FLOW OF CONDUCTING BINGHAM PLASTICS THROUGH AN ANNULUS WITH SUCTION AND INJECTION UNDER A RADIAL MAGNETIC FIELD

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The present paper deals with the flow of conducting Bingham Plastics between two non-conducting co-axial cylinders with suction and injection under a radial magnetic field. The exact solution has been obtained and then several particular cases have been deduced,

1. Introduction. The study of