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AXISYMMETRIC PLANE-STRAIN THERMOELASTIC MIXED BOUNDARY VALUE PROBLEM OF AN ELASTIC STRIP

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The present paper seeks to solve the steady-state axisymmetric plain strain thermoelastic problem of an elastic strip with one face stress free and the other face resting on a rigid frictionless foundation. The free surface of the strip is subjected to arbitrary flux on the part $a < |x| < 1$, whereas the rest of the surface is at zero temperature. The surface in contact with the foundation is insulated. The problem is reduced into triple integral equations with cosine kernel and a weight function. These equations are solved by using finite Hilbert transform techniques.

1. INTRODUCTION

Steady-state thermoelastic problem for the elastic layer has been considered by Sneddon and Locket [1], Martin and Payton [2] and Dhaliwal [3]. The authors [1,2] restricted their analysis to the cases in which the temperature is prescribed on both faces of the layer, and both faces are stress free or one face is stress free and the other rests on a rigid foundation. Dhaliwal [3] considered an axisymmetric problem with mixed thermal boundary conditions on the stress free surface. These mixed boundary value problems lead to dual integral equations of the type considered by Lebedev and Uflyand [4], Erdelyi and Sneddon [5], Noble [6], and Love [7].

In this paper the plane-strain thermoelastic mixed problem of an elastic strip has been considered. Section **2** gives the Fourier transformed solution of the plane-strain steady-state thermoelastic equations. Section 3 gives the boundary conditions and derive the appropriate triple integral equations. The triple integral equations are reduced to a single Fredholm integral equation of the second kind in section 4. The iterative solution of the integral equation is obtained in section 5 for $d \gg 1$ up to the order d^{-10}. The analytical expressions up to the order d^{-10} are obtained in section 6.

2. SOLUTION OF THE GOVERNING EQUATIONS

The steady-state thermoelastic equations for plane strain may be written as (see Nowacki [8])

$$2(1-\nu)\,\partial^2 u/\partial x^2+(1-2\nu)\,\partial^2 u/\partial y^2+\partial^2 v/\partial x\,\partial y=2\alpha\,(1+\nu)\,\partial T/\partial x\,, \quad (1)$$

$$(1-2\nu)\,\partial^2 v/\partial x^2+2(1-\nu)\,\partial^2 v/\partial y^2+\partial^2 u/\partial x\,\partial y=2\alpha\,(1+\nu)\,\partial T/\partial y\,, \quad (2)$$

$$\partial^2 T/\partial x^2+\partial^2 T/\partial y^2=0\,, \quad (3)$$

where

(x, y) = cartesian coordinate system

u, v = x, y components of displacement vector respectively

ν = Poisson's ratio

α = coefficient of linear thermal expansion

T = temperature distribution.

The components of the stress tensor associated with the displacement field are given by

$$\sigma_{xx}=2\mu\,[(1-\nu)\,\partial u/\partial x+\nu\,\partial v/\partial y-\alpha\,(1+\nu)\,T]/(1-2\nu)\,, \quad (4)$$

$$\sigma_{yy}=2\mu\,[(1-\nu)\,\partial v/\partial y+\nu\,\partial u/\partial x-\alpha\,(1+\nu)\,T]/(1-2\nu)\,, \quad (5)$$

$$\sigma_{xy}=\mu\,[\partial u/\partial y+\partial v/\partial x]\,, \quad (6)$$

where $\mu=E/[2(1+\nu)]$ represents the modulus of rigidity and E is the Young's modulus of the Elastic material. To solve these equations of thermoelasticity, we introduce the following Fourier sine and cosine transforms :

$$\bar{f}(x, y)=F_s[f(x, y)\,;x\rightarrow k]=(2/\pi)^{\frac{1}{2}}\int_0^\infty f(x, y)\sin(kx)\,dx\,, \quad (7)$$

$$\tilde{f}(x, y)=F_c[f(x, y)\,;x\rightarrow k]=(2/\pi)^{\frac{1}{2}}\int_0^\infty f(x, y)\cos(kx)\,dx. \quad (8)$$

