ON WAVE SOLUTIONS OF THE FIELD EQUATIONS OF EINSTEIN'S, BONNOR'S AND SCHRODINGER'S NON-SYMMETRIC UNIFIED THEORIES

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In the present paper the field equations of EINSTEIN's, BONNOR's and SCHRÖDINGER's non-symmetric unified field theories are investigated. It has been found that the field equations of non-symmetric unified field theories of EINSTEIN and BONNOR yield wave solutions under certain conditions, whereas for the field equations of SCHRODINGER's theory such solutions do not exist.

1. Introduction. The present paper is in continuation to author's paper [1] [1)] in which the wave solutions of field equations of general relativity in the space-time, represented by the metric

$$(1.1)\quad ds^2 = -A\,dx^2 - 2D\,dxdy - \mathrm{B}\,dy^2 - (C-E)\,dz^2 - 2E\,dzdt + (C+E)\,dt^2,$$

where A, B, D are functions of the single variable $Z = Z(z-t)$, C is any function of (z, t), and E is any function of x, y, z and t are investigated. In this paper, with the help of line element (1.1) the attempts have been made to obtain the wave solutions of the field equations of non-symmetric unified field theories of EINSTEIN, BONNOR and SCHRÖDINGER. The field equations of A. EINSTEIN's unifield theory [6] are

$$(1.2)\qquad g_{ij;k} \equiv g_{ij,k} - g_{sj}\,\Gamma^s_{ik} - g_{is}\,\Gamma^s_{kj} = 0,$$

$$(1.3)\qquad \Gamma_t \equiv \Gamma^s_{\underset{\vee}{ts}} = 0,$$

1) Numbers in brackets refer to the references at the end of the paper.

$$\text{(1.4)}\quad \text{a)}\quad R_{kl} \equiv \Gamma^{s}_{kl,s} - \Gamma^{s}_{ks,l} - \Gamma^{s}_{tl}\,\Gamma^{s}_{ks} + \Gamma^{s}_{ts}\,\Gamma^{t}_{kl} = 0,$$

$$\text{b)}\quad (i)\; R_{\underline{ij}} = 0, \qquad (ii)\; R_{\underset{\vee}{ij},k} + R_{\underset{\vee}{jk},i} + R_{\underset{\vee}{ki},j} = 0,$$

where a comma followed by an index denotes ordinary partial differentiation and latin indices take the values 1, 2, 3, 4. A bar and a hook under two indices denote respectively symmetry and anti-symmetry between them.

Following above notation the field equations of W.B. BONNOR [5] are given by (1.2), (1.3) and

$$\text{(1.5)}\quad \text{a)}\quad R_{\underline{ij}} + p^{2}\, U_{\underline{ij}} = 0,$$

$$\text{b)}\quad (R_{\underset{\vee}{ij},k} + R_{\underset{\vee}{jk},i} + R_{\underset{\vee}{ki},j}) + p^{2}(U_{\underset{\vee}{ij},k} + U_{\underset{\vee}{jk},i} + U_{\underset{\vee}{ki},j}) = 0,$$

where R_{ij} is the Ricci tensor, p is an arbitrary real or imaginary constant and U_{ik} is given by

$$\text{(1.6)}\qquad U_{ik} = g_{\underset{\vee}{ki}} - g^{\underset{\vee}{mn}}\, g_{im}\, g_{nk} + \tfrac{1}{2}\, g^{\underset{\vee}{mn}}\, g_{nm}\, g_{ik}\,.$$

On use of similar notations the field equations of E. SCHRÖDINGER ([2], [7]) are given by (1.2), (1.3) and

$$\text{(1.7)}\quad \text{a)}\quad R_{\underline{ij}} + \lambda\, g_{\underline{ij}} = 0,$$

$$\text{b)}\quad (R_{\underset{\vee}{ij}} + \lambda\, g_{\underset{\vee}{ij}})_{,k} + (R_{\underset{\vee}{jk}} + \lambda\, g_{\underset{\vee}{jk}})_{,i} + (R_{\underset{\vee}{ki}} + \lambda\, g_{\underset{\vee}{ki}})_{,j} = 0,$$

where λ is a non-vanishing constant.

