ON WAVE SOLUTIONS OF GAUGE INVARIANT GENERALIZATION OF FIELD THEORIES WITH ASYMMETRIC FUNDAMENTAL TENSOR IN A GENERALIZED PERES SPACE-TIME

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Lal and Pandey [5] [1]) have investigated the solutions of non-symmetric unified field theories in a generalized Peres space-time. In this paper we have considered the gauge invariant generalization of Einstein's field equations as given by Buchdahl ([8], [4]) in a generalized Peres space-time and it has been found that under certain conditions the solutions exist.

1. Introduction. By combining the field theories of Weyl [1] and Einstein [2], Buchdahl ([3], [4]) introduced another geometry based on an asymmetric covariant tensor g_{ij} and covariant vector K_i, relating these to an asymmetric linear connection Γ^i_{jk} in such a way that the geometry could be regarded equivalently as 'Gauge invariant generalization of Einstein's theory'.

Buchdahl's gauge invariant generalization of field theories ([3], [4]) are based upon an asymmetric tensor $g_{ij} = h_{ij} + f_{ij}$ (where h_{ij} is the symmetric part of g_{ij} which is coincident with the fundamental tensor of the metric space and f_{ij} is the skew-symmetric part of g_{ij}), a covariant vector k_i and a linear connection Γ^i_{jk} defined by

$$g_{lj} L^l_{lk} + g_{il} L^l_{kj} - g_{ij,k} = 0 \quad (,k = \partial_k = \partial/\partial x^k\,,\ x^k \equiv x, y, z, t)\,, \tag{1.1}$$

$$g_{lj} V^l_{ik} + g_{il} V^l_{kj} - g_{ij} K_k = 0\,, \tag{1.2}$$

$$\Gamma^i_{jk} = L^i_{jk} - V^i_{jk}\,, \tag{1.3}$$

where L^i_{jk} is the linear connection of Einstein's non-symmetric theory [2]. The indices i, j, k take the values 1, 2, 3, 4; a comma (,) before an index i denotes its partial differentiation with respect to x^i. The simplest field equations based upon this theory, as given by Buchdahl [4], are

[1]) Numbers in brackets refer to the references at the end of the paper.

$$\Gamma_i \equiv \Gamma^i_{\underset{\vee}{ij}} = 0\,, \tag{1.4}$$

$$G_{ij} = N_{ij} + \overset{k}{V^0}_{\underset{+-}{ij;k}} - (K_{\underset{+}{i;j}} + K_{\underline{j;i}}) + V^k{}_{il}\, V^l{}_{kj} - 2\, V^k{}_{ij}\,,\ \ K_k = 0\,, \tag{1.5}$$

where

$$N_{ij} = -\,L^k{}_{ij,k} + \frac{1}{2}\,(L^k{}_{\underline{ik},j} + L^k{}_{\underline{kj},i}) + L^k{}_{il}\, L^l{}_{kj} - L^k{}_{ij}\, L^l{}_{\underline{kl}}\,, \tag{1.6}$$

and a semi-colon (;) denotes covariant differentiation with respect to $L^i{}_{jk}$. A '+', '−' or '0' sign below an index fixes the position of covariant index k in connection as $L^{\cdot}{}_{\cdot k}$, $L^{\cdot}{}_{k\cdot}$, $L^{\cdot}{}_{\cdot\underline{k}}$ and a bar (—) and a hook (v) below the indices denote the symmetry and skew symmetry respectively between those indices. Tensor G_{ij} and N_{ij} are the hermitian tensor of gauge invariant and Einstein's usual asymmetric theory respectively.

Lal and Pandey considered Einstein's field equations in a generalized Peres space-time [5], given by

$$ds = -\,A\, dx^2 - B\, dy^2 - (1 - E)\, dz^2 - 2E\, dzdt + (1 + E)\, dt^2, \tag{1.7}$$

where $A = A\,(z, t)$, $B = B\,(z, t)$, and $E = E\,(x, y, z, t)$.

The non-symmetric g_{ij} [5] is given by

$$g_{ij} = \begin{bmatrix} -A & 0 & -\rho & -\rho \\ 0 & -B & \sigma & -\sigma \\ -\rho & -\sigma & -(1-E) & -E \\ \rho & \sigma & -E & (1+E) \end{bmatrix}, \tag{1.8}$$

where ρ, σ are functions of x, y, $z - t$.

