THE JACOBIAN CONJECTURE II *)

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Let k be a field of characteristic zero, and $n > 0$ an integer. Write $X = (X_1, \ldots, X_n)$ for the sequence in n indeterminates $X_1, \ldots, X_n$. $k[X] = k[X_1, \ldots, X_n]$ denotes the ring of polynomials in $X_1, \ldots, X_n$.

Let $F = (F_1, \ldots, F_n)$ be an endomorphism of $k[X]$, where F sends X_i to F_i. Hyman BASS has shown that the Jacobian Conjecture will follow once it is shown for all $F = (F_1, \ldots, F_n)$ of the form $F_i = X_i - N_i$, where each N_i is a cubic homogeneous polynomial and the matrix $J(N) = (\partial N_i / \partial X_j)$ is nilpotent. An $F = X - N$ above has an analytic inverse $G = (G_1, \ldots, G_n)$ near the origin. For each i, $G_i(X)$ is a power series such that $G_i(F) = X_i$, where $G_i(X) \in k[[X]]$, the ring of formal power series. Jacobian Conjecture asserts that these power series G_i are polynomials. Let us denote by $G_i^{(d)}$, the homogeneous components of degree $2d + 1$ of G_i. H. BASS has shown that,

G_i are polynomials iff $G_i^{(d)} = 0$ for $d > \delta^{n-1} - 1/\delta - 1$ where $\delta = \deg N_i$.

In this work, I have improved the recurrence formula for $G^{(d)}$ which I had developed in my previous work [1]. The improved formula is more convenient to produce fruitfull results toward the solution of the Jacobian problem. In fact, if, for $F = X - N$, N being cubic homogeneous and $J(F) = I - J(N)$ invertible with $J^2(N) = 0$, then F is invertible, whose inverse is $G = X + N$. Although H. BASS obtained this result, by means of the improved formula, this result is obtained in a much simpler way.

Furthermore, the recursive character of the formula enables me to give estimations of d, for $G^{(d)} = 0$. For arbitrary n with $J^n(N) = 0$, G is a polynomial whenever it is shown that, $G^{(d)} = 0$ for

$$1 + \sum_{s=0}^{n-2} 3^s \leq d \leq \sum_{s=0}^{n-1} 3^s .$$

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Since the recurrence formula of $G^{(d)}$ for $d \geq 1 + \sum_{s=0}^{n-1} 3^s$ consists only of the terms $G^{(d)}$ for

$$1 + \sum_{s=0}^{n-2} 3^s \leq d \leq \sum_{s=0}^{n-1} 3^s$$

where the terms for $d < 1 + \sum_{s=0}^{n-2} 3^s$ occur in the formula with higher derivatives than their degrees, disappearing from the formula. The JACOBIAN CONJECTURE states that :

$$J(F) \text{ is invertible} \Longrightarrow F \text{ is invertible.}$$

REFERENCE

[1] KİREZCİ, M. : *The Jacobian Conjecture I,* Bulletin of the Technical University of Istanbul 43 (1990), 3, 421-436.

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