2. g_{ij} **corresponding to plane waves.** We have obtained [1] the transverse-electromagnetic wave solutions F_{ij} of the generalized MAXWELL's equations in the space-time (1.1) which are given by

$$\text{(2.1)}\qquad F_{ij} = \begin{bmatrix} 0 & 0 & -\sigma_{1} & \sigma_{1} \\ 0 & 0 & \rho_{1} & -\rho_{1} \\ \sigma_{1} & -\rho_{1} & 0 & 0 \\ -\sigma_{1} & \rho_{1} & 0 & 0 \end{bmatrix},$$

where ρ_1 and σ_1 are arbitrary functions of x, y and $z - t$, which satisfy the conditions $\partial\rho_1 / dx + \partial\sigma_1 / \partial y = 0$ and
$(A\, \partial\rho_1 / \partial y - B\, \partial\sigma_1 / \partial x) - D(\partial\rho_1 / \partial x - \partial\sigma_1 / \partial y) = 0$.

In this section we will find the non-symmetric g_{ij} corresponding to above F_{ij}. Let us assume that

$$(2.2) \qquad g_{ij} = g_{\underline{ij}} + g_{\underset{\vee}{ij}} = h_{ij} + f_{ij},$$

where $g_{\underline{ij}} = h_{ij}$ is the symmetric part coinciding with the metric tensor of the Riemannian space-time defined by the line element (1.1) and $g_{\underset{\vee}{ij}}$ is the anti-symmetric part of g_{ij} corresponding to the electromagnetic field (2.1). Thus using (1.1) and (2.2) g_{ij} are given by

$$(2.3) \qquad (g_{ij}) = \begin{bmatrix} -A & -D+f_{12} & f_{13} & f_{14} \\ -D-f_{12} & -B & f_{23} & f_{24} \\ -f_{13} & -f_{23} & -(C-E) & -E+f_{34} \\ -f_{14} & -f_{24} & -E-f_{34} & (C+E) \end{bmatrix}.$$

To connect F_{ij} with g_{ij}, we shall use the equation

$$(2.4) \qquad F_{ij} = \tfrac{1}{2}\, \varepsilon_{ijkl} \sqrt{-g}\, g^{kl} (g = \det (g_{ij}))$$

introduced by IKEDA [8], where g^{ij} is the contravariant tensor of g_{ij} and $\varepsilon_{ijkl} = +1$ or -1 according as i, j, k, l have even or old permutations. From (2.1) we have $F_{12} = F_{34} = 0$. Hence from (2.4) we have

$$g^{12} = g^{21}, \quad g^{34} = g^{43}.$$

Using $f_{12} = f_{34} = 0$, the values of g, g_{ij} and g^{ij} finally obtained under the assumption $f_{13}\, f_{24} - f_{14}\, f_{23} = 0$ are given by

$$(2.5) \qquad (g_{ij}) = \begin{bmatrix} -A & -D & \rho & -\rho \\ -D & -B & \sigma & -\sigma \\ -\rho & -\sigma & -(C-E) & -E \\ \rho & \sigma & -E & (C+E) \end{bmatrix},$$

$$g = -m\, C^2,$$

where $m = AB - D^2$, $\rho = (A\rho_1 + D\sigma_1)/\sqrt{m}$ and $\sigma = (D\rho_1 + B\sigma_1)/\sqrt{m}$, and

$$(2.6) \qquad (g^{ij}) = \begin{bmatrix} -B/m & D/m & U & U \\ D/m & -A/m & V & V \\ -U & -V & -1/C+W & W \\ -U & -V & W & 1/C+W \end{bmatrix},$$

where $U = (B\rho - D\sigma/mC$, $V = (A\sigma - D\rho)/mC$,

$$W = [(A\sigma^2 - 2D\rho\sigma + B\rho^2)/m - E]/C^2.$$

3. Connections Γ^k_{ij} corresponding to g_{ij}. We put

$$(3.1) \qquad \Gamma^k_{ij} = p^k_{ij} + q^k_{ij},$$

where $p^k_{ij} = \Gamma^k_{\underline{ij}}$ and $q^k_{ij} = \Gamma^k_{\underset{\vee}{ij}}$.