The wave solutions of field equations of Buchdahl given by (1.1) - (1.5), have been investigated earlier in a $V_2 \times V_2$ space-time by Lai and Khare [6] and in a cyiindrically symmetric space-time by Lai and Singh [7] and the solutions have been found to exist under certain conditions. In this paper we propose to consider the field equations (1.1)-(1.5) in the generalized Peres space-time (1.7) with the assumptions

$$A = A\,(z - t)\,,\, B = B\,(z - t)\,. \tag{1.9}$$

Equation (1.1) is the first equation of Einstein's unified field theory and it has been solved in [5]. In what follows, the components of $L^i{}_{jk}$ are obtained from those given in [5] by using (1.9), which together with $V^i{}_{jk}$ give connection coefficients in gauge invariant theory.

2. Solution of the Field Equation (1.2). We shall use Hlavatý's [8] method to solve equation (1.2). Mishra [9] has proved that $V^k{}_{ij}$ can be put in the form

$$V^k{}_{ij} = H^k{}_{ij} + S^k{}_{ij} + U^k{}_{ij}\,, \tag{2.1}$$

where

$$U^k{}_{ij} = 2\, h^{km} \quad S^l{}_{m\,(i}{}^f{}_{j)\,l}\,, \tag{2.2}$$

$$H^k{}_{ij} = \frac{1}{2}\, h^{kl}\,(K_i\, h_{jl} + K_j\, h_{il} - K_l\, h_{ij})\,, \tag{2.3}$$

$$S^k{}_{ij} = V^k{}_{[ij]} = h^{kl}\,(K'{}_{ijl} + 2\; U_l{}^m{}_{[i}{}^f{}_{j]m})\,, \tag{2.4}$$

$$K'{}_{ijk} = K_{ijk} + 2 f_{n\,[i}\, H_j{}^n{}_{]\,k}\,, \tag{2.5a}$$

$$K_{ijk} = \frac{1}{2}\,(K_j f_{ik} + K_k f_{ij} - K_i f_{jk})\,. \tag{2.5b}$$

Since f_{ij} obtained from (1.8) are of class third in the sense of Hlavatý [8], we have solution of (2.4) as

$$S^k{}_{ij} = h^{kl}\,(K'{}_{ijl} - 2 f^m{}_{[j}\, K_i'{}_{]mn} f_l{}^n)\,. \tag{2.6}$$

Using (1.8) in equations (2.3) and (2.5b), we get the components of $H^k{}_{ij}$ $(= H^k{}_{ji})$ and K_{ijl} $(= -K_{jil})$ as follows :

$$\begin{aligned}
&H^k{}_{11}\,[K_1/2,\; -K_2A/2B,\; (K_3\,A/2 - EA\,(K_3 + K_4)/2)\,,\\
&\qquad (K_4A/2 - E\,A\,(K_3 + K_4)/2)],\\
&H^k{}_{22}\,[-K_1B/2A,\; K_2/2,\; (-K_3B/2 - EB\,(K_3 + K_4)/2)\,,\\
&\qquad (K_4B/2 - EB\,(K_3 + K_4)/2)]\,,\\
&H^k{}_{33}\,[-K_1\,(1 - E)/2,\; -K_2(1 - E)/2B,\; (K_3/2 - K_4E/2 + E^2\,(K_3 + K_4)/2)\,,\\
&\qquad (K_4/2 - K_3E/2 - K_4\,E + E^2\,(K_3 + K_4)/2)]\,,\\
&H^k{}_{44}\,[K_1\,(1 + E)/2,\; K_2\,(1 + E)/2,\; (K_3/2 + K_3E + K_4E/2 + E^2\,(K_3 + K_4)/2)\,,\\
&\qquad (K_4/2 + K_3E/2 + E^2\,(K_3 + K_4)/2)]\,,\\
&H^k{}_{12}\,[K_2/\,2,\; K_1/\,2,\; 0,\; 0]\,,
\end{aligned} \tag{2.7}$$

similar expressions for $H^k{}_{13}$, $H^k{}_{14}$, $H^k{}_{23}$, $H^k{}_{24}$, $H^k{}_{34}$ are omitted for brevity's sake, where k takes the values 1, 2, 3, 4. And

