SOME STATIC AND TIME DEPENDENT SOLUTIONS OF ZERO MASS MESON FIELD EQUATIONS

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The present authors have taken up investigation of some static and time dependent solutions of

$$G_{ij} \equiv R_{ij} - \frac{1}{2} g_{ij} R = -8\pi T_{ij}$$

and

$$g^{ij} V_{;ij} = 0$$

for the case of a metric used by Kompaneets.

1. Introduction. Recently the solutions of Einstein's field equations containing zero-mass meson fields have been subject to investigation by various authors. In particular, Janis, Newman and Winicour [1] considered spherically symmetric solution of these field equations with a view to present an extension of Israel's [2] idea of singular event horizons. Buchdahl [3] has constructed reciprocal static solutions for axially and spherically symmetric fields and studied the physical interpretation of these solutions. Penny [4] and Gautreau [5] have further extended the study to the case of axially symmetric static fields and have found that the scalar field obeys a flat space Laplace equation such that a large class of solutions exist. Exact cylindrical wave solutions of Einstein's field equations containing zero-rest-mass scalar fields have been discussed by Lai and Singh [6]. Recently plane symmetric zero-mass meson solutions of Einstein's equations have been presented by Patel [7].

The field equations of general relativity containing zero-mass meson fields are given by

$$G_{ij} \equiv R_{ij} - \frac{1}{2} g_{ij} R = - 8\pi T_{ij} , \tag{1.1}$$

$$g^{ij} V_{;ij} = 0 , \tag{1.2}$$

where V is the scalar field having zero-rest-mass and T_{ij}, the energy momentum tensor of this field is given by

$$T_{ij} = V_{,i} V_{,j} - \frac{1}{2} g_{ij} V_{,1} V_{,m} g^{1m} . \tag{1.3}$$

A semicolon indicates covariant differentiation and a comma followed by an index i denotes partial differentiation with respect to x^i. The present authors in this paper have taken up investigation of some static and time dependent solutions of (1.1) and (1.2) for the case of a metric used by Kompaneets [8]. Our investigation also includes a static solution of coupled Einstein-Maxwell-scalar fields. The solutions in the static case have been shown to exhibit the results obtained by Patel [7] and Taub [9] as particular cases. As regards the time dependent case, a method has been given by which one can obtain, under certain conditions, solutions of the field equations (1.1) and (1.2) from known cylindrically symmetric solutions of the empty space field equations of Einstein's theory of gravitation. It is also found that one of the solutions of the zero-mass meson field equations is non singular in the sense of Bonnor.

2. Metric and the field equations. We consider a space time whose geometry is defined by the metric [8]

$$ds^2 = -A(dx^1)^2 - C(dx^2)^2 - D(dx^3)^2 - 2Bdx^2 dx^3 + A(dx^4)^2, \tag{2.1}$$

where A, B, C, D are functions of x^1 and x^4 only and x^i, $i = 1,2,3$, denote space coordinates whereas x^4 corresponds to time coordinate t. If $B = 0$, (2.1) corresponds to the cylindrically symmetric Einstein-Rosen metric and if in addition $C = D$, it refers to plane symmetric metric of Taub [9].

The non vanishing components of the Ricci tensor R_{ij} corresponding to (2.1) are found to have the following values :

$$\begin{aligned} R_{11} &= \alpha_{11}/2\alpha - \alpha_1^2/4\alpha^2 + (L_{11} - L_{44}) + (B_1^2 - C_1 D_1)/2\alpha - (A_1 \alpha_1 + A_4 \alpha_4)/4A\alpha, \\ R_{44} &= \alpha_{44}/2\alpha - \alpha_4^2/4\alpha^2 - (L_{11} - L_{44}) + (B_4^2 - C_4 D_4)/2\alpha - (A_1 \alpha_1 + A_4 \alpha_4)/4A\alpha, \end{aligned} \tag{2.2}$$

$$R_{14} = \alpha_{14}/4\alpha - \alpha_1\alpha_4/4\alpha^2 + (2B_1 B_4 - C_1 D_4 - C_4 D_1)/4\alpha - (A_1\alpha_4 + A_4\alpha_1)/4A\alpha,$$

$$R_{22} = (2A)^{-1} (C; \alpha, P),\ R_{33} = (2A)^{-1} (D; \alpha, P),\ R_{23} = (2A)^{-1} (B; \alpha, P),$$

where the notations used are as follows:

$$(C\,; \alpha, P) = [C_{11} + C_{44} - (2\alpha)^{-1}] \{C_1\alpha_1 - C_4\alpha_4 + 2CP\} , \ldots \text{etc} ,$$

$$P \equiv (B_1^2 - B_4^2 - C_1 D_1 - C_4 D_4),$$

and we have used

$$A = e^{2L} , \alpha \equiv (CD - B^2) . \tag{2.3}$$

A simple calculation shows that (1.1) together with (1.3) yields, on contraction,

$$R = -8\pi V_{,i} V^{,i} ,$$

consequently (1.1) takes the form

$$R_{ij} = -8\pi V_{,i} V_{,j} . \tag{2.4}$$

Taking the scalar field V as a function of x^1 and x^4 and using (2.1), equations (1.2) and (2.4) simplify to

$$V_{11} - V_{44} + (\alpha_1/2\alpha) V_1 - (\alpha_4/2\alpha) V_4 = 0 , \tag{2.5}$$

$$R_{11} = -8\pi V_1{}^2 , R_{44} = -8\pi V_4{}^2 , R_{14} = -8\pi V_1 V_4 , R_{22} = R_{33} = R_{23} = 0, \tag{2.6}$$

where $V_i = V_{,i}$ and the lower suffixes 1 and 4 after unknown functions correspond to partial differentiation with regard to x^1 and x^4 respectively. Using (2.6), multiplying R_{33} by D, R_{33} by C and R_{23} by - $2B$ and adding, in view of α being expressed as in (2.3), we obtain

$$(\sqrt{\alpha})_{11} - (\sqrt{\alpha})_{44} = 0 . \tag{2.7}$$

3. A static solution. In this section we restrict ourselves to the static case. Thus assuming that the unknown functions involved in the above equations are independent of x^4 and depend on x^1 only and substituting the values of R_{ij} from (2.2), equations (2.5), (2.6) finally simplify to

$$V'' - V' \alpha'/2\alpha = 0, L'' - L'\alpha'/2\alpha - (B'^2 - C'D')/2\alpha = -8\pi V'^2 ,$$

$$L'' + L'\alpha'/2\alpha = 0, C'' - C'\alpha'/2\alpha - (C/\alpha) Q = 0, D'' - D'\alpha'/2\alpha - (D/\alpha) Q = 0, \tag{3.1}$$

$$B'' - B'\alpha'/2\alpha - (B/\alpha) Q = 0,$$

where $Q = (B'^2 - C'D')$ and a dash overhead indicates differentiation with respect to x^1.

Equation (2.7) for the static case yields a solution

$$\alpha = (k_1 x^1 + k_2)^2. \tag{3.2}$$

On the other hand equations (3.1) by virtue of (3.2) exhibit on integration

$$V = (k_3/k_1) \log (k_1 x^1 + k_2) + k_4 , L = (k_5/k_1) \log (k_1 x^1 + k_2) + k_6,$$

$$C e^{-k} = D e^k = (k_1 x^1 + k_2) \operatorname{Cosh} \{(k_7/k_1) \log \{(k_1 x^1 + k_2)\}, \tag{3.3}$$

$$B = (k_1 x^1 + k_2) \operatorname{Sinh} \{(k_7/k_1) \log (k_1 x^1 + k_2)\},$$

where k and all k_i's are constants of integration and k_1, k_3, k_5 and k_7 are related as

$$k_1{}^2 + 4 k_1 k_5 = 16\pi k_3{}^2 + k_7{}^2 , \qquad k_1 \neq 0. \tag{3.4}$$

Thus (3.3) and (3.4) along with (2.1) and (2.3) characterize a static solution of (1.1) and (1.2). However, on analyzing this solution following interesting cases arise.

(*i*) If $k_4 = k_7 = 0$, (3.3) reveals that $B = 0$, $C = e^{2k} D$ which by a suitable coordinate transformation can be reduced to $C = D$. In this case the metric (2.1) transforms to static plane symmetric metric of Taub, and the corresponding solution is given by (2.1), where

$$\begin{aligned} L &= 1/2 \log A = (k_5/k_1) \log (k_1 x^1 + k_2) + k_6, \\ C &= D = (k_1 x^1 + k_2),\ B = 0, \end{aligned} \tag{3.5a}$$

and the scalar field V is expressed in the form

$$V = \sqrt{(k_1 + 4k_5)/16\pi k_1} \log (k_1 x^1 + k_2) + k_4. \tag{3.5b}$$

Thus (3.5a) and (3.5b), together with the metric (2.1), on proper identification with static plane symmetric metric of Taub, describe the solution of zero-mass meson fields presented by Patel [7].