By the above substitution the field equation (1.2) will give 64 equations involving 24 q's and 40 p's. Following the method of H. TAKENO, M. IKEDA and S. ABE to solve these 64 equations we first express all p's in terms of q's and $\{^{\,k}_{ij}\}$ by the formula [9],

$$(3.2) \qquad p^k_{ij} = \{^{\,k}_{ij}\} + h^{kl}(q^m_{li}\, f_{jm} + q^m_{lj}\, f_{im}),$$

where $\{^{\,k}_{ij}\}$ are CHRISTOFFEL symbols of the second kind formed from $h_{ij} = g_{\underline{ij}}$ given by (1.1) and h^{ij} is the corresponding contravariant tensor of h_{ij}. After calculations, we find that the non-vanishing components of p's are given by

$$(3.3) \qquad p^1_{11} = -2D\rho(q^3_{12} - q^4_{13})/m, \quad p^2_{11} = 2A\rho(q^3_{12} - q^4_{12})/m,$$

$$p^3_{11} = -\bar{A}/2C + 2\rho\,\{(C + E)\,(q^3_{13} - q^4_{13}) + E(q^3_{14} - q^4_{14})\}/C^2,$$

$$p^4_{11} = -\bar{A}/2C - 2\rho\,\{(C - E)\,(q^3_{14} - q^4_{14}) - E(q^3_{13} - q^4_{13})\}/C^2,$$

$$p_{12} = -\{B\rho(q^3_{12} - q^4_{12}) + D\sigma(q^3_{12} - q^4_{12})\}/m,$$

$$p^2_{12} = \{Dp(q^3_{12} - q^4_{12}) + A\sigma(q^3_{12} - q^4_{12})\}/m,$$

....

....

other non-vanishing components have heen omitted for the sake of brevity.

On using the values of p's from (3.3) in (3.2) the non-vanishing components of q's are given by

$$
\begin{aligned}
(3.4)\qquad q^1_{13} &= -q^1_{14} = \{-B\,\partial\rho/\partial x + \tfrac{1}{2}D(\partial\rho/\partial y + \partial\sigma/\partial x)\}/m,\\
q^2_{13} &= -q^2_{14} = \{-D\,\partial\rho/\partial x + \tfrac{1}{2}A(\partial\rho/\partial y + \partial\sigma/\partial x)\}/m,\\
q^1_{23} &= -q^1_{24} = \{\ \ D\,\partial\sigma/\partial y - \tfrac{1}{2}B(\partial\rho/\partial y + \partial\sigma/\partial x)\}/m,\\
q^2_{23} &= -q^2_{24} = \{-A\,\partial\sigma/\partial y + \tfrac{1}{2}D(\partial\rho/\partial y + \partial\sigma/\partial x)\}/m,\\
q^3_{12} &= q^4_{12} = -(\partial\rho/\partial y - \partial\sigma/\partial x)/2C,\\
q^3_{13} &= q^4_{13} = -q^3_{14} = -q^4_{14} = \lambda/C,\\
q^3_{23} &= q^4_{23} = -q^3_{24} = -q^4_{24} = \mu/C,
\end{aligned}
$$

where

$$
\begin{aligned}
\lambda &= [-\bar\rho + \rho\{P + (E_3 + E_4 + C_3 - C_4)/2C\} + \sigma S],\\
\mu &= [-\bar\sigma + \sigma\{T + (E_3 + E_4 + C_3 - C_4)/2C\} + \rho Q_1], \text{ and}\\
P &= (\bar{A}B - D\bar{D})/2m, \qquad S = (A\bar{D} - \bar{A}D)/2m,\\
Q_1 &= (B\bar{D} - \bar{B}D)/2m, \qquad T = (A\bar{B} - D\bar{D})/2m.
\end{aligned}
$$