$$\begin{aligned}
&K_{131} = -K_{141} = \rho K_1,\; K_{232} = -K_{242} = \sigma K_2\,,\\
&K_{342} = -\sigma(K_3 + K_4)/2,\; K_{132} = -K_{142} = K_{231} = -K_{241} = (\sigma K_1 + \rho K_2)/2\,,\\
&K_{133} = \rho K_3,\; K_{341} = -\rho(K_3 + K_4)/2,\; K_{233} = \sigma K_3\,,\\
&K_{134} = K_{143} = \rho(K_4 - K_3)\,/\,2,\; K_{234} = K_{243} = \sigma(K_4 - K_3)/2\,,\\
&K_{123} = -K_{124} = (\rho K_2 - \sigma K_1)/2,\; K_{144} = -\rho K_4\,,\\
&K_{244} = -\sigma K_4/2\,.
\end{aligned} \tag{2.8}$$

Using equations (1.8), (2.7) and (2.8) in (2.5a), we find the values of $K'_{ijk} (= - K'_{jik})$ as follows :

$$\begin{aligned}
&K'_{131} = - K'_{141} = (\sigma K_2 A/2B), K'_{232} = - K'_{242} = (\rho K_1 B/2A) , \\
&K'_{143} = - K'_{144} = (\rho K_3/2 + \rho E (K_3 + K_4)/2) , \\
&K'_{243} = - K'_{244} = (\sigma K_3/2 + \sigma E (K_1 + K_4)/2) , \\
&K'_{133} = - K'_{134} = (\rho K_4/2 - \rho E (K_3 + K_4)/2) , \\
&K'_{323} = - K'_{324} = (- \sigma K_4/2 + \sigma E (K_3 + K_4)/2) , \qquad (2.9)\\
&K'_{343} = - K'_{344} = (- \rho K_1/2A - \sigma K_2/2B) , \\
&K'_{132} = - K'_{142} = (\sigma K_1 - \rho K_2/2) , K'_{231} = - K'_{241} = (\rho K_2 - \sigma K_1/2) , \\
&K'_{121} = - \sigma(K_3 + K_4) A/2, K'_{212} = - \rho B (K_3 + K_1)/2 , \\
&K'_{341} = K'_{342} = K'_{123} = K'_{124} = 0 .
\end{aligned}$$

Substituting from equations (1.8) and (2.9) in (2.6) we find that the last term on the right hand side of equation (2.6) is identically zero and equation (2.6) reduces to

$$S^k{}_{ij} = h^{kl} K'_{ijl} . \qquad (2.10)$$

Equation (2.2) reduces to

$$U^k{}_{ij} = 0 \qquad (2.11)$$

with the help of equations (1.8), (2.9) and (2.10).

Thus we have solved equations (2.2), (2.3) and (2.4) in terms of g_{ij} and K_i. Using equations (2.7), (2.9), (2.10) and (2.11) in equation (2.1) we find the components of $V^k{}_{ij}$ as follows :

$$\begin{aligned}
&V^k{}_{11} [K_1/2, - K_2 A/2B, - (K_3 A/2 - EA (K_3 + K_4)/2), \\
&\qquad (K_4 A/2 - EA (K_3 + K_4)/2] , \\
&V^k{}_{22} [- K_1 B/2A, K_2/2, (- K_3 B/2 - EB (K_3 + K_4) / 2), \\
&\qquad (K_4 B/2 - EB (K_3 + K_4)/2)] , \qquad (2.12)\\
&V^k{}_{33} [- K_1 (1 - E)/2A, - K_2 (1 - E)/2B, (K_3/2 - K_4 E/2 + E^2 (K_3 + K_4)/2), \\
&\qquad (K_4/2 - K_3 E/2 - K_4 E + E^2 (K_3 + K_4)/2] , \\
&V^k{}_{44} [K_1 (1 + E)/2A, K_2 (1 + E)/2B, (K_3/2 + K_3 E + K_4 E/2 + E^2 (K_3 + K_4)/2), \\
&\qquad (K_4/2 + K_3 E/2 + E^2 (K_3 + K_4)/2)], \\
&V^k{}_{\substack{12\\21}} [(K_2/2 \pm \sigma (K_3 + K_4)/2), (K_1/2 \mp \rho (K_3 + K_4)/2), 0, 0],
\end{aligned}$$

similar expressions for $V^k{}_{\substack{13\\31}}$, $V^k{}_{\substack{14\\41}}$, $V^k{}_{\substack{23\\32}}$, $V^k{}_{\substack{24\\42}}$, $V^k{}_{\substack{34\\43}}$ are ommitted for brevity's sake.