Now it can be shown that equations (1) - (6) may be written in the form

$$(D^2-m^2k^2)\,\bar{u}-(m^2-1)\,k\,D\tilde{v}=-\beta\,k\,\tilde{T}\,, \quad (9)$$

$$(m^2D^2-k^2)\,\tilde{v}+(m^2-1)\,k\,D\bar{u}=\beta\,D\,\tilde{T}\,, \quad (10)$$

$$(D^2-k^2)\,\tilde{T}=0\,, \quad (11)$$

$$\tilde{\sigma}_{xx}=\mu\,[(m^2-2)\,D\tilde{v}+m^2k\,\bar{u}-\beta\,\tilde{T}]\,, \quad (12)$$

$$\tilde{\sigma}_{yy}=\mu\,[(m^2-2)\,k\,\bar{u}+m^2D\tilde{v}-\beta\,\tilde{T}]\,, \quad (13)$$

$$\bar{\sigma}_{xy}=\mu\,[D\bar{u}-k\,\tilde{v}]\,, \quad (14)$$

where

$$D=d/dy\,;\beta=2(1+\nu)\,\alpha/(1-2\nu)\;;m^2=2(1-\nu)/(1-2\nu)\,. \quad (15)$$

The solution of the set of simultaneous ordinary differential equations (9)-(11) may be written as

$$\tilde{T} = A \cosh(ky) + B \sinh(ky) \,; \tag{16}$$

$$\bar{u} = (A_1 + ky\, A_2) \cosh(ky) + (B_1 + ky\, B_2) \sinh(ky) \,, \tag{17}$$

$$\tilde{v} = (A_3 + ky\, A_4) \cosh(ky) + (B_3 + ky\, B_4) \sinh(ky) \,, \tag{18}$$

where $A, B, A_i, B_i (i = 1, 2, 3, 4)$ are arbitrary functions of k, although not all of these are independent. Substituting $\tilde{T}, \bar{u}, \tilde{v}$ from equations (16) - (18) into equation (9) and equating the coefficients of cosh (ky), sinh (ky), ky cosh (ky) and ky sinh (ky) from both sides, we obtain

$$\left.\begin{aligned} A_2 &= -B_4 = [k(m^2 - 1)(A_3 + B_1) - \beta B] / k(m^2 + 1) \\ A_4 &= -B_2 = -[k(m^2 - 1)(A_1 + B_3) - \beta A] / k(m^2 + 1) \end{aligned}\right\} . \tag{19}$$

Substituting from equations (16) - (18) into equations (12) - (14), the following expressions for the components of stress are obtained :

$$\begin{aligned} \tilde{\sigma}_{xx} = \mu \, [[\, & m^2 (A_1 + ky\, A_2 + A_4)\, u - 2\, k\, A_4 - \beta A + \\ & + (m^2 - 2)(B_3 + ky\, B_4)\, k\,] \cosh(ky) + \\ & + [m^2 (B_1 + ky\, B_2 + B_4)\, k - 2\, k\, B_4 + (m^2 - 2)(A_3 + ky\, A_4)\, k - \\ & - \beta B\,] \sinh(ky)\,] \,, \end{aligned} \tag{20}$$

$$\begin{aligned} \tilde{\sigma}_{yy} = \mu \, [\, & [(m^2 - 2)(A_1 + ky\, A_2)\, k + m^2 (A_4 + B_3 + ky\, B_4)\, k - \\ & - \beta A\,] \cosh(ky) + [(m^2 - 2)(B_1 + ky\, B_2)\, k - \beta B + \\ & + m^2 (B_4 + A_3 - ky\, A_4)\, k\,] \sinh(ky)\,] \,, \end{aligned} \tag{21}$$

$$\begin{aligned} \bar{\sigma}_{xy} = \mu\, k\, [& (A_1 + 2\, ky\, A_2 + B_2 - B_3) \sinh(ky) + \\ & + (B_1 + 2\, ky\, B_2 + A_2 - A_3) \cosh(ky)\,] \,, \end{aligned} \tag{22}$$

where A_2, A_4, B_2 and B_4 are given by equation (19).