On substituting the values of q's from (3.4) into (3.3) we obtain the 40 values of p's in terms of A, B, C, D, E, p and σ. Then using the values of p's and q's into (3.1), the components of Γ^k_{ij} finally obtained are given by

$$
\begin{aligned}
(3.5)\qquad \Gamma^k_{11} &= [0,0,\ -\bar{A}/2C,\ -\bar{A}/2C],\ \Gamma^k_{22} = [0,0,\ -\bar{B}/2C,\ -\bar{B}/2C],\\
\Gamma^k_{12} &= [0,0,\ -\bar{D}/2C - (\partial\rho/\partial y - \partial\sigma/\partial x)/2C,\\
&\qquad -\bar{D}/2\bar{C} - (\partial\rho/\partial y - \partial\sigma/\partial x)/2C],\\
\Gamma^k_{21} &= [0,0,\ -\bar{D}/2C + (\partial p/\partial y - \partial\sigma/\partial x)/2C,\\
&\qquad -\bar{D}/2C + (\partial\rho/\partial y - \partial\sigma/\partial x)/2C],
\end{aligned}
$$

$$\Gamma^k_{13} = -\Gamma^k_{14} = [P + \{-B\,\partial\rho/\partial x + \tfrac{1}{2}D(\partial\rho/\partial y + \partial\sigma/\partial x)\}/m,$$

$$S - \{-D\,\partial\rho/\partial x + \tfrac{1}{2}A(\partial\rho/\partial y + \partial\sigma/\partial x)\}/m,$$

$$(\lambda - \tfrac{1}{2}\,\partial E/\partial x)/C + \{(B\rho - D\sigma)\,\partial\rho/\partial x + \tfrac{1}{2}(A\sigma - D\rho)(\partial\rho/\partial y + \partial\sigma/\partial x)\}/Cm,$$

$$\text{,,} \qquad \text{,,} \qquad \text{,,} \qquad],$$

$$\Gamma^k_{31} = -\Gamma^k_{41} = [P - \{-B\partial\rho/\partial x + \tfrac{1}{2}D(\partial\rho/\partial y + \partial\sigma/\partial x)\}/m,$$

$$S + \{-D\partial\rho/\partial x + \tfrac{1}{2}A(\partial\rho/\partial y + \partial\sigma/\partial x)\}/m,$$

$$-(\lambda + \tfrac{1}{2}\,\partial E/\partial x)/C + \{(B\rho - D\sigma)\,\partial\rho/\partial x + \tfrac{1}{2}(A\sigma - D\rho)(\partial\rho/\partial y + \partial\sigma/\partial x)\}/Cm,$$

$$\text{,,} \qquad \text{,,} \qquad \text{,,} \qquad],$$

$$\Gamma^k_{23} = -\Gamma^k_{24} = [Q_1 + \{D\partial\rho/\partial y - \tfrac{1}{2}B(\partial\rho/\partial y + \partial\sigma/\partial x)\}/m,$$

$$T + \{-A\partial\sigma/\partial y + \tfrac{1}{2}D(\partial\rho/\partial y + \partial\sigma/\partial x)\}/m,$$

$$(\mu - \tfrac{1}{2}\,\partial E/\partial y/C + \{(A\sigma - D\rho)\,\partial\sigma/\partial y + \tfrac{1}{2}(B\rho - D\sigma)(\partial\rho/\partial y + \partial\sigma/\partial x)\}/Cm,$$

