SYLOW 2-SUBGROUPS OF 2'-R GROUPS

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Summary : In this note we shall determine the Sylow 2-subgroups for groups all whose irreducible characters are rational valued on the 2'-elements, and having a strongly embedded subgroup.

2'-R GRUPLARIN 2-SYLOW ALT GRUPLARI

Özet : Bu çalışmada, 2'-elemanlar üzerinde bütün ingdirgenemez karakterleri rasyonel değerli olan ve kuvvetli yatırılmış bir alt grubu bulunan grupların 2-Sylow alt grupları belirlenmektedir.

All groups will be finite. The notations and terminology are standard (see [3, 4]).

Definition (see [1]). A 2'-r group is a group all whose irreducible characters are rational valued on the 2'-elements.

Theorem 1 (Bender, see [4], p. 391). Let G be a group having a strongly embedded subgroup H. Then, we have one of the following alternatives:

(1) Every 2-Sylow subgroup of G contains exactly one element of order 2. Thus a 2-Sylow subgruop of G is either a cyclic group or a generalized quternion group.

(2) The group G possesses a normal series $G > L > M > \{1\}$ such that both G/L and M are groups of odd order, and such that the factor group L/M is isomorphic to one of the simple groups PSL(2, q), Sz(q), or PSU(3, q), where q is a power of 2.

In the first case (1), let t be any element of order two. Then, $C_G(t)$ is a proper subgroup of G, and any proper subgroup of G containing $C_G(t)$ is strongly embedded in G. In the second case (2), every strongly embedded subgroup H of G is of the form $H = N_G(S)\, O_{2'}(G)$ for some 2-Sylow subgroup S of G.

Theorem 2 (see [4], p. 393). Let H be a strongly embedded subgroup of a group G. Let u be an element of $I(H)$, and let $C = C_G(u)$. Then, the following propositions hold:

(1) The set $I(G) = \{x \in G \mid x^2 = 1\}$ is a conjugacy class of G. In other words, all involutions of G are conjugate.

(2) The set $I(H)$ is a conjugacy class of H. Furthermore, if $b = a^x$ for $a, b \in I(H)$ and $x \in G$, then we have $x \in H$.

Theorem 3 (see [2]). In the conditions of Theorem 1 part 2, a 2-Sylow subgroup of G is either homocyclic or a Suzuki 2-group and $G/O_{2}.(G)$ is a 2-normal group.

Theorem 4. Let G be a solvable group such that $G/O_{2'}(G)$ is a 2'-r group having a strongly embedded subgroup. Then a Sylow 2-subgroup of G is cyclic group.

Proof. To study the Sylow 2-subgroups of G we can suppose that $O_{2'}(G)$ is trivial. Let $A = \langle I(G) \rangle$. Then A is a normal subgroup of G and since G is solvable, it contains an abelian minimal normal subgroup of G.

Since $O_{2'}(G)$ is trivial it follows that $A = I(G)$ and A is abelian. Let H be a strongly embedded subgroup of G such that $A < H$. Then G has only one 2-Sylow subgroup S. Then G/S is also a 2'-r group, hence $G = S$ and S contains only one involution. By Theorem 3, then S is either a cyclic group or a generalized quaternion group.

If S is a generalized quaternion group, then by Glauberman Z^*-Theorem (see [4]) $Z(G)$ contains an involution, therefore G can't have a strongly embedded subgroup.

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