Thus the solutions of equation (1.2) in the space-time (1.7) under the assumption (1.9) are given by (2.12).

3. Tensors $\mathbf{G}_{ij}$ and $\mathbf{N}_{ij}$. The gauge invariant Einstein tensor G_{ij} as given by Buchdahl [4], is

$$G_{ij} = N_{ij} - (K_{i,j} + K_{j,i} - 2K_l L^l{}_{ij}) - 2V^l{}_{ij} K_l + V^l{}_{ij,l} + L^l{}_{\underline{ml}} V^m{}_{ij} - L^m{}_{ij} V^l{}_{mj} - L^m{}_{ij} V^l{}_{lm} + V^m{}_{il} V^l{}_{mj} \tag{3.1}$$

and N_{ij}, the Einstein tensor of Einstein's usual asymmetric theory with $L^k{}_{ij}$ as linear connection, is given by

$$N_{ij} = - L^l{}_{ij,l} + \frac{1}{2}(L^l{}_{\underline{il},j} + L^l{}_{\underline{lj},i}) + L^l{}_{mj} L^m{}_{il} - L^l{}_{ml} L^m{}_{ij}. \tag{3.2}$$

To calculate G_{ij} and N_{ij} more easily we find out $L^j{}_{\underline{ij}}$ as follows :

$$L^j{}_{\underline{1j}} = L^j{}_{\underline{2j}} = 0, \quad L^j{}_{\underline{3j}} = - L^j{}_{\underline{4j}} = (\bar{A}/2A + \bar{B}/2B). \tag{3.3}$$

Using (3.3) and the components of $L^k{}_{ij}$ from [5] in view of (1.9) in (3.2), we get the components of N_{ij} as follows :

$$\begin{aligned} &N_{11} = N_{22} = N_{12} = N_{21} = 0, \\ &N_{13} = - N_{14} = \partial_x w + P, N_{31} = - N_{41} = \partial_x w - P, \\ &N_{23} = - N_{24} = \partial_y w + Q, N_{32} = - N_{42} = \partial_y w - Q; \end{aligned} \tag{3.4}$$

$$\begin{aligned} &N_{33} = M + R(1 - E), \\ &N_{34} = - M + ER, \\ &N_{44} = M - R(1 + E), \end{aligned} \tag{3.5}$$

where

$$\begin{aligned} &w = (\partial_z + \partial_t) E, P = (\partial_{xx}\rho/A + (\partial_{yy}\rho + \partial_{xy}\sigma)/2B - \rho R), \\ &Q = (\partial_{yy}\sigma/B + (\partial_{xx}\sigma + \partial_{xy}\rho)/2A - \sigma R), R = (\partial_z + \partial_t) w/2, \\ &M = \bar{\bar{A}}/2A - (\bar{A}/2A)^2 + \bar{\bar{B}}/2B - (\bar{B}/2B)^2 - (\partial_x\rho/A)^2 - (\partial_y\sigma/B)^2 - \\ &- (\partial_y\rho + \partial_x\sigma)^2/2AB - (\bar{A}/2A + \bar{B}/2B) w/2 - \partial_{xx} E/2A - \partial_{yy}E/2B + \\ &+ (2/A)\, \partial\alpha/\partial x + (2/B)\, \partial\beta/\partial y, \\ &\alpha = (\rho/A)\, \partial_x\rho + (\sigma/2B)(\partial_y \rho + \partial_x \sigma), \\ &\beta = (\sigma/B)\, \partial_y\sigma + (\rho/2A)(\partial_y\rho + \partial_x\sigma). \end{aligned}$$

Using (2.12), (3.3) and the components of $L^k{}_{ij}$ from [5] in equation (3.1), we get the components of G_{ij} as follows :