$$\text{,,} \qquad \text{,,} \qquad \text{,,} \qquad],$$

$$\Gamma^k_{32} = -\Gamma^k_{42} = [Q_1 - \{D\partial\sigma/\partial y - \tfrac{1}{2}B(\partial\rho/\partial y + \partial\sigma/\partial x)\}/m,$$

$$T - \{-A\partial\sigma/\partial y + \tfrac{1}{2}D(\partial\rho/\partial y + \partial\rho/\partial x)\}/m,$$

$$-(\mu + \tfrac{1}{2}\,\partial E/\partial y)/C + \{(A\sigma - D\rho)\,\partial\sigma/\partial y + \tfrac{1}{2}(B\rho - D\sigma)(\partial\rho/\partial y + \partial\sigma/\partial x)\}/Cm,$$

$$\text{,,} \qquad \text{,,} \qquad \text{,,} \qquad],$$

$$\Gamma^1_{33} = \Gamma^1_{44} = -\Gamma^1_{34} = \{(B\,\partial E/\partial x - D\,\partial E/\partial y)/2m + 2(B\alpha - D\beta)/m\},$$

$$\Gamma^2_{33} = \Gamma^2_{44} = -\Gamma^2_{34} = -\{(D\partial E/\partial x - A\partial E/\partial y)/2m + 2(D\alpha - A\beta)/m\},$$

$$\Gamma^3_{33} = S_1\,E/2C^2 + C_3/2C - E_3/2C,$$

$$\Gamma^4_{33} = S_1\,E/2C^2 + C_4/2C - E_3/C - E_4/2C,$$

$$\Gamma^3_{44} = S_1\,E/2C^2 + C_3/2C + E_3/2C + E_4/C,$$

$$\Gamma^4_{44} = S_1\, E/2C^2 + C_4/2C + E_4/2C,$$

$$\Gamma^3_{34} = \Gamma^3_{43} = -\, S_1\, E/2C^2 + C_4/2C - E_4/2C,$$

$$\Gamma^4_{34} = \Gamma^4_{43} = -\, S_1\, E/2C^2 + C_3/2C + E_3/2C,$$

where

$$S_1 = E_3 + E_4 + C_3 - C_4\,,$$

$$\alpha = -\, \bar{m}^1 [(B\rho - D\sigma)\, \partial\rho/\partial x + \tfrac{1}{2}(A\sigma - D\rho)(\partial\rho/\partial y + \partial\sigma/\partial x)],$$

$$\beta = -\, \bar{m}^1 [(A\sigma - D\rho)\, \partial\sigma/\partial y + \tfrac{1}{2}(B\rho - D\sigma)(\partial\rho/\partial y + \partial\sigma/\partial x)],$$

4. Solution of the field equation (1.3). On substituting of the values of q's from (3.4) in the equation $\underset{\vee}{\Gamma^s_{ts}} = 0$, we find that it is identically satisfied for $t = 1$ and 2 whereas for $t = 3$ and 4 it is satisfied subject to the condition

$$B\, \partial\rho/\partial x + A\, \partial\sigma/\partial y = D(\partial\rho/\partial y + \partial\sigma/\partial x). \tag{4.1}$$

5. Solution of the field equation (1.4). To solve the field equation (1.4) the components of the generalized RICCI tensor using the formula

$$R_{ij} = \Gamma^s_{ij,s} - \Gamma^s_{is,j} - \Gamma^s_{ij}\, \Gamma^t_{is} + \Gamma^s_{is}\, \Gamma^t_{ij} \tag{5.1}$$

are calculated and then on substitution of the values of Γ^i_{ij} from (3.5) into (5.1) and using (4.1), the non-vanishing components of RICCI tensor R_{ij} are given by

