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SUPERSOLVABLE 2'-R GROUPS

Ion ARMEANU
University of Bucharest, Physics Faculty, Mathematics Dept., Bucharest-Magurele,
P.O. Box MG-11, ROMANIA

Summary : In this note we shall study the structure of the supersolvable groups ail whose irreducible characters are rational valued on the 2-regular elements.

SÜPERÇÖZÜLEBİLİR 2'-R GRUPLAR

Özet : Bu çalışmada, 2-regüler elemanlar üzerinde bütün indirgenemez karakterleri rasyonel değerli olan süperçözülebilir grupların yapısı incelenmektedir.

Definition. A 2'-r group is a finite group all whose characters are rational valued on the 2-regular elements (see [1]).

Theorem 1. A finite group G is a 2'-r group if and only if for every 2-regular $x \in G$, $N_G(\langle x \rangle)/C_G(\langle x \rangle) \simeq \mathrm{Aut}(\langle x \rangle)$ (see [1]).

Theorem 2 (see [2], p. 716). Let G be a supersolvable group. Then:

(1) G' is nilpotent.

(2) Let p be a maximal prime number such that $p \mid |G|$. Then, a p-Sylow subgroup of G is normal in G.

(3) Let q be a minimal prime number such that $q \mid |G|$. Then, G is q-nilpotent.

Theorem 3. Let G be a supersolvable 2'-r group and let S be a Sylow 2-subgroup of G. Then:

i) No two elements of S are fused in S by G and $Foc_G(S) = S' = G' \cap S$.

ii) Let P be a p-Sylow subgroup of G for an odd prime p. Then P is normal in G and is a p-rational group.

iii) G is 2-nilpotent.

iv) If $P \in Syl_p(G)$ is abelian, then P is elementary abelian.

v) $G' = R \times P$ where $R = G' \cap S$ is a 2-Sylow subgroup of G'.

Proof. G/G' is an abelian 2'-r group (see [1]), hence by Theorem 1, G/G' is an abelian 2-group.

Let P be a maximal 2'-subgroup of G. Then $P \leq G'$ by [1]. By Theorem 2 G' is nilpotent, so that $G = R \times P$ where R is a Sylow 2-subgroup of G'. By Theorem 2, P is a normal subgroup of G and thus G is 2-nilpotent. Since $G' = R \times P$ it follows that P is p-rational for every odd prime p. Since $S \simeq G/P$, by Wielandt Theorem (see [3], p. 258) no two elements of S are fused by G in S and $Foc_G(S) = S' = G' \cap S$ (see [3], p. 255).

The other statements follow now easy from the previous.

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