$$G_{11} = -2(K_1)_{,1} - \bar{A}(K_3+K_4)/4 + K_2^2 A/2B + EA(K_3+K_4)^2/2 + A(K_3^2-K_4^2)/2 -$$
$$- \bar{B}A(K_3+K_4)/4B - \rho^2(K_3+K_4)/2 + (K_1/2)_{,1} - (K_2A/2B)_{,2} - (K_3A/2)_{,3} -$$
$$- (EA(K_3+K_4)/2)_{,3} + (K_4A/2)_{,4} - (EA(K_3+K_4)/2)_{,4} ,$$
$$G_{22} = -2(K_2)_{,2} - \bar{B}(K_3+K_4)/4 + K_1^2 B/2A + EB(K_3+K_4)^2/2 + B(K_3^2-K_4^2)/2 -$$
$$- \bar{A}B(K_3+K_4)/4A - \sigma^2(K_3+K_4)/2 + (K_2/2)_{,2} - (K_1B/2A)_{,1} - (K_3B/2)_{,3} -$$
$$- (EB(K_3+K_4)/2)_{,3} + (K_4B/2)_{,4} - (EB(K_3+K_4)/2)_{,4} , \qquad (3.6)$$
$$G_{\substack{12\\21}} = -(K_1/2)_{,1} - (K_2/2)_{,1} - (\partial_y\rho - \partial_x\sigma)(K_3+K_4)/2 - K_1K_2/2 -$$
$$- \rho\sigma(K_3+K_4)^2/2 \pm (\sigma(K_3+K_4)/2)_{,1} \mp (\rho(K_3+K_4)/2)_{,2}$$
$$G_{33} = N_{33} - 2(K_3)_{,3} - K_4^2/2 + K_3E(K_3+K_4) - K_3(\bar{A}/2A + \bar{\bar{B}}/2B) -$$
$$-(K_4\partial_z E + K_3\partial_t E + 2K_4\partial_t E) - (K_3+K_4)\partial_z E -$$
$$-E((K_4)_{,3} + (K_3)_{,4} + 2(K_4)_{,4})/2 + T_1 + J_1 + T_{11} ,$$

similar expressions for G_{44}, $G_{\substack{34\\43}}$, $G_{\substack{13\\31}}$, $G_{\substack{14\\41}}$, $G_{\substack{23\\32}}$, $G_{\substack{24\\42}}$ are omitted for the sake of brevity, where

$$T_1 = (\bar{A}/2A + \bar{B}/2B)((K_3 - K_4)/2 + E(K_3+K_4)/2) - K_1^2E/2A - K_2^2E/2B -$$
$$- E^2(K_3+K_4)^2/2 + (EK_1/2A)_{,1} + (EK_2/2B)_{,2} + (E^2(K_3+K_4)/2)_{,3} +$$
$$+ (E^2(K_3+K_4)/2)_{,4} + (K_3/2)_{,3} + (K_4/2)_{,4} -$$
$$- ((\rho K_1/A)^2 + (\sigma K_3/B)^2 + \rho\sigma K_1 K_2/AB)/2,$$
$$J_1 = -(K_1/2A)_{,1} - (K_2/2B)_{,2} + K_1^2/2A + K_2^2/2B ,$$
$$T_{11} = \rho K_1 \partial_y\sigma/AB + \sigma K_2 \partial_x\rho/AB - (\partial_y\rho + \partial_x\sigma)\sigma K_1/2AB - (\partial_y\rho + \partial_x\sigma)\rho K_2/2AB -$$
$$- K_1(\rho\partial_x\rho/A + \sigma(\partial_y\rho + \partial_x\sigma)/2B) - K_2(\sigma\partial_y\sigma/B + \rho(\partial_y\rho + \partial_x\sigma)/2A) ,$$

similar expressions for T_2, T_3, T_4, J_2, J_3, J_4, T_{22}, T_{33}, T_{44}, J_{22}, J_{33}, J_{44} are omitted for brevity sake.

A single and double overhead bar (—) denote the partial differentiation with respect to (z-t) once and twice respectively.

4. Solution of the Field Equation (1.4). Using the components of $L^k{}_{ij}$ from [5] and $V^k{}_{ij}$ from (2.12) in equation (1.4), we find that when $i = 1, 2$, equation $\underset{v}{\Gamma}{}^j{}_{ij} = 0$ gives

$$K_3 + K_4 = 0 , \qquad (4.1)$$

while when $i = 3, 4$ equation $\underset{v}{\Gamma}{}^j{}_{ij} = 0$ gives

$$-(\rho K_1 B + \sigma K_2 A) + B\,\partial_x \rho + A\,\partial_y \sigma = 0\,. \tag{4.2}$$

Thus (4.1) and (4.2) are necessary conditions in order that the gauge invariant generalized second field equations be satisfied in a generalized Peres space-time.

