Journal of Mathematics, Physics and Astronomy, New Series Volume 2 (2006-2007)

Stability Analysis of Systems In Two Degrees of Freedom With Internal And External Damping Subject To Harmonic Excitation

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(Accepted 15 July 2007)

The statement of fundamental problems and solution of systems in two degrees of freedom with internal and external damping are studied in the present paper. Theoretical results are applied to the systems with harmonic excitation. The effect of damping coefficients on the character of the solution is investigated. The conditions of stability, asymptotic stability and instability of the solutions are obtained and the orbits of the motion for the different values of parameters are driven by using Maple 10 program.

1. INTRODUCTION

The statement of fundamental problems and solution of systems in two degrees of freedom with internal and external damping are studied in the present paper [1]. The solutions of dynamical problems with two degrees of freedom are used in a number of areas of science and technology [1-4]. Event of obtaining the solutions of the systems is an actual problem. There are some investigations of this kind in the paper, too.

Since the power of exponential kernel function is related with the damping coefficient and harmonic excitation, the dependence of the solutions obtained on this parameter is studied. Stability, asymptotic stability and instability [5-7] of the solutions are solved by using Maple 10 program for the different values of parameters (Fig. 1-3). There are some orbits in two and three dimensions. Investigating the graphs the critical points are obtained and can be used by researchers in various fields.

We take motion equations given by general dynamical systems studied in [1].

2. THE STATEMENT OF THE PROBLEM

In the present paper

$$a_{11}\ddot{\varphi}+a_{12}\ddot{\psi}+\nu_1\left(a_{11}\dot{\varphi}+a_{12}\dot{\psi}\right)+c_{11}\varphi+c_{12}\psi=e^{-\sigma t}\left(A_0\sin\omega t+B_0\cos\omega t\right)$$

$$a_{21}\ddot{\varphi}+a_{22}\ddot{\psi}+\nu_2\left(a_{21}\dot{\varphi}+a_{22}\dot{\psi}\right)+c_{21}\varphi+c_{22}\psi=e^{-\sigma t}\left(C_0\sin\omega t+D_0\cos\omega t\right) \quad (1)$$

$t=0:\ \varphi=\dot{\varphi}=0,\quad \psi=\dot{\psi}=0$

(where a_{11}, a_{12}, a_{21}, a_{22}, c_{11}, c_{12}, c_{21}, c_{22}, A_0, B_0, C_0, D_0, $0<\sigma$ are constants $\left(a_{11}\cdot a_{22}-a_{12}\cdot a_{21}\neq 0\right)$, $\nu_1 a_{11}$, $\nu_1 a_{12}$, $\nu_2 a_{21}$, $\nu_2 a_{22}$ are damping coefficients, ω is the frequency of excitation) dynamical systems in two degrees of freedom with internal and external damping subject to harmonic excitation is studied with the initial conditions.

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Substituting $\omega t = t_1$ and using the notation

$$\frac{a_{12}}{a_{11}} = A_1,\ \frac{a_{21}}{a_{22}} = A_2,\ \frac{c_{11}}{\omega^2 a_{11}} = n_1^2,\ \frac{c_{22}}{\omega^2 a_{22}} = n_2^2,\ \frac{c_{12}}{\omega^2 a_{11}} = B_1,\ \frac{c_{21}}{\omega^2 a_{22}} = B_2$$

$$\frac{A_0}{a_{11}}\omega^2 = A,\ \frac{B_0}{\omega^2 a_{11}} = B,\ \frac{C_0}{\omega^2 a_{22}} = C,\ \frac{D_0}{\omega^2 a_{22}} = D,\ \frac{\sigma}{\omega} = \mu$$

then (1) becomes as follows:

$$\begin{aligned} &\ddot{\varphi} + A_1\ddot{\psi} + \omega^{-1}\nu_1\left(\dot{\varphi} + A_1\dot{\psi}\right) + n_1^2\varphi + B_1\psi = e^{-\mu t_1}\left(A\sin t_1 + B\cos t_1\right) \\ &\ddot{\psi} + A_2\ddot{\varphi} + \omega^{-1}\nu_2\left(\dot{\psi} + A_2\dot{\varphi}\right) + n_1^2\psi + B_2\varphi = e^{-\mu t_1}\left(C\sin t_1 + D\cos t_1\right) \end{aligned} \tag{2}$$

$$t_1 = 0:\ \varphi = \dot{\varphi} = 0,\quad \psi = \dot{\psi} = 0.$$

We are to examine the case $\omega^{-1}\nu_1 = \omega^{-1}\nu_2 = 2\nu$ and substituting $\varphi = e^{-\nu t_1}\Phi,\ \psi = e^{-\nu t_1}\Psi$ in (2), we obtain

$$\begin{aligned} &\ddot{\Phi} + A_1\ddot{\Psi} + +\left(n_1^2 - \nu^2\right)\Phi + \left(B_1 - A_1\nu^2\right)\Psi = e^{(\nu-\mu)t_1}\left(A\sin t_1 + B\cos t_1\right) \\ &\ddot{\Psi} + A_2\ddot{\Phi} + +\left(n_2^2 - \nu^2\right)\Psi + \left(B_2 - A_2\nu^2\right)\Phi = e^{(\nu-\mu)t_1}\left(C\sin t_1 + D\cos t_1\right). \end{aligned} \tag{3}$$

