ELECTROMAGNETIC TENSOR FIELD OF FIRST AND SECOND CLASS

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The author and R.S. Mishra [1] established the identities in Nijenhuis tensor for first, second and third class of electromagnetic tensor fields. The identities for first class were non zero. In this paper I have obtained the identities which are zero for first class in tensor $M(X,Y)$. The identities for the second class have also been obtained in $M(X,Y)$ with respect to electromagnetic tensor field in four dimensional space-time.

Introduction Our operational space is the four-dimensional space-time V_4 of general relativity. Let F be the electromagnetic tensor field, which is vector valued linear function on V_4. Then F satisfies its own characteristic equation [2]

$$^{(4)}X + 2K \ ^{(2)}X + kX = 0 \tag{1.1}$$

for arbitrary X, where

$$^{(p)}X \overset{\text{def}}{=} F \ ^{(p-1)}X \, , \ ^{(0)}X \overset{\text{def}}{=} X \tag{1.2}$$

$$k \overset{\text{def}}{=} \det |F| \tag{1.3}$$

$$4K \overset{\text{def}}{=} - c_1^1 \ F(F) \, . \tag{1.4}$$

Let X be a vector field, then the operation by which $^{(1)}X = F(X)$ is obtained is called a F- operation. The electromagnetic field F is said to be of the

(i) First class if $Kk \neq 0$

(ii) Second class if $K \neq 0 \, , \, k = 0$

(iii) Third class (null field) if $K = 0 \, , k = 0 \, , {}^{(2)}X \neq 0$

(iv) Fourth class if $^{(2)}X = 0$.

The equation (1.1) is the lowest recurrence relation for first class and for second class it is given by [2]

$$^{(3)}X + 2K\ ^{(1)}X = 0\,. \tag{1.5}$$

Let D be the Riemannian connesion in V_4, then

$$[X,Y] = D_X\,Y - D_Y\,X \tag{1.6}$$

where $[X,Y]$ is a lie bracket.

Considering the tensor $M(X, Y)$

$$M(X,Y) \overset{\text{def}}{=} D_{{}^{(1)}X}\,{}^{(1)}Y + {}^{(2)}D_X\,Y - {}^{(1)}D_{{}^{(1)}X}\,Y - {}^{(1)}D_X\,{}^{(1)}Y \tag{1.7}$$

such that, the Nijenhuis tensor is expressed as

$$N(X,Y) = M(X,Y) - M(Y,X)\,. \tag{1.8}$$

Making use of the equation

$$D_{ax+b\bar{x}}\,cY + d\overline{Y} = ac\,D_X\,Y + ad\,D_X\,\overline{Y} + bc\,D_{\bar{X}}\,Y + bd\,D_{\bar{X}}\,\overline{Y} \tag{1.9}$$

the identities in M's are obtained.

2. SECOND CLASS

Theorem (2.1). The following identity holds for second class :

$$^{(2)}M\,({}^{(2)}X, {}^{(1)}Y) - {}^{(2)}M\,({}^{(1)}X, {}^{(2)}Y) = 0. \tag{2.1}$$

Proof. Using equations (1.5), (1.7) and (1.9) we get the result.

Theorem (2.2). We have the following identity for second class :

$$^{(2)}M\,({}^{(2)}X,Y) - 2K\,{}^{(1)}M\,({}^{(1)}X,Y) = 0. \tag{2.2}$$

Theorem (2.3). The following identity holds for second class :

$$^{(2)}M\,({}^{(1)}X,Y) + {}^{(1)}M\,({}^{(1)}X, Y) = 0. \tag{2.3}$$

Theorems (2.2) and (2.3) can be proved like theorem (2.1).

Note. By the repeated application of F operation to the vector fields X and Y we can have six different theorems from each of the theorems (2.2) and (2.3).

3. FIRST CLASS

Theorem (3.1). The following identities hold for first class :

$$ {}^{(3)}M\,({}^{(2)}X,Y) - {}^{(3)}M\,(X,{}^{(2)}Y) + {}^{(2)}M\,({}^{(3)}X,Y) - {}^{(2)}M\,(X,{}^{(3)}Y) - k\,\{M\,({}^{(1)}X,Y) - M\,(X,{}^{(1)}Y)\} = 0 \qquad (3.1)\text{a} $$

$$ {}^{(2)}M\,({}^{(3)}X,{}^{(2)}Y) - {}^{(2)}M\,({}^{(2)}X,{}^{(3)}Y) - k\,\{{}^{(1)}M\,(X,{}^{(2)}Y) + M\,({}^{(1)}X,{}^{(2)}Y) - {}^{(1)}M\,({}^{(2)}X\,Y) - M\,({}^{(2)}X,{}^{(1)}Y)\} = 0 \qquad (3.1)\text{b} $$

$$ {}^{(3)}M\,({}^{(2)}X,{}^{(1)}Y) - {}^{(3)}M\,({}^{(1)}X,{}^{(2)}Y) + {}^{(2)}M\,({}^{(3)}X,{}^{(1)}Y) - {}^{(2)}M\,({}^{(1)}X,{}^{(3)}Y) - k\,\{{}^{(1)}M\,(X,{}^{(1)}Y) - {}^{(1)}M\,({}^{(1)}X,Y)\} = 0 \qquad (3.1)\text{c} $$

Proof. Using equations (1.1), (1.7) and (1.9) we get the result.

Theorem (3.2). We have the following identity for first class :

$$ {}^{(3)}M\,({}^{(2)}X,Y) + {}^{(2)}M\,({}^{(3)}X,Y) - k\,\{{}^{(1)}M\,(X,Y) + M\,({}^{(1)}X,Y)\} = 0. \qquad (3.2) $$

Proof. Making use of (1.1), (1.7) and (1.9) we get the result.

Note. By the repeated application of F-operation to the vector field X and Y, we can have six different theorems like (3.2).

REFERENCES

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ÖZET

Yazar ve R.S. Mishra bundan önceki bir çalışmada [1], birinci, ikinci ve üçüncü sınıf elektromanyetik tansör alanları için Nijenhuis tansörü cinsinden özdeşlikler vermişlerdi. Birinci sınıfa ait özdeşlikler sıfırdan farklı idiler. Bu çalışmada $M(X,Y)$ tansörü cinsinden birinci sınıf için sıfır olan özdeşlikler bulunduğu gibi, $M(X,Y)$ cinsinden dört boyutlu uzay-zamandaki elektromanyetik tansör alanına göre ikinci sınıf için de özdeşlikler verilmiştir.