 \leq u_n(t,a,b) = \int_{r=0}^{t}\int_{\rho=0}^{t^2(1-r^2)} r^b e^{-a\rho/2}u_{n-1}(r\sqrt{\rho/(1-r^2)},a,b)\frac{drd\rho}{2(1-r^2)} \tag{21}$$

(21) can be written as follows for $n = 1$:

$$0 \leq u_1(t,a,b) = \int_{r=0}^{1}\int_{\rho=0}^{t^2(1-r^2)} \frac{r^b e^{-a\rho/2}}{2(1-r^2)}drd\rho. \tag{22}$$

For to find an upper bound to the function u_1 we take $t^2(1-r^2) = s(t,r) > 0$ and integrate (21) with respect to p then we obtain

$$0 \le u_1(t,a,b) \le \frac{t^2}{2}\int_{r=0}^{1} \frac{r^b}{as} 2(1-e^{-a\rho/2})dr.$$

Since $1-e^{-a\rho/2} \le as/2$ for $as \ge 0$ and from (22) it can be written

$$0 \le u_1(t,a,b) \le \frac{t^2}{2}\int_{r=0}^{1} r^b dr \le \frac{t^2}{2(b+1)}$$

where $b+1>0$. Thus the function (20) is equivalent to the sum of the series (15) for $n=1$ under the conditions $a>0, b+1>0$.

Now we consider the double integral (21) for $n=2$. It can be written

$$\begin{aligned} 0 \le u_2(t,a,b) &= \int_{r=0}^{1}\int_{\rho=0}^{s} r^b e^{-a\rho/2} u_1(r\sqrt{\frac{\rho}{1-r^2}},a,b)\frac{1}{2(1-r^2)drd\rho} \\ &\le t^4 \frac{}{4(b+1)}\int_{r=0}^{1}\int_{\rho=0}^{s} \frac{r^{b+2}}{s^2}\rho e^{-a\rho/2} drd\rho \\ &= \frac{t^4}{4(b+1)}\int_{r=0}^{1}\frac{r^{b+2}}{a^2s^2}4[1-(1-as/2)e^{-a\rho/2}]dr. \end{aligned}$$

where $s = t^2(1-r^2) > 0$. Let $m = as/2 \ge 0$. From the following inequality we obtain

$$0 \le u_2(t,a,b) \le \frac{t^4}{2^2 2!(b+1)(b+3)}.$$

We assume that

$$0 \le u_{n-1}(t,a,b) \le \frac{t^{2n-2}}{2^{n-1}(n-1)!(b+1)(b+3)\ldots(b+2n-3)} \tag{23}$$

where $a > 0, b+1 > 0$. From (20) and (23) it can be written as follows for $s = t^2(1-r^2)$,

$$\begin{aligned} 0 \le u_n(t,a,b) &\le \frac{t^{2n}}{2^{n-1}(n-1)![b]_{(b+2n-3)}}\int_{r=0}^{1}\int_{\rho=0}^{s}\frac{r^{b+2n-2}}{s^n}p^{n-1}e^{-a\rho/2}drd\rho \\ &= \frac{t^{2n}}{2^{n-1}(n-1)![b]_{(b+2n-3)}}\int_{r=0}^{1}\frac{r^{b+2n-2}}{s^n}(n-1)![1-\sum_{\rho=0}^{n-1}\frac{s^{n-\rho-1}e^{-s}}{\Gamma(n-p)}]dr. \end{aligned}$$

where $[b]_{(2n-1)} = (b+1)(b+3)\ldots(b+2n-1)$. From the following inequality which is easily proved with mathematical induction principle

$$n!\{1-\sum_{p=0}^{n-1}\frac{s^{n-p-1}e^{-s}}{\Gamma(n-p)}\} \le s^n$$

we obtain,

$$0 \leq u_n(t,a,b) \leq \frac{t^{2n}}{2^n n!(b+1)(b+3)\dots(b+2n-1)} \quad (n \in N). \tag{24}$$

