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ALGEBRAIC CHARACTERIZATION OF SOME FUZZY TOPOLOGICAL SPACES

M. S. EROĞLU

Let (X, τ) be a topological space and $C(X, \tau)$ the semigroup of continuous functions $f: (X, \tau) \longrightarrow (X, \tau)$. In [1] and [2], Magill defined S- and S^*-spaces and proved that two topological S- (S^*-) spaces (X, τ) and (X^*, τ^*) are homeomorphic iff the semigroups $C(X, \tau)$ and $C(X^*, \tau^*)$ are isomorphic.

For every $f \in C(X, \tau)$ and $z \in X$, we define $H(f) := \{x \in X \mid f(x) = x\}$, and $H(z, f) := f^{-1}(z)$. (X, τ) will be called an a- (a*-) space iff

$$\{H(f) \mid f \in C(X, \tau)\} \ (\{H(z, f) \mid z \in X, f \in C(X, \tau)\})$$

is a subbase for the closed sets of (X, τ).

Now let $(X, \mathfrak{A})$ be a fuzzy topological space and $C(X, \mathfrak{A}) := f$ $f: C(X, \mathfrak{A}) \longrightarrow C(X, \mathfrak{A})$ fuzzy continuous. For every topology τ on X, $w(\tau)$ denotes the wellknown Lowen fuzzy topology on X. For every $f: X \longrightarrow [0, 1]$, we set $s_0(f) := f^{-1}((0, 1))$, and define $k(\tau) := \{f \mid f: X \longrightarrow [0, 1]$ and $s_0(f) \in \tau\}$. We can prove the following

Theorem 1. Let (X_1, τ_1) and (X_2, τ_2) be two non empty a- (a*-) spaces. $(X_1, w(\tau_1))$ and $(X_2, w(\tau_2))$ are fuzzy homeomorphic iff $C(X_1, w(\tau_1))$ and $C(X_2, w(\tau_2))$ are isomorphic semigroups. Every isomorphism

$$\theta: C(X_1, w(\tau_1)) \longrightarrow C(X_2, w(\tau_2))$$

is of the form $0(f) = \varphi \circ f \circ \varphi^{-1}$ for every $f \in C(X_1, w(\tau_1))$ with a fuzzy homeomorphism $\varphi: (X_1, w(\tau_1)) \longrightarrow (X_2, w(\tau_2))$. Particularly every automorphism of $C(X_1, w(\tau_1))$ is inner.

The same is true if we take $k(\tau_1)$, $k(\tau_2)$ for $w(\tau_1)$, $w(\tau_2)$ respectively.

A fuzzy topological space $(X, \mathfrak{A})$ will be called a **fuzzy T_2-space** iff for every different points $a, b \in X$ and for every $\alpha, \beta \in (0,1)$ there are $f, g \in \mathfrak{A}$ with $f \wedge g = 0$, $\alpha < f(a)$ and $\beta < g(b)$. For $A \subset X$, χ_A denotes the characteristic function of A in X. For every $\lambda \in [0, 1]$, λ_x will denote the constant function on X with value λ. A fuzzy topological space $(X, \mathfrak{A})$ will be called a **fuzzy a-space** iff $\{\lambda_x \mid \lambda \in [0,1]\} \cup \{\chi_{H(f)} \mid f \in C(X, \mathfrak{A})\}$ is a subbase for fuzzy closed sets of $(X, \mathfrak{A})$. Then we have the following

Theorem 2. Two fuzzy a-spaces $(X, \mathfrak{A})$ and $(Y, \mathfrak{B})$ are fuzzy homeomorphic iff the semigroups $C(X, \mathfrak{A})$ and $C(Y, \mathfrak{B})$ are isomorphic. Every isomorphism $\theta : C(X, \mathfrak{A}) \longrightarrow C(Y, \mathfrak{B})$ is of the form

$$\theta(f) = \varphi \circ f \circ \varphi^{-1} \quad \forall f \in C(X, \mathfrak{A})$$

with a fuzzy homeomorphism $\varphi : C(X, \mathfrak{A}) \longrightarrow C(Y, \mathfrak{B})$. Particularly every automorphism of $C(X, \mathfrak{A})$ is inner.

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DEPARTMENT OF MATHEMATICS
FACULTY OF ARTS AND SCIENCES
KARADENIZ TECHNICAL UNIVERSITY
61080 TRABZON-TURKEY