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SUPERSOLVABLE Q-GROUPS

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Summary: One important problem in **Q**-gruops theory is to classify particular classes of **Q**-groups. In this note we will classify the supersolvable **Q**-groups.

SÜPER ÇÖZÜLEBİLİR Q-GRUPLARI

Özet: **Q**-grupları teorisinin önemli bir problemi, özel **Q**-gruplarının sınıflandırılmasıdır. Bu çalışmada süper çözülebilir **Q**-grupları sınıflandırılmaktadır.

Definition. A **Q**-group is a finite group whose all characters are rational valued.

Theorem 1. A finite group G is a **Q**-group if and only if for every $x \in G$, $N_G(<x>) / C_G(<x>) \simeq \mathrm{Aut}(<x>)$ (see [3]).

Theorem 2 (see [2], pg. 716). Let G be a supersolvable group. Then:

(1) G' is nilpotent.

(2) Let p be a maximal prime number such that $p \mid |G|$. Then, a p-Sylow subgroup of G is normal in G.

(3) Let q be a minimal prime number such that $q \mid |G|$. Then, G is q-nilpotent.

Theorem 3. Let G be a supersolvable **Q**-group. Then:

(1) $|G| = 2^a 3^b$.

(2) Let H be a 2-Sylow subgroup of G. Then H is also a **Q**-group, no two elements of H are fused in H by G and $\mathrm{Foc}_G(H) = H' = G' \cap H$ (the notations are those of [4]).

(3) Let P be a 3-Sylow subgroup of G. Then P is normal in G and is a 3-rational group (Therefore G is 2-nilpotent).

(4) If $P \in \mathrm{Syl}_3(G)$ is abelian, then P is elementary abelian.

(5) $G' = R \times P$ where $R = G' \cap H$ is a 2-Sylow subgroup of G'.

Proof. By Gow Theorem (see [1]) if G is a solvable **Q**-group, then $|G| = 2^a 3^b 5^c$. Since G/G' is an abelian **Q**-group, by Theorem 1,

$$G/G' \simeq \mathbf{Z}_2 \times \dots \times \mathbf{Z}_2 .$$

Let H be a 2-Sylow subgroup of G, P a 3-Sylow subgroup of G and T be a 5-Sylow subgroup of G. By Theorem 2 G' is nilpotent, so that $G = R \times P \times T$ where R is a 2-Sylow subgroup of G'. Let $x \in T$. Then $|\mathrm{Aut}(<x>)| = 5^{k-1} 4$ and since $\exp(G/G') = 2$, by Theorem 1 it follows that T is trivial and $|G| = 2^a 3^b$. By Theorem 2, P is a normal subgroup of G and G is 2-nilpotent. Since $G' = R \times P$ it follows that P is 3-rational. Since a factor group of a **Q**-group is also a **Q**-group, it follows that $H = G/P$ is a **Q**-group. By Welandt Theorem (see [4], *pg.* 258) no two elements of H are fused by G in H and $\mathrm{Foc}_G(H) = H' = G' \cap H$ (see [4], pg. 255).

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