3. STATEMENT OF THE PROBLEM AND APPROPRIATE TRIPLE INTEGRAL EQUATIONS

The plane-strain problem of an infinite, homogeneous, isotropic, compressible elastic strip occupying the region $0 \leq y \leq d$ is considered, such that the surface $y = 0$ is stress free and the surface $y = d$ is resting on a rigid, frictionless foundation. By considering plane strain perpendicular to the z - axis, the stated mechanical boundary conditions may be written as follows :

$$\begin{aligned} \sigma_{yy} &= \sigma_{xy} = 0 \quad \text{on } y = 0 \\ v &= \sigma_{xy} = 0 \quad \text{on } y = h. \end{aligned} \tag{23}$$

Using equations (18), (19), (21) and (22), that boundary conditions (23) will be satisfied if

$$\begin{aligned} B_1 &= \beta\, [\, k\, \delta\,(kd)\,]^{-1}\, [\, A\, [\, \sinh\,(kd) + kd \cosh\,(kd)\,] \sinh\,(kd) + \\ &\quad + B\, [\, kd \cosh^2 (kd) - m^2\, kd - (m^2 - 1) \sinh\,(kd) \cosh\,(kd)\,]\,]\ , \\ B_3 &= \beta\, [\, k\, \delta\,(kd)\,]^{-1}\, [\, A\, [m^2\, kd + kd \sinh^2 (kd) + \\ &\quad + m^2 \sinh\,(kd) \cosh\,(kd)\,] + B\, [\, kd \sinh\,(kd) \cosh\,(kd)\,]\,]\ , \\ A_1 &= m^2\, B_3 - (\beta\, A\, k^{-1}) / 2\ , \\ A_3 &= m^2\, B_1 - (\beta\, B\, k^{-1}) / 2\ , \end{aligned} \tag{24}$$

where

$$\delta\,(x) = m^2\,(1 - m^2)\,[\, 2x + \sinh\,(2x)\,]\,.$$

Equations (16) - (22) and (24) determine $\tilde{T}$, $\bar{u}$, $\tilde{v}$, $\tilde{\sigma}_{xx}$, $\tilde{\sigma}_{yy}$ and $\bar{\sigma}_{xy}$ in terms of only two unknown functions, namely $A(k)$ and $B(k)$ which are to be determined from the thermal boundary conditions.

If the surface $y = d$ is insulated and on the surface $y = 0$ the temperature is zero on the part $0 < |x| < a$, $|x| > 1$, whereas the flux is prescribed on the part $a < |x| < 1$, the thermal boundary conditions may be written as

$$\left.\begin{aligned} &T = 0,\ 0 < |x| < a,\ |x| > 1,\ \text{at } y = 0 \\ &\frac{\partial T}{\partial y} = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} f(x)\,,\ a < |x| < 1,\ \text{at } y = 0 \end{aligned}\right\} \tag{25}$$

$$\frac{\partial T}{\partial y} = 0,\ 0 < |x| < \infty,\ \text{at } y = d\,, \tag{26}$$

where $f(x)$ is assumed as a continuously differentiable function for x in $(a, 1)$.

The boundary condition (26) will be identically satisfied on taking

$$A(k) = -\, B(k) \coth\,(kd). \tag{27}$$

From (19), (24) and (27) the following results are obtained:

$$\left.\begin{aligned} &A_2 = 0\ ;\ A_4 = 0\ ;\ B_2 = 0\ ;\ B_4 = 0 \\ &A_3 = B_1 = -\,\beta\, k^{-1}\, A(k) \tanh\,(kd) / 2\,(m^2 - 1) \\ &A_1 = B_3 = \beta\, k^{-1}\, A(k) / 2\,(m^2 - 1) \end{aligned}\right\} \tag{28}$$

and hence

$$\left.\begin{aligned} \tilde{T} &= A(k) \cosh k\,(y-d) \,/\, \cosh (kd) \\ \bar{u} &= \beta\, A(k) \cosh k\,(y-d) \,/\, 2\,(m^2-1)\, k \cosh (kd) \\ \tilde{v} &= \beta\, A(k) \sinh k\,(y-d) \,/\, 2\,(m^2-1)\, k \cosh (kd) \end{aligned}\right\} \tag{29}$$

$$\bar{\sigma}_{xx} = 0\,,\ \bar{\sigma}_{yy} = 0\,,\ \tilde{\sigma}_{xy} = 0\,,$$

which is in agreement with results of Sneddon and Locket [1].