(*ii*) If $k_4 = k_5 = k_6 = k_7 = 0$, ,in view of (3.3), (3.4) and the discussions held in (i), we have $k_5/k_1 = -1/4$, so that the corresponding solution is obtained as

$$A = (k_1 x^1 + k_2)^{-1/2},\ C = D = (k_1 x^1 + k_2),\ B = 0. \tag{3.6}$$

Thus the metric (2.1) together with (3.6) characterizes an empty space-time discussed by Taub [9].

4. A static solution of the coupled Einstein-Maxwell equations. Following the method described by Pandey [10], we can construct a solution of coupled Einstein-Maxwell equations containing zero-mass meson fields from the solutions we have obtained in Sec. 3. Pandey has shown that if a solution of the field equations containing zero-mass meson field is given by the metric

$$ds^2 = e^{2v}(dx^4)^2 - e^{-2v} h_{ij}\, dx^i\, dx^j \qquad (i, j = 1,2,3), \tag{4.1}$$

then the metric

$$ds^2 = e^{2w} (dx^4)^2 - e^{-2w} h_{ij}\, dx^i\, dx^j, \tag{4.2}$$

where v, w and h_{ij} are functions of x^i and $e^w = \lambda \operatorname{Sech} v$, λ being a constant, describes a static solution of the field equations representing coupled gravitational, electromagnetic and zero-mass meson fields. Also the electromagnetic field tensor F_{ij} is given by

$$F_{4j} = U_{,j}, \text{ where } U = \lambda \operatorname{Tanh} v. \tag{4.3}$$

This electromagnetic field is nonvanishing.

It is easy to witness that the metric (2.1) in view of (2.3) can be expressed as

$$\begin{aligned} ds^2 = e^{2L} (dx^4)^2 - e^{-2L} [e^{4L}(dx^1)^2 + e^{2L} \{C (dx^2)^2 + \\ + 2B dx^2 dx^3 + D (dx^3)^2\}]. \end{aligned} \tag{4.4}$$

As already discussed in Sec. 3, the metric (4.4) with L, B, C, D and V given by (3.3) and (3.4) describes a static solution of the zero-mass meson field equations and is of the form (4.1). Therefore applying the technique mentioned above, we can obtain a solution of the coupled Einstein-Maxwell equations. The geometry of the solution thus obtained is described by the metric

$$ds^2 = e^{2w}(dx^4)^2 - e^{-2w}\,[e^{4L}(dx^1)^2 + + e^{2L}\{C(dx^2)^2 + 2B dx^2\,dx^3 + D(dx^3)^2\}], \tag{4.5}$$

where $e^w = \lambda$ Sech L and L, B, C, D and V are given by (3.3) and (3.4). It is evident from (4.3) that the only surviving component of F_{ij} is obtained as

$$F_{41} = (\lambda \tanh L)_{,1} = \lambda k_5\,(k_1 x^1 + k_2)^{-1} \times \times \text{Sech}^2\,\{k_5/k_1 \log\,(k_1 x^1 + k_2) + k_6\}. \tag{4.6}$$

If we take $k_4 = k_7 = 0$ in (3.3), (4.5) and (4.6), we easily observe in the light of the discussion held in (i) Sec. 3, that the solution of the coupled Einstein-Maxwell fields is given by the metric

$$ds^2 = e^{2w}(dx^4)^2 - e^{-w}\,[e^{4L}(dx^1)^2 + e^{2(L+M)}\,\{(dx^2)^2 + (dx^3)^2\}], \tag{4.7}$$

where

$$e^w = \lambda \text{ Sech } L,\; L = k_5/k_1 \log\,(k_1 x^1 + k_2) + k_6,\; C = D = e^{2M} = (k_1 x^1 + k_2), \tag{4.8}$$

$$V = k_3/k_1 \log(k_1 x^1 + k_2),\; F_{41} = \lambda k_5 (k_1 x^1 + k_2)^{-1} \times \times \text{Sech}^2\,\{(k_5/k_1) \log\,(k_1 x^1 + k_2) + k_6\}. \tag{4.9}$$

Thus (4.7), (4.8) and (4.9) describe the static plane symmetric solution of the coupled Einstein-Maxwell equations obtained by Patel [7].