5. Solution of equation (1.5). Using equation (4.1) in (3.6), equation (1.5) yields

$$\left.\begin{aligned}
&-2(K_1)_{,1}+K_2^2B/2A+(K_1/2)_{,1}-(K_2A/2B)_{,2}-(K_3A/2)_{,3}+(K_4A/2)_{,4}=0,\\
&-2(K_2)_{,2}-K_1^2B/2A+(K/2)_{,2}-(K_1B/2A)_{,1}-(K_3B/2)_{,3}+(K_4B/2)_{,4}=0,\\
&-(K_1/2)_{,2}-(K_2/2)_{,1}-K_1K_2/2=0\,,
\end{aligned}\right\} \tag{5.1}$$

$$\left.\begin{aligned}
N_{33} &= 2\,(K_3)_{,3}+K_4^2/2+K_3\,(\bar{A}/2A+\bar{B}/2B)+\\
&\quad +(K_4\partial_z E+K_3\,\partial_t E+2K_4\,\partial_t E)/2+\\
&\quad +E\,((K_4)_{,3}+(K_3)_{,4}+2\,(K_4)_{,4})/2-T_1'-J_1'-T_{11}'\,,\\
N_{44} &= 2\,(K_4)_{,4}+K_3^2/2-K_4\,(\bar{A}/2A+\bar{B}/2B)-\\
&\quad -(K_3\,\partial_z E+K_4\,\partial_z E+2K_1\,\partial_z E)/2-\\
&\quad -E\,((K_3)_{,4}+(K_4)_{,3}+2\,(K_3)_{,3})/2-T_1'-\mathbf{J}_1'-T_{11}'\,,\\
N_{34} &= (K_3/2)_{,4}+(K_4/2)_{,3}+K_3K_1/2+E\,((K_3)_{,3}-(K_4)_{,4})/2+\\
&\quad +(K_3\,\partial_z E-K_4\,\partial_t E)/2-T_2'-J_2'-T_{22}'+J_{22}'\,,\\
N_{43} &= (K_3/2)_{,4}+(K_4/2)_{,3}+K_3K_4/2+E\,((K_3)_{,3}-(K_4)_{,4})/2+\\
&\quad +(K_3\,\partial_z E-K_4\,\partial_t E)/2-T_2'+J_2'-T_{22}'+J_{22}'\,,
\end{aligned}\right\} \tag{5.2}$$

$$\left.\begin{aligned}
N_{\substack{13\\31}} &= (K_1/2)_{,3}+(K_3/2)_{,1}+K_1K_3/2\pm(\rho K_4/2)_{,3}\pm(\rho K_4/2)_{,4}-T_3'\mp J_3',\\
N_{\substack{14\\41}} &= (K_1/2)_{,4}+(K_4/2)_{,1}+K_1K_4/2\pm(\rho K_3/2)_{,3}\pm(\rho K_3/2)_{,4}+T_3'\pm J_3',\\
N_{\substack{23\\32}} &= (K_2/2)_{,3}+(K_3/2)_{,2}+K_2K_3/2\pm(\sigma K_4/2)_{,3}\pm(\sigma K_4/2)_{,4}-T_4'\mp J_4',\\
N_{\substack{24\\42}} &= (K_2/2)_{,4}+(K_4/2)_{,2}+K_2K_4/2\pm(\sigma K_3/2)_{,3}\pm(\sigma K_3/2)_{,4}+T_4'\pm J_4'.
\end{aligned}\right\} \tag{5.3}$$

Here dashed T_i's and J_i's are the corresponding values of T_i's and J_i's under the condition (4.1).

There are various possibilities under which the solution of field equation (1.5) may be considered. However in this paper we shall consider the solution in the case when

$$K_1 = K_2 = 0 \tag{5.4}$$

and

$$K_3 = - K_4 = K(z, t). \tag{5.5}$$

Here (5.5) is not assumed but is implied by the condition (4.1). Using (5.4) and (5.5), equation (5.1) yields

$$\begin{aligned} (KA)_{,3} + (KA)_{,4} &= 0\,, \\ (KB)_{,3} + (KB)_{,4} &= 0\,. \end{aligned} \tag{5.6}$$

Equation (5.6) gives

$$(K)_{,3} + (K)_{,4} = 0\,, \tag{5.7}$$

which implies that

$$K = - K(z - t) \tag{5.8}$$

i.e. K is a function of $(z - t)$.

Further using (5.4) and (5.8) in (5.2) and (5.3), we get

$$N_{33} = - N_{34} = - N_{43} = N_{44} = \bar{K} + K^2/2 - Kw/2 \tag{5.9}$$

and

$$N_{13} = N_{31} = N_{14} = N_{41} = N_{23} = N_{32} = N_{24} = N_{42} = 0 \tag{5.10}$$

respectively.

Equating the expressions for the same components of N_{ij} from (3.4), (3.5) and (5.9), (5.10), we find that

$$\begin{aligned} M + R(1 - E) &= \bar{K} + K^2/2 - K(\partial_z + \partial_t)\, E/2\,, \\ M - RE &= \bar{K} + K^2/2 - K(\partial_z + \partial_t)\, E/2\,, \\ M - R(1 + E) &= \bar{K} + K^2/2 - K(\partial_z + \partial_t)\, E/2 \end{aligned} \tag{5.11}$$

and

$$\begin{aligned} \partial_x w + P = 0, \quad \partial_x w - P = 0\,, \\ \partial_y w + Q = 0, \quad \partial_y w - Q = 0\,. \end{aligned} \tag{5.12}$$

Simplifying (5.11), we get

$$(\partial_z + \partial_t)\, w = 0\,, \tag{5.13}$$

$$\begin{aligned} &\bar{\bar{A}}/2A - (\bar{A}/2A)^2 + \bar{\bar{B}}/2B - (\bar{B}/2B)^2 - (\partial_x \rho/A)^2 - (\partial_y \sigma/B)^2 - \\ &- (\partial_y \rho + \partial_x \sigma)^2/2AB - (\bar{A}/2A + \bar{B}/2B)\, w/2 - \partial_{xx} E/2A - \\ &- \partial_{yy} E/2B + (2/A)\, \partial_x \alpha + (2/B)\, \partial_y \beta = \bar{K} + K^2/2 - Kw/2\,. \end{aligned} \tag{5.14}$$

Equation (5.12) implies that $\partial_x w = 0$, $P = 0$ and $\partial_y w = 0$, $Q = 0$ i.e.

$$\begin{aligned} &\text{a) } (\partial_z + \partial_t)\,\partial_x E = 0\,, \\ &\text{b) } (\partial_z + \partial_t)\,\partial_y E = 0 \end{aligned} \tag{5.15}$$

and

$$\begin{aligned} &\text{a) } \partial_{xx}\rho/A + (\partial_{yy}\rho + \partial_{xy}\sigma)/2\,B - \rho\,(\partial_z + \partial_t)\,w = 0\,, \\ &\text{b) } \partial_{yy}\sigma/B + (\partial_{xx}\sigma + \partial_{xy}\rho)/2\,A - \sigma\,(\partial_z + \partial_t)\,w = 0\,. \end{aligned} \tag{5.16}$$

Integrating (5.13), (5.15) a), b) we find that the form of E is given by

$$\begin{aligned} &E = (x + y + z)\,f_1\,(z - t) + f(x, y)\,f_2\,(z - t) \\ \text{or}\quad &E = f(x, y, z - t) + z f_1\,(z - t)\,. \end{aligned} \tag{5.17}$$

Using equation (4.2) and (5.13), equation (5.16) is further simplified as

$$\begin{aligned} &\partial_{xx}\rho/A + \partial_{yy}\rho/B = 0\,, \\ &\partial_{xx}\sigma/A + \partial_{yy}\sigma/B = 0\,. \end{aligned} \tag{5.18}$$

Thus K_i given by $K_1 = K_2 = 0$ and $K_3 = -K_4 = K(z - t)$, the g_{ij} given by (1.8) along with the equations (5.14), (5.17) and (5.18) can be said to constitute the wave solutions of Buchdahl's field equations in the generalized Peres space-time.

When $K_1 = K_2 = 0$ and $K_3 = -K_4 = K(z - t)$, then all the components of the skew symmetric $F_{ij} = \partial K_i/\partial x^j - \partial K_j/\partial x^i$, defined in [3] reduce to zero, thereby showing that $\underset{\vee}{g_{ij}}$ may be associated with the electromagnetic field tensor in the Buchdahl's gauge invariant theory.

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ÖZET

Bu çalışmada Buchdahl tarafından verilen, Einstein alan denklemlerinin ölçüyü sabit bırakan bir genelleştirilmesi incelenmektedir.