The boundary conditions (25) lead to the triple integral equations

$$\int_0^\infty A(k) \cos (kx)\, dk = 0\,,\ 0 < |x| < a\,, |x| > 1\,, \tag{30}$$

$$\int_0^\infty k\, A(k)\, [1 + H(kd)] \cos (kx)\, dk = (2/\pi)^{\frac{1}{2}} f(x),\ \ a < |x| < 1\,, \tag{31}$$

where

$$H(2x) = -\,2\, e^{-x}\, (1 + e^{-x})^{-1}\,.$$

4. THE SOLUTION OF THE TRIPLE INTEGRAL EQUATIONS

For solving the set of triple integral equations (30) and (31) the method of Lowengrub and Srivastava [9] has been adopted. Let

$$A(k) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} k^{-1} \int_0^\infty h(t^2) \sin (kt)\, dt\,, \tag{32}$$

where $h(t^2)$ is the solution of the Fredholm integral epuation of second kind

$$h(x^2) + \int_a^1 h(t^2)\, K_1\,(x^2\,, t)\, dt = M\,(x^2),\ \ a < |x| < 1, \tag{33}$$

satisfying the condition

$$\int_a^1 h\,(t^2)\, dt = 0\,, \tag{34}$$

and

$$K_1\,(x, t) = -\,\frac{4}{\pi^2} \left(\frac{x^2-a^2}{1-x^2}\right)^{\frac{1}{2}} \int_a^1 \left(\frac{1-y^2}{y^2-a^2}\right)^{\frac{1}{2}} \frac{y\, K_2\,(y, t)}{y^2-x^2}\, dy \tag{35}$$

with

$$K_2(y,t) = \int_0^\infty H(kd)\cos(ky)\sin(kt)\,dk, \tag{36}$$

and

$$M(x^2) = -\frac{4}{\pi^2}\left(\frac{x^2-a^2}{1-x^2}\right)^{\frac{1}{2}} \int_a^1 \left(\frac{1-y^2}{y^2-a^2}\right)^{\frac{1}{2}} \frac{yf(y)}{y^2-x^2}\,dy + $$
$$+ C'\,[(x^2-a^2)(1-x^2)]^{-1/2}, \tag{37}$$

where C' is an arbitrary constant to be determined from condition (34). Now integrating (33) with respect to x from a to 1 and using (34), we find that

$$C' = \frac{1}{F}\int_a^1 h(t^2)\left[\int_a^1 K_1(x^2,t)\,dx\right]dt + \frac{4}{\pi^2}\int_a^1 W(x^2)\,dx, \tag{38}$$

where

$$W(x^2) = \left(\frac{x^2-a^2}{1-x^2}\right)^{\frac{1}{2}} \int_a^1 \left(\frac{1-y^2}{y^2-a^2}\right)^{\frac{1}{2}} \frac{yf(y)}{y^2-x^2}\,dy, \tag{39}$$

and F is an elliptic integral of the first kind.

Hence from equations (33), (37) and (38), d must satisfy the integral equation

$$h(x^2) + \int_a^1 h(t^2)\,K(x^2,t)\,dt = P(x^2),\ a < |x| < 1, \tag{40}$$

where

$$K(x^2,t) = K_1(x^2,t) - \frac{1}{F}[(x^2-a^2)(1-x^2)]^{-1/2}\int_a^1 K_1(x^2,t)\,dx, \tag{41}$$

$$P(x^2) = \frac{4}{\pi^2}\left[W(x^2) - \frac{1}{F}[(x^2-a^2)(1-x^2)]^{-1/2}\int_a^1 W(x^2)\,dx\right]. \tag{42}$$

5. ITERATIVE SOLUTION OF THE INTEGRAL EQUATION

If the case $d \gg 1$ is considered then by substituting $kd = \varphi$ and expanding $\cos(\varphi y/d)$ and $\sin(\varphi y/d)$ in series, we may write (36) in the form

$$K_2(y,t) = \sum_{n=0}^{\infty} \frac{I_n}{d^{2n+2}}\,M_n(t,y), \tag{43}$$

where

$$M_n(t, y) = \frac{1}{2} [(t+y)^{2n+1} + (t-y)^{2n+1}]$$

and

$$I_n = \frac{(-1)^n}{(2n+1)!} \int_0^\infty H(\varphi)\, \varphi^{2n+1}\, d\varphi .$$