$$\begin{aligned} B_{13} = -\, R_{14} &= -\, \Delta\rho/2m + (\lambda_3 + \lambda_4)/C - (E_{13} + E_{14})/2C, \\ R_{31} = -\, R_{41} &= \quad \Delta\rho/2m - (\lambda_3 + \lambda_4)/C - (E_{13} + E_{14})/2C, \\ R_{23} = -\, R_{24} &= -\, \Delta\sigma/2m + (\mu_3 + \mu_4)/C - (E_{23} + E_{24})/2C, \\ R_{32} = -\, B_{42} &= \quad \Delta\sigma/2m - (\mu_3 + \mu_4)/C - (E_{23} + E_{24})/2C, \end{aligned} \tag{5.2}$$

$$R_{33} = -\xi + (\Delta E + 4Q)/2m + \eta/2m^2 + N(C - E)/2C^2,$$

$$R_{44} = -\xi + (\Delta E + 4Q)/2m + \eta/2m^2 - N(C + E)/2C^2,$$

$$R_{34} = R_{43} = \xi - (\Delta E + 4Q)/2m - \eta/2m^2 + EN/2C^2,$$

where

$$(5.3) \quad \Delta = (B\,\partial^2/\partial x^2 - 2D\,\partial^2/\partial x\,\partial y + A\,\partial^2/\partial y^2),$$

$$\xi = \{\bar{\bar{m}} - \bar{m}^2/2m - (\bar{A}\,\bar{B} - \bar{D}^2) - \bar{m}(E_3 + E_4 + C_3 - C_4)/2C\}/2m,$$

$$\eta = \{(B\partial\rho/\partial x - A\partial\sigma/\partial y)^2 + (AB - 2D^2)(\partial\rho/\partial y + \partial\sigma/\partial x)^2 + 4D(\partial\rho/\partial x\ \ \partial\sigma/\partial y)\},$$

$$Q = (B\partial\alpha/\partial x - D\partial\beta/\partial x) - (D\partial\alpha/\partial y - A\partial\beta/\partial y),$$

$$N = (C_3^2 - C_4^2)/C - (C_{33} - C_{44}) + (C_3 + C_4)(E_3 + E_4)/C - E_{33} - E_{44} - 2E_{34}$$

$$= -2C(\lambda_3 + \lambda_4)/\rho = -2C(\mu_3 + \mu_4)/\rho.$$

The indices 3 and 4 attached to λ and μ denote differentiation with respect to z and t respectively and α, β are the same as given in equation (3.5).

Now using the values of (5.2) into the strong field equation (1.4) a) we have

$$(5.4) \quad -\Delta\rho/2m + (\lambda_3 + \lambda_4)/C - (E_{13} + E_{14})/2C = 0,$$

$$-\Delta\rho/2m + (\lambda_3 + \lambda_4)/C + (E_{13} + E_{14})/2C = 0,$$

$$-\Delta\sigma/2m + (\mu_3 + \mu_4)/C - (E_{23} + E_{24})/2C = 0,$$

$$-\Delta\sigma/2m + (\mu_3 + \mu_4)/C + (E_{23} + E_{24})/2C = 0,$$

$$(5.5) \quad M - N(C - E)/2C^2 = M + N(C + E)/2C^2 = M + NE/2C^2 = 0,$$

where

$$M = \xi - (\Delta E + 4Q)/2m - \eta/2m^2.$$

The equations (5.5) is satisfied if and only if

(5.6) $M = 0$ and

(5.7) $N = 0,$

while the equations in (5.4), with the help of (5.7) are satisfied if and only if

(5.8) $\Delta\rho = 0, \quad \Delta\sigma = 0,$

(5.9) $E_{13} + E_{14} = 0, \quad E_{23} + E_{24} = 0$

which on integration yields the forms of E which are given by

(5.10)
$$E = (x + y + z)\ \phi_1(z - t) + \phi(x, y)\ \phi_2(z - t),$$
$$E = (x + y + t)\ \phi_1(z - t) + \phi(x, y)\ \phi_2(z - t),$$
$$E = \phi(x, y, z - t) + z\ \phi_1(z - t),$$
$$E = \phi(x, y, z - t) + t\ \phi_1(z - t).$$

Hence g_{ij} given by (2.5) are the solutions of the field equation (1.4) a) under the conditions (5.6), (5.7), 5.8) and (5.10).