5. Some time dependent solutions of (1.1) and (1.2). We now consider the equations (2.5) - (2.7) when $A, B, C, D,$ and V are functions of x^1 and x^4. Following the method used by Einstein-Rosen [11] we choose a coordinate system given by

$$\overline{x}^i = \overline{x}^i\,(x^1, x^4),\; i = 1, 4 \quad \text{and} \quad \overline{x}^j = x^j,\; j = 2{,}3. \tag{5.1}$$

It is easy to see that the metric (2.1) remains form invariant under the transformation (5.1) if $\overline{x}^1$ and $\overline{x}^4$ satisfy

$$\begin{aligned} &\text{a)}\quad \partial_1 \overline{x}^1 = \partial_4\, \overline{x}^4, \\ &\text{b)}\quad \partial_1 \overline{x}^4 = \partial_4 \overline{x}^1 \quad (\partial_i \equiv \partial/\partial x^i) \end{aligned} \tag{5.2}$$

or that $\overline{x}^1$ may be choosen arbitrarily to satisfy

$$\partial_{11}\bar{x}^1 - \partial_{44}\bar{x}^1 = 0, \tag{5.3}$$

whereas $\bar{x}^4$ is to be determined from (5.2). On the other hand α must satisfy (2.7). Hence, comparing (2.7) with (5.3), we can take

$$\sqrt{\alpha} = \bar{x}^1. \tag{5.4}$$

Then $\bar{x}^4$ is obtained from (5.1).

Thus there exists a transformation (5.1) associated with (5.2) and (5.4) for which the metric (2.1) readjusts to

$$ds^2 = -\bar{A}(d\bar{x}^1)^2 - C(dx^2)^2 - D(dx^3)^2 - 2Bdx^2\,dx^3 + A\,(d\bar{x}^4)^2, \tag{5.5}$$

where $A = \bar{A}\,[(\partial_1\bar{x}^1)^2 - (\partial_1\bar{x}^4)^2]$. Consequently equations (2.5)-(2.7) with R_{ij} given by (2.2) suffer only in A being replaced by $\bar{A}$, as a result L, in view of (2.3) being changed to $L = 1/2 \log \bar{A}$. Therefore using (5.4) equations (2.5)-(2.7) in the new coordinate system after little simplification reduce to (omitting bars now for the sake of simplicity)

$$V_{11} - V_{44} + V_1/x^1 = 0, \tag{5.6}$$

$$(x^1)^{-1} L_1 - (2x^1)^{-2}(B_1{}^2 + B_4{}^2 - C_1D_1 - C_4D_4) = 4\pi\,(V_1{}^2 + V_4{}^2), \tag{5.7}$$

$$(x^1)^{-1} L_4 - (2x^1)^{-2}(2B_1\,B_4 - C_1D_4 - C_4D_1) = 8\pi V_1\,V_4, \tag{5.8}$$

$$(C\,;P) = (D\,;P) = (B\,;P) = 0, \tag{5.9}$$

where we have used the notations as follows :

$$(Y\,;P) = [Y_{11} - Y_{44} - Y_1/x^1 - Y/(x^1)^2\,P]\,;\,P \equiv (B_1{}^2 - B_4{}^2 - C_1D_1 + C_4D_4),$$

and the suffixes 1 and 4 correspond to partial differentiation with respect to new x^1 and x^4 coordinates. First we solve the three equations given by (5.9). Thus taking

$$B = (CD - (x^1)^2)^{1/2}, \tag{5.10}$$

as obtained from (2.3) and (5.4) and introducing two new variables given by

$$\sigma = C\,(CD - (x^1)^1)^{-1/2},\ \delta = D\,(CD - (x^1)^2)^{-1/2}, \tag{5.11}$$

the equations (5.9) resume the form

$$\begin{aligned}\sigma_{11} - \sigma_{44} + \sigma_1/x^1 - (\sigma\delta - 1)^{-1}[\sigma(\sigma_1\,\delta_1 - \sigma_4\,\delta_4) + \delta\,(\sigma_1{}^2 - \sigma_4{}^2)] = 0,\\ \sigma_{11} - \sigma_{44} + \delta_1/x^1 - (\sigma\delta - 1)^{-1}[\delta\,(\sigma_1\,\delta_1 - \sigma_4\,\delta_4) + \sigma\,(\delta_1{}^2 - \delta_4{}^2) = 0.\end{aligned} \tag{5.12}$$