Under this substitution the initial conditions become

$$t_1 = 0:\ \Phi = \dot{\Phi} = 0,\ \Psi = \dot{\Psi} = 0. \tag{4}$$

The solution of the homogeneous systems

$$\begin{aligned} &\ddot{\Phi} + A_1\ddot{\Psi} + +\left(n_1^2 - \nu^2\right)\Phi + \left(B_1 - A_1\nu^2\right)\Psi = 0 \\ &\ddot{\Psi} + A_2\ddot{\Phi} + +\left(n_2^2 - \nu^2\right)\Psi + \left(B_2 - A_2\nu^2\right)\Phi = 0 \end{aligned} \tag{5}$$

Is

$$\begin{aligned} &\Phi = a\sin\left(k_1 t_1 + \beta_1\right) + b\sin\left(k_2 t_1 + \beta_2\right) \\ &\Psi = a\alpha_1\sin\left(k_1 t_1 + \beta_1\right) + b\alpha_2\sin\left(k_2 t_1 + \beta_2\right), \end{aligned} \tag{6}$$

there a, b, β_1, β_2's are the functions of weak changed time, k_1 and k_2 $\left(0 < k_1 < k_2\right)$ are the main frequencies that are determined by the principal frequency equation

$$\delta k^4-\left(n_1^2+n_2^2-2\delta\nu^2-A_1B_2-A_2B_1\right)k^2+$$
$$+\left(n_1^2-\nu^2\right)\left(n_2^2-\nu^2\right)-\left(B_1-A_1\nu^2\right)\left(B_2-A_2\nu^2\right)=0 \tag{7}$$

(where $\delta=1-A_1A_2$) [1].

α_1 , α_2 are distribution coefficients determined by formulas

$$\alpha_1=\frac{A_2k_1^2-\left(B_2-A_2\nu^2\right)}{\left(n_2^2-\nu^2\right)-k_1^2}=\frac{\left(n_1^2-\nu^2\right)-k_1^2}{A_1k_1^2-\left(B_1-A_1\nu^2\right)},$$

$$\alpha_2=\frac{A_2k_2^2-\left(B_2-A_2\nu^2\right)}{\left(n_2^2-\nu^2\right)-k_2^2}=\frac{\left(n_1^2-\nu^2\right)-k_2^2}{A_1k_2^2-\left(B_1-A_1\nu^2\right)}. \tag{8}$$

To find a particular solution of the non-homogeneous systems (3), substitute

$$\Phi=e^{(\nu-\mu)t_1}\left(a\sin t_1+b\cos t_1\right)$$

$$\Psi=e^{(\nu-\mu)t_1}\left(c\sin t_1+d\cos t_1\right) \tag{9}$$

into the left members. The result must be identically equal to right side of (3), hence we find

$$a=\frac{\Delta_1}{\Delta},\ b=\frac{\Delta_2}{\Delta},\ c=\frac{\Delta_3}{\Delta},\ d=\frac{\Delta_4}{\Delta}\ ,$$

where

$$\Delta=\begin{vmatrix} \xi+n_1^2 & -2(\nu-\mu) & A_1\xi+B_1 & -2A_1(\nu-\mu) \\ 2(\nu-\mu) & \xi+n_1^2 & 2A_1(\nu-\mu) & A_1\xi+B_1 \\ A_2\xi+B_2 & -2A_2(\nu-\mu t) & \xi+n_2^2 & -2(\nu-\mu) \\ 2A_2(\nu-\mu) & A_2\xi+B_2 & 2(\nu-\mu t) & \xi+n_2^2 \end{vmatrix},\ .$$

$$\Delta_1=\begin{vmatrix} A & -2(\nu-\mu) & A_1\xi+B_1 & -2A_1(\nu-\mu) \\ B & \xi+n_1^2 & 2A_1(\nu-\mu) & A_1\xi+B_1 \\ C & -2A_2(\nu-\mu) & \xi+n_2^2 & -2(\nu-\mu) \\ D & A_2\xi+B_2 & 2(\nu-\mu) & \xi+n_2^2 \end{vmatrix},$$