Next putting the values of R_{ij} from (5.2) into (1.4) b) (*i*), we find that it is satisfied if and only if the conditions (5.6), (5.7) and (5.10) bold, while the field equation (1.4) b) (*ii*) is satisfied when

(5.11) $\Delta(\partial\rho/\partial y - \partial\sigma/\partial x) = 0$ and $N = 0,$

Therefore wave solutions of the EINSTEIN's weak field equations (1.4) b) (*i*) and (1.4) b) (*ii*) are composed of g_{ij} given by (2.5) under the conditions (5.6), (5.7), (5.10) and (5.11) respectively.

6. Wave solutions of the field equations of BONNOR's non-symmetric unified theory. The first two of the field equations of above theory [5] are the same as the field equations (1.2) and (1.3) of EINSTEIN's unified field theory. Hence their solutions in the space - time (1.1) will be given by (2.5) and (4.1).

On substituting the values of g_{ij} and g^{ij} from (2.5) and (2.6) into (1.6), the non-vanishing components of U_{ik} are given by

$$(6.1) \quad U_{13} = - U_{31} = - U_{14} = U_{41} = - \rho - C(AU + DV),$$

$$U_{23} = - U_{32} = - U_{24} = U_{42} = - \sigma - C(DU + BV),$$

$$U_{33} = - U_{34} = U_{43} = U_{44} = - 2C(\rho U + \sigma V).$$

When the values of U_{ik} from (6.1) and $R_{\underline{ij}}$ with the help of (5.2) are substituted into the field equation (1.5) a), we find that it is satisfied under the condition

$$(6.2) \quad m\, M + 2p^2(B\rho^2 - 2D\rho\sigma + A\sigma^2) = 0$$

along with the conditions given by the equations (5.7) and (5.10), where M is given by (5.5). Therefore, the wave solutions of the field equation (1.5) a) is composed of g_{ij} given by (2.5) under the conditions (4.1), (5.7), (5.10) and (6.2).

Now substituting the values of $\underset{\vee}{R_{ij}}$ and $\underset{\vee}{U_{ik}}$ into the field equation (1.5) b) we find that it is satisfied if and only if

$$(6.3) \quad (\Delta + 4mp^2)(\partial\rho/\partial y - \partial\sigma/\partial x) = 0 \quad \text{and} \quad N = 0.$$

Thus g_{ij} given by (2.5) constitute the wave solutions of the field equation (1.5) b) under the conditions (4.1) and (6.3).

7. Solution of SCHRÖDINGER's field equations. On substituting the values of R_{ij} from (5.2) and g_{ij} from (2.5) into (1.7) b), we find that they cannot be satisfied and other equations are the same as in the first two field equations of EINSTEIN's and BONNOR's unified field theories, which have the solution (2.5) under the condition (4.1).

It is interesting to note that the wave solutions of the field equations of EINSTEIN's and BONNOR's non-symmetric unified field theories as found by H. TAKENO [2] and [10], LAL and ALI [3], and LAL and SRIVASTAVA [11] can easily be derived from the solutions obtained in this paper by taking the functional character of A, B, C, D, E, ρ and σ as assumed by them.

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ÖZET

Bu çalışmada EINSTEIN'in, BONNOR'un ve SCHRÖDINGER'in simetrik olmayan Birleşik Alan Teorileri incelenmekte, EINSTEIN ve BONNOR'un teorilerinin bazı şartlar altında dalga çözümleri verdikleri, fakat SCHRÖDINGER teorisinde bu tür çözümlerin mevcut olmadığı sonucu elde edilmektedir.