Equations (5.12) correspond to two non-linear interacting cylindrical waves. It is difficult to get the most general solution of the equation (5.12). However we make some simplifying assumptions. As such we assume that

$$\sigma = e^{\beta} f(\psi),\ \delta = e^{-\beta} f(\psi), \tag{5.13}$$

where β is a constant and ψ is a function satisfying

$$\psi_{11} - \psi_{44} + \psi_1/x^1 = 0. \tag{5.14}$$

Equations (5.12) in view of (5.13) and (5.14) reduce to

$$[f'' - 2ff'/(f^2 - 1)]\,(\psi_1{}^2 - \psi_4{}^2) = 0, \tag{5.15}$$

where a dash overhead denotes differentiation with respect to ψ. In view of (5.15) we have two cases :

$$\text{a)}\quad \psi_1{}^2 - \psi_4{}^2 = 0 \quad \text{or} \quad \text{b)}\quad f'' - 2ff'/(f^2 - 1) = 0. \tag{5.16}$$

By inspecting (5.16) a) it is evident that ψ is function of $(x^1 - x^4)$ or $(x^1 + x^4)$, which in combination with (5.14) implies that ψ is a constant. This is a trivial case.

On the other hand (5.16) b) on integration yields $f = \text{Coth}\,(K\psi)$, where K is a constant. Consequently (5.10), (5.11) and (5.13) in this case present the values of B, C, D in the form

$$C = x^1\, e^{\beta}\, \text{Cosh}\,(K\psi)\,,\; D = x^1\, e^{-\beta}\, \text{Cosh}\,(K\psi)\,,\; B = x^1\, \text{Sinh}\,(K\psi), \tag{5.17}$$

where ψ is given by (5.14). Applying (5.17) two of the remaining equations, i.e. (5.7) and (5.8) transform to

$$\begin{aligned} L_1 &= (K^2/4)\, x^1\, (\psi_1{}^2 + \psi_4{}^2) + 4\pi\,(V_1{}^2 + V_4{}^2) - 1/4\,(x^1)^{-1}, \\ L_4 &= (K^2/2)\, x^1\, \psi_1\, \psi_4 + 8\pi x^1\, V_1\, V_4\,. \end{aligned} \tag{5.18}$$

Equations (5.6) and (5.14) which determine V and ψ respectively, serve as the integrability condition of (5.18).

There are infinite number of possible combinations of V and ψ that can be used to integrate the equation (5.18) and thus to generate a solution of the zero-mass meson field equation. However, if we restrict ourselves to the case when V and ψ are functionally related then (5.6) and (5.14) yield

$$\psi = a\,V + b, \tag{5.19}$$

where a and b are constants. In view of (5.19) equation (5.14) reduces to (5.6) which determines the form of the scalar field V. Now making the substitution

$$L + (1/4)\log x^1 = M, \quad \text{and} \quad V = \theta/d, \tag{5.20}$$

where $d^2 = (Ka^2/2 + 8\pi)$ is a constant, equation (5.6) and (5.18) resume the form

$$\theta_{11} - \theta_{44} + \theta_1/x^1 = 0, \tag{5.21}$$

$$M_1 = (1/2)\, x^1\, (\theta_1{}^2 + \theta_4{}^2), \tag{5.22}$$

$$M_4 = x^1\, \theta_1\, \theta_4\,. \tag{5.23}$$

The integrability condition of (5.22) and (5.23) is satisfied by virtue of (5.21). Hence whenever ψ is known from (5.21), V is determined from (5.20), B, C, D

and L are obtained from (5.17), (5.2θ), (5.22) and (5.23) in terms of ψ, which can be written in the form

$$\psi = \left(a \,/\, \sqrt{K(a^2/2) + 8\pi}\right) \theta + b, \qquad (5.24)$$

where θ is given by (5.21).

Consider now the cylindrically symmetric Einstein-Rosen metric [11]

$$ds^2 = e^{M-\theta} \left[(dx^4)^2 - (dx^1)^2\right] - (x^1)^2 e^{-\theta} (dx^2)^2 - e^{\theta} (dx^3)^2, \qquad (5.25)$$

where M and θ are functions of x^1 and x^4, x^i $(i = 1,2,3,4)$ correspond to r, ϕ, z, t. The empty space field equations of Einstein's theory corresponding to (5.25) reduce to (5.21), (5.22) and (5.23). Thus we established the following result :

"For every solution of (5.21) - (5.23) corresponding to the Einstein-Rosen metric (5.25) and representing empty space-time in Einstein's theory of gravitation, we have a solution for a more general case given by (2.1), (5.17), (5.2θ), (5.24), θ and M remaining the same, which represent coupled gravitational and zero-mass meson fields.