$$\Delta_2 = \begin{vmatrix} \xi + n_1^2 & A & A_1\xi + B_1 & -2A_1(\nu-\mu) \\ 2(\nu-\mu) & B & 2A_1(\nu-\mu) & A_1\xi + B_1 \\ A_2\xi + B_2 & C & \xi + n_2^2 & -2(\nu-\mu) \\ 2A_2(\nu-\mu) & D & 2(\nu-\mu) & \xi + n_2^2 \end{vmatrix},$$

$$\Delta_3 = \begin{vmatrix} \xi + n_1^2 & -2(\nu-\mu) & A & -2A_1(\nu-\mu) \\ 2(\nu-\mu) & \xi + n_1^2 & B & A_1\xi + B_1 \\ A_2\xi + B_2 & -2A_2(\nu-\mu) & C & -2(\nu-\mu) \\ 2A_2(\nu-\mu) & A_2\xi + B_2 & D & \xi + n_2^2 \end{vmatrix},$$

$$\Delta_4 = \begin{vmatrix} \xi + n_1^2 & -2(\nu-\mu) & A_1\xi + B_1 & A \\ 2(\nu-\mu) & \xi + n_1^2 & 2A_1(\nu-\mu) & B \\ A_2\xi + B_2 & -2A_2(\nu-\mu) & \xi + n_2^2 & C \\ 2A_2(\nu-\mu) & A_2\xi + B_2 & 2(\nu-\mu) & D \end{vmatrix}$$

$\left(\xi = (\nu-\mu)^2 - \nu^2 - 1\right)$.

The general solution of (3) is

$$\begin{aligned} \Phi &= a\sin\left(k_1 t_1 + \beta_1\right) + b\sin\left(k_2 t_1 + \beta_2\right) + e^{(\nu-\mu)t_1}\left(a\sin t_1 + b\cos t_1\right) \\ \Psi &= a\alpha_1 \sin\left(k_1 t_1 + \beta_1\right) + b\alpha_2 \sin\left(k_2 t_1 + \beta_2\right) + e^{(\nu-\mu)t_1}\left(c\sin t_1 + d\cos t_1\right), \end{aligned} \tag{10}$$

where a, b, c, d are as above.

From the substitution $\varphi = e^{-\nu t_1}\Phi$, $\psi = e^{-\nu t_1}\Psi$, and $t_1 = \omega t$, (10) becomes

$$\begin{aligned} \varphi &= e^{-\nu\omega t}\left[a\sin\left(k_1\omega t + \beta_1\right) + b\sin\left(k_2\omega t + \beta_2\right)\right] + e^{-\sigma t}\left(a\sin\omega t + b\cos\omega t\right) \\ \psi &= e^{-\nu\omega t}\left[a\alpha_1 \sin\left(k_1\omega t + \beta_1\right) + b\alpha_2 \sin\left(k_2\omega t + \beta_2\right)\right] + e^{-\sigma t}\left(c\sin\omega t + d\cos\omega t\right). \end{aligned} \tag{11}$$

3. EXAMPLE

Let consider the following dynamic systems.

$$\ddot{\varphi} - 1.2\ddot{\psi} + 2\nu\left(\dot{\varphi} - 1.2\dot{\psi}\right) + 8\varphi - 0.3\psi = e^{-0.2t}\left(0.2\sin t - 0.7\cos t\right)$$

$$\ddot{\psi} + 0.9\ddot{\varphi} + 2\nu\left(\dot{\psi} + A_2\dot{\varphi}\right) + 8.7\psi - 0.32\varphi = e^{-0.2t}\left(0.2\sin t - 0.5\cos t\right) \qquad (12)$$