Thus the expression of the series form and the integral form of the functions $u_n(t,a,b)$, $n \in N$ are equivalent when $a > 0, b+1 > 0$. Now we can study for the convergence problem of the solution of the Cauchy problem (5), (6) with respect to the above results. Let us

$$u(x,t,a,b) = \sum_{n=0}^{\infty} u_n(t,a,b)\Delta^n f \tag{25}$$

where $u_n(t,a,b)$ is given by (15). For our aim we can write the upper bounds of the functions as follows:

$$(b+1)(b+3)\dots(b+2n-1) \geq (b+1)(b+2)\dots(b+n)$$

where $b+1 > 0$. But for every $n \in N$ we can find a number d which is written as follows:

$$d^n(b+1)(b+2)\dots(b+n) > n!$$

where $d > 1$. Hence

$$\frac{t^{2n}}{2^n n!(b+1)(b+3)\dots(b+2n-1)} \leq \frac{d^n t^{2n}}{2^n n! n!} \leq \frac{d^n t^{2n}}{(2n)!} \quad (n = 1, 2, \dots).$$

¿From (24) and the above inequality we can write:

$$0 \leq u_n(t,a,b) \leq \frac{d^n t^{2n}}{(2n)!} \quad (n = 1, 2, \dots). \tag{26}$$

4 The Solution of The Singular CP (5), (6)

We have seen that the functions $u_n(t,a,b)$ have represented with the power series (15) when $a > 0, b+1 > 0$. And it has seen that the functions $u_n(t,a,b)$ have bounded as in (26). Thus the series which is defined as follows,

$$v(f,t,a,b) = \sum_{n=0}^{\infty} \frac{d^n t^{2n}}{(2n)!}|\Delta^n f|$$

can be taken a majorant for the function (25) that is a solution of the singular Cauchy problem (5), (6). And then we can express the following theorem.

Theorem 1. 1 *Let $f(x_1, x_2, \dots, x_n)$ is differentiable continuously infinitely with respect to its variables and let $a > 0, b+1 > 0$. The function $u(f,t,a,b)$,* *which is a solution of the singular Cauchy problem* (5), (6), *is given by the following series:*

$$u(f,t,a,b) = \sum_{n=0}^{\infty} u_n(t,a,b)\Delta^n f \tag{27}$$

where the functions $u_n(t,a,b)$, $(n \in N)$ *are expressed with* (15). *The series* (27) *is absolutely and uniformly convergent in the space* $R^n \times R$ *when the following property is satisfied for the initial function* $f(x)$

$$|\Delta^n f| = o((2n)!), \quad (n \in N).$$

The series (27) *is absolutely and uniformly convergent in the subspace of* $R^n \times R$ *which contains the plane* $t = 0$ *when the following property is satisfied for the initial function* $f(x)$

$$|\Delta^n f| = O((2n)!), \quad (n \in N).$$

Corollary 1. 1 *If it is written* $a = 0$ *in the solution* (27) *we obtain the solution of the singular problem of the EPD Equation which is given [10] as follows;*

$$u(f,t,b) = \Gamma(b+1) \sum_{n=0}^{\infty} \frac{(b)_{2n-2} t^{2n}}{2^n n! \Gamma(b+2n)} \Delta^n f.$$

Corollary 1. 2 *If it is written* $b = 0$ *in the equation* (5) *we obtain the series solution of the singular Cauchy problem*

$$\Delta u = u_{tt} + atu_t$$

$$u(x,0,a) = f(x), u_t(x,0,a) = 0$$

which is given by Dernek [6]. The solution is

$$u(f,t,a) = \sum_{n=0}^{\infty} \{\sum_{r=0}^{\infty} (-1)^r a^r \frac{2.4\ldots(2n+2r-2)}{(2r+2n)!} \varphi_{n,2r} t^{2r+2n}\} \Delta^n f$$

where $\varphi_{n,2r}$ *is defined by* (14).

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