Now from (43), (35) and (41), we find that

$$K(x^2, t) = \frac{t}{8\pi y} \sum_{n=1}^{\infty} \frac{B_n(x^2, t^2)}{d^{2n}} , \tag{44}$$

where

$$\begin{aligned} Y &= [(x^2 - a^2)(1 - x^2)]^{1/2} , \\ B_1 &= I_0 A_0(x^2) , \\ B_2 &= I_1 [A_0(x^2) t^2 + 8 A_1(x^2)] , \\ B_3 &= I_2 [A_0(x^2) t^4 + 10 A_1(x^2) t^2 + A_2(x^2)] , \\ B_4 &= I_3 [A_0(x^2) t^6 + 21 A_1(x^2) t^4 + 7 A_2(x^2) t^2 + A_3(x^2)] \end{aligned} \tag{45}$$

with

$$\begin{aligned} A_0(x^2) &= 16 (x^2 - E/F) , \\ A_1(x^2) &= 8 (2x^4 + \alpha_0 x^2 + \alpha_1) , \\ A_2(x^2) &= 10 (8 x^6 + 4 \alpha_0 x^4 - \alpha^4 x^2 + \alpha_2) , \\ A_3(x^2) &= 7 (16 x^8 + 8 \alpha_0 x^6 - 2 \alpha^4 x^4 + \alpha_0 \alpha^4 x^2 + \alpha_3) , \end{aligned} \tag{46}$$

where

$$\begin{aligned} \alpha &= 1 - a^2 , \\ \alpha_0 &= -(1 + a^2) , \\ \alpha_1 &= \alpha^2 E/F - 2 I_1' , \\ \alpha_2 &= \alpha^2 (1 + 3a^2) E/F + 4 \alpha^2 I_1' - 8 I_2' , \\ \alpha_3 &= \alpha^2 (1 + 2a^2 + 5a^4) E/F + 2\alpha^2 (1 + 3a^2) I_1' + \\ &\quad + 8 \alpha^2 I_2' - 16 I_3' , \end{aligned}$$

$$I_n' = \frac{1}{F} \int_a^1 \left[\frac{x^2 - a^2}{1 - x^2} \right]^{1/2} x^{2n}\, dx ,$$

we find that

$I_0' = -a^2 + E / F$,

$I_1' = [-a^2 + (2 - a^2) E / F] / 3$,

$I_2' = [a^2 (a^2 - 4) + (8 - 3 a^2 - 2 a^4) E / F] / 15$,

$I_3' = [a^2(4a^4 + 5a^2 - 2i) + (48 - 16 a^2 - 9 a^4 - 8 a^6) E / F] / 105$,

where F and E are elliptic integrals of the first and second kind respectively defined by

$$F = F(\alpha, \pi / 2) = \int_a^1 dx / Y,$$

$$E = E(\alpha, \pi / 2) = \int_a^1 x^2 \, dx / Y.$$

For $f(x) = T_0$ we find from (39) that

$$W(x^2) = \pi T_0 [(x^2 - a^2) / (1 - x^2)]^{\frac{1}{2}} / 2 ,$$

and hence from (42) we find that

$$P(x^2) = T_0 A_0 (x^2) / (8 \pi Y) . \tag{47}$$

Since $d \gg 1$, $|K(x^2, t)| < o$ where $\sigma < 1$, the solution to (40) may be taken in the form

$$h(x^2) = \sum_{h=0}^{\infty} M_n (x^2) / d^{2n} . \tag{48}$$

Now substituting for $K(x^2, t)$ and $h(x^2)$ respecting from (44) and (48) in (40) and equating the various powers of d from both sides, we obtain

$$M_0 (x^2) = P(x^2) ,$$

$$M_n (x^2) = - \frac{1}{8 \pi Y} \sum_{m=1}^{n} \int_a^1 t \, B_m (x^2, t^2) \, M_{n-m} (t^2) \, dt ,$$

$$n = 1, 2, 3, \dots .$$

By carrying out the above iteration process up to M_4 , we find that

$$h(x^2) = \frac{T_0}{4 \pi Y} (\beta_0 + \beta_1 x^2 + \beta_2 x^4 + \beta_3 x^6 + \beta_4 x^8) + (\alpha^{-10}), \tag{49}$$

where

$\beta_0 = - 8 E / F + 4E I_0 b_0 d^{-2} + c_0 d^{-4} + d_0 d^{-6} + e_0 d^{-8}$,