$$\varphi(0) = \psi(0) = \dot{\varphi}(0) = \dot{\psi}(0) = 0$$

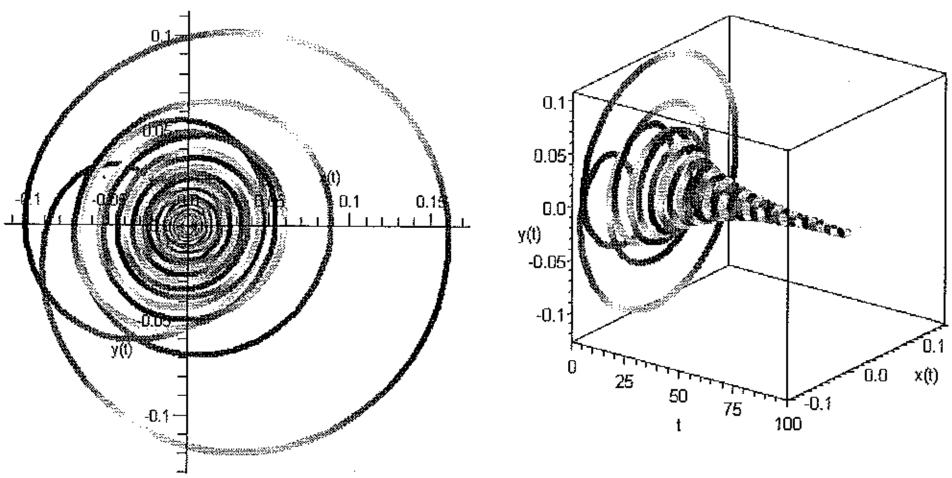

Figure 1. $\nu = 1.1$ two and three dimensions

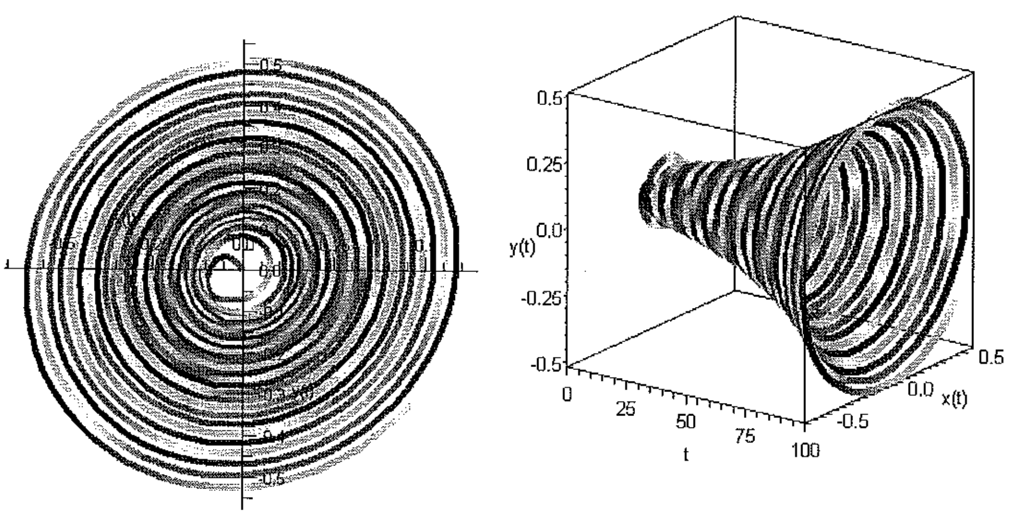

Figure 2. $\nu = 1.025$ two and three dimensions

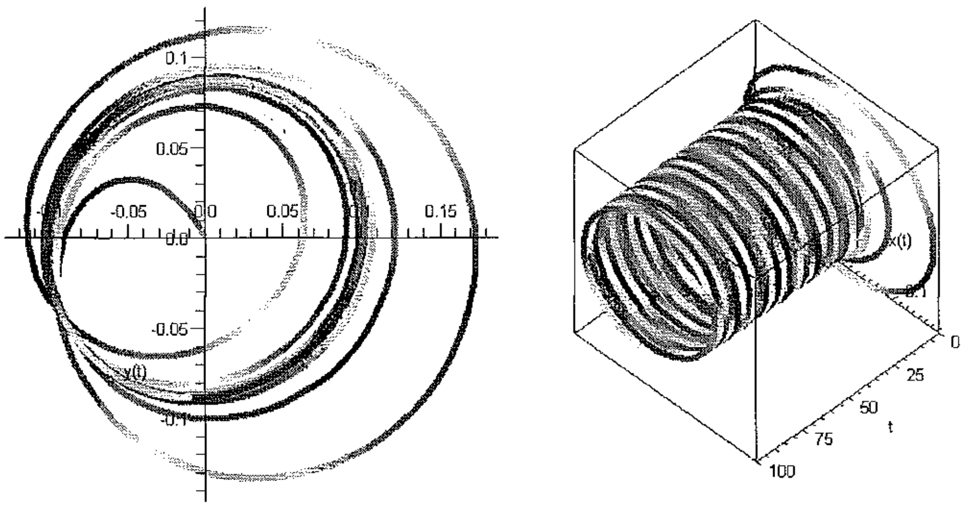

Figure 3. $\nu = 1.048$ two and three dimensions

As shown in figures, the value $\nu = 1.048$ is the critical point for the systems (12). For this value solution is appropriate to stable harmonic motion and stable but not asymptotic stable (Fig. 3). When the value is greater than 1.048, the solution is asymptotic stable (Fig. 1); when is less than 1.048 the solution is unstable (Fig. 2).

4. CONCLUSION

Viscoelastic dynamic systems with two degrees of freedom can be adapted to the equations of motion of two binding pendulums spring to each other. Also can be used by engineers in various fields.

ACKNOWLEDGMENT:

The Author would like to express his gratitude to adviser Prof. Dr. Musa Ilyasov for his support and advice.

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