6. A non-singular solution of (1.1) and (1.2). Einstein and Rosen [11] and Rosen [12] have obtained solutions of the wave equations of the type (5.21) corresponding to progressive or stationary gravitational waves. These solutions contain singularity along the axis of z, presumably representing the source of the gravitational waves. Later Bonnor [13] obtained a non-singular solution of (5.21) by adopting the procedure applied by Synge [14]. Bonnor has shown that equations (5.21) - (5.23) have a non-singular solution given by

$$\begin{aligned}\theta &= 2\sqrt{2}\, c \left(p + \sqrt{p^2 + q^2}\right)^{1/2} (p^2 + q^2)^{-1/2}, \\ M &= -2c^2 (x^1)^2 (p^2 - q^2)(p^2 + q^2)^{-2} + \\ &\quad + (c^2/m^2)\left[\{(x^1)^2 - (x^4)^2 - m^2\}(p^2 + q^2)^{-1/2} + 1\right],\end{aligned} \qquad (6.1)$$

where $p = (x^1)^2 - (x^4)^2 + m^2$, $q = 2\,mx^4$, c and m being arbitrary constants. Therefore, corresponding to a non-singular solution of Einstein's empty space field equations given by (5.25) and (6.1), we have a solution for the more general case given by (2.1), (5.17), (5.19), (5.2θ) and (6.1), which is also non-singular in the sense of Bonnor [13].

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REFERENCES

[1] JANIS, A.I., NEWMAN, E.T. and WINICOUR, J. : Reality of the Schwarzschild singularity, Phys. Rev. Letters, **29** (1968), 878-880.

[2] ISRAEL, W. : Event horizons in static vacuum space-times, Phys. Rev., 164 (1967), 1776.

[3] BUCHDAHL, H.A. : Reciprocal static metrics and scalar fields in the general theory of relativity, Phys. Rev., **115** (1959), 1325-28.

[4] PENNY, R. : Axially symmetric zero-mass meson solutions of Einstein's equations, Phys. Rev., 174 (1968), 1578-1579.

[5] GAUTREAU, R. : Coupled Weyl gravitational and zero-rest-mass scalar fields, Nuovo Cem., B **62** (1969), 360-370.

[6] LAL, K.B. and SINGH, T. : Cylindirical wave solutions of Einstein's field equations of general relativity containing zero-rest-mass scalar fields I, Tensor, N.S., 27 (1973), 211-216.

[7] PATEL, L.K. : Plane-symmetric zero-mass meson solutions of Einstein's equations, Tensor, N.S., **29** (1975), 237-240.

[8] KOMPANEETS, A.S. : Propagation of strong electromagnetic gravitational wave in vacuo, Soviet Physics, JETP., **37** (1960), 1218-1220.

[9] TAUB, A.H. : Empty space-times admitting a three parameter group of motions, Ann. Math., **53** (1951), 472-490.

[10] PANDEY, D.B. : Contributions to the problems of axisymmetric fields in general relativity, Ph. D. Thesis, University of Gorakhpur, India (1970).

[11] EINSTEIN, A. and ROSEN, N. : On gravitational waves, J. Franklin Inst., **223** (1937), 43-54.

[12] ROSEN, N. : Some cylindrical gravitational waves, Bull. Res. Counc. Israel, **3** (1954), 328-332.

[13] BONNOR, W.B. : Non-singular fields in general relativity. J. Math. Mech., **6** (1957), 203-214.

[14] SYNGE, J.L. : Relativity-the special theory (North Holland, Amsterdam, 1956), Chap. IX, pp. 358-61.

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ÖZET

Bu çalışmada

$$G_{ij} \equiv R_{ij} - \frac{1}{2} g_{ij} R = -8\pi T_{ij}$$

ve

$$g^{ij} V_{;ij} = 0$$

denklemlerinin, Kompaneets tarafından kullanılan bir metrik altında istikrarlı ve zamana bağlı bazı çözümleri incelenmektedir.