$\beta_1 = 8 - 4 I_0 b_0 d^{-2} + c_1 d^{-4} + d_1 d^{-6} + e_1 d^{-8}$,

$$\beta_2 = C_2 d^{-4} + d_2 d^{-6} + e_2 d^{-8} , \tag{50}$$
$$\beta_3 = d_3 d^{-6} + e_3 d^{-8} ,$$
$$\beta = e_4 d^{-8}$$

with

$$b_0 = 4 (r_1 - r_0 E / F) / \pi ,$$
$$b_1 = 16 (r_2 - r_1 E / F) / \pi ,$$
$$b_2 = 32 (r_3 - r_2 E / F) / \pi ,$$
$$b_3 = 64 (r_4 - r_3 E / F) / \pi , \tag{51}$$
$$b_4 = 32 (c_0 r_1 + c_1 r_2 + c_2 r_3) / \pi ,$$
$$b_5 = 16 (c_0 r_0 + c_1 r_1 + c_2 r_2) / \pi ,$$
$$c_0 = - 2 I_0^2 b_0^2 E / F + I_1 b_1 E / F - 6 I_1 b_0 \alpha_1 ,$$
$$c_1 = 2 I_0^2 b_0^2 - I_1 b_1 - 6 I_1 b_0 \alpha_0 , \tag{52}$$
$$c_2 = - 12 I_1 b_0 ,$$
$$d_0 = E [(I_2 b_2 - I_0 I_1 b_1^2) / 2 F] - 5 I_2 b_1 \alpha_1 - 5 I_2 b_0 \alpha_2 / 2 + + I_0 I_1 b_0 b_1 \alpha_1 / 3 + E b_5 / 8 F ,$$
$$d_1 = - I_2 (b_2 + 10 b_1 \alpha_0 - 35 \alpha^2 b_0) / 2 + + I_0 I_1 b_1 (b_1 + 6 b_0 \alpha_0) / 2 - I_0 b_5 / 8 , \tag{53}$$
$$d_2 = - 10 I_2 (b_1 + b_0 \alpha_0) + 6 I_0 I_1 b_0 b_1 ,$$
$$d_3 = - 20 I_2 b_0 ,$$
$$e_0 = I_3 (2 b_3 E / F - 42 b_2 \alpha_1 - 35 b_1 \alpha_2 - 14 b_3 \alpha_3) / 8 - - I_0 I_2 b_1 (b_2 E / F - 10 b_1 \alpha_1 - 5 b_0 \alpha_2) / 4 + + 2 I_0 E (d_0 r_0 + d_1 r_1 + d_2 r_2 + d_3 r_3) / \pi F + + I_1 (b_4 E / F - 3 b_5 \alpha_1) / 16 ,$$
$$e_1 = - I_3 (2 b_3 + 42 b_2 \alpha_0 - 24 \alpha^2 b_1 + 14 \alpha^4 b_0 \alpha_0) / 8 \pi + + I_2 I_2 b_1 (b_2 + 10 b_1 \alpha_0 - 35 \alpha^4 b_0) / 4 - - 2 I_0 (d_0 r_0 + d_1 r_1 + d_2 r_2 + d_3 r_3) / \pi - - I_1 (b_4 + 3 b_5 \alpha_0) ,$$
$$e_2 = - 7 I_3 (3 b_2 + 5 b_1 \alpha_0 - \alpha^2 b_0) / 2 + 5 I_0 I_2 b_1 (b_1 + b_0 \alpha_0) - - 3 I_1 b_5 / 8 , \tag{54}$$
$$e_3 = - 7 I_3 (5 b_1 + 2 b_0 \alpha_0) / \pi + 10 I_0 I_2 b_0 b_1 ,$$
$$e_4 = - 28 I_3 b_0 ,$$

and

$$r_n = \int_a^1 \frac{x^{2n+1}}{Y}\, dx, \quad n = 0, 1, 2, \ldots \tag{55}$$

so that

$$\begin{aligned}
r_0 &= \pi / 2 , \\
r_1 &= \pi (1 + a^2) / 4 , \\
r_2 &= \pi (3 + 2 a^2 + 3 a^4) / 16 , \\
r_3 &= \pi (5 + 3 a^2 + 3 a^4 + 5 a^6) / 32 , \\
r_4 &= \pi (35 + 20 r^2 + 18 r^4 + 20 r^6 + 35 r^8) / 256 .
\end{aligned}$$

6. EXPRESSIONS FOR THE PHYSICAL QUANTITIES

From equation (29) we find that

$$T(x,y) = (2 / \pi)^{\frac{1}{2}} \int_0^\infty [A(k) \cosh (y - d) k / \cosh (kd)]\, dk , \tag{56}$$

$$u(x,y) = \frac{\beta}{\sqrt{2\pi}\,(m^2 - 1)} \int_0^\infty [A(k) \cosh (y - d) k / \cosh (kd)]\, dk , \tag{57}$$

$$v(x,y) = \frac{\beta}{\sqrt{2\pi}\,(m^2 - 1)} \int_0^\infty [A(k) \sinh (y - d) k / k \cosh (kd)]\, dk . \tag{58}$$

Now the boundary values at $y = 0$ for the physical quantities for the case of $f(x) = T_0$ are obtained. From equations (32) and (56) we find that the temperature on the boundary $y = 0$ is given by

$$T(x, 0) = \frac{\pi}{2} \int_a^1 t\, h(t^2)\, dt , \tag{59}$$

since

$$f(x) = T_0 , \quad \int_0^1 h(t^2)\, dt = 0 .$$

Substituting for $h(t^2)$ from (49) in (59), we obtain

$$T(x, 0) = \frac{T_0}{8} \sum_{n=0}^{\infty} \beta_n r_n + 0 (d^{-10}) . \tag{60}$$

The total quantity of heat passing per second through the edge $y = 0$, $a < x < 1$ is given by

$$Q = -\varkappa \int_a^1 (\partial t / \partial y)_{y=0} \, dx \,, \tag{61}$$

where $\varkappa$ denotes the coefficient of conductivity of the material. From equations (32), (56) and (61), we find that for $f(x) = T_0$

$$Q = \left(\frac{2}{\pi}\right) \varkappa \int_a^1 [t\, h(t^2) / (t^2 - x^2)]\, dt - (2/\pi)\, \varkappa \int_a^1 h(t^2)\, k_2(x, t)\, dt \,,$$

where

$$\int_a^1 [t\, h(t^2) / (t^2 - x^2)]\, dt = \frac{T_0}{8} \begin{cases} R(x^2) / Y_1 + N(x^2) + 0(d^{-10}), & 0 < x < a, \\ R(x^2) / Y_2 + N(x^2) + 0(d^{-10}), & x > 1 \,, \end{cases}$$

where

$$R(x^2) = \sum_{n=0}^{4} \beta_n x^{2n},$$

$$N(x^2) = \left[\frac{2}{\pi} r_0 \sum_{n=1}^{4} \beta_n x^{2n-2} + r_1 \sum_{n=2}^{4} \beta_n x^{2n-4} + r_2 (\beta_3 + \beta_4 x^2) + r_3 \beta_4 \right],$$

$$Y_1 = [(a^2 - x^2)(1 - x^2)]^{\frac{1}{2}},$$

$$Y_2 = [(x^2 - a^2)(x^2 - 1)]^{\frac{1}{2}}.$$

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ÖZET

Bu çalışmada bir yüzü serbest, diğer yüzü sürtünmesiz ve katı bir temel üzerinde bulunan bir elastik şeridin kararlı, eksen simetrili, düzlem gerilimli termoelastik probleminin çözümü araştırılmaktadır.