İstanbul Üniv. Fen Fak. Mat. Der., 54 (1995), 119-120

2'-R GROUPS HAVING A FAITHFUL MONOMIAL IRREDUCIBLE CHARACTER

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Summary : In this note we shall study the structure of the finite 2'-r groups having a nonlinear faithful monomial irreducible character of minimal degree.

BİR SADIK MONOMİYAL İNDİRGENEMEZ KARAKTERE SAHİP 2'-R GRUPLAR

Özet : Bu çalışmada minimal dereceli, lineer olmayan bir sadık monomiyal indirgenemez karaktere sahip sonlu 2'-r grupların yapısı incelenmektedir.

The notations and terminology are standard (see for example [2] and [3]). All groups will be finite.

Definition (see [1]). A 2'-r group is a group all whose irreducible characters are rational valued on the 2-regular elements.

Theorem 1. Let G be an 2'-r group having a nonlinear faithful monomial irreducible character of minimal degree. Then G is a 2-nilpotent group, $G=O(G)\,S$, where $S \in Syl_2(G)$ and G' is abelian.

Proof. We shall prove first that G' is abelian. Suppose the contrary, thus $G'' \neq 1$. For a 2'-r group G/G' is an abelian 2-group (see [1]). Let χ be a faithful monomial nonlinear irreducible character of minimal degree. Let $H \leq G$ and $\mu \in Irr(G)$ a linear character such that $\mu^G = \chi$. Let λ be any irreducible constituent of $(1_H)^G$. It is clear that

$$\chi(1) = \mu^G(1) = (1_H)^G(1) > 1_G(1),$$

hence $\lambda(1) < \chi(1)$ and λ must be linear by the minimality of the degree of χ. Then $ker(\lambda) > G'$ (see [2], p. 25). Thus

$$G' \leq \cap\, ker(\lambda) = ker\,(i_H)^G = \cap_{g \in G}\, g^{-1} Hg \leq H.$$

Since G/G' is an abelian 2-group and G' is characteristic in G, it follows that H is normal in G and the inertia group $I_G(\lambda)=H$. By Clifford's theorem (see [2]) $\chi_H = \sum_{j=1}^{k} \mu_j$, where $\mu_j \in Irr(H)$ are the distinct conjugates of μ in G. Since μ_j are linear we have $ker(\mu_j) \geq H' \geq G''$ and hence $ker(\chi) = \cap_j ker(\mu_j) \geq G'' \neq 1$ which contradicts the faithfulness of χ.

Now let $O(G)$ be the maximal odd order normal subgruop of G. Since G is a 2'-r gruop, $O(G) \leq G'$ (see [1]) and hence $O(G)$ is abelian and $G'=O(G)\times P$ with $P \in Syl_2(G')$. Let $S \in Syl_2(G)$ such that $P \leq S$. Then $S \simeq G/O(G)$ so that G is 2-nilpotent. Since G is a 2'-r group and G' is abelian, it follows that $G/G' \simeq P$ where P is an abelian 2-group which inverts all elements of G'.

REFERENCES

[1] ARMEANU, I. : *The structure of the groups all whose characters are rational valued on the* 2'-r *elements* (to appear).

[2] ISAACS, I.M. : **Character Theory of Finite Groups**, Academic Press, 1976.

[3] KURZWEIL, H. : **Endliche Gruppen**, Springer-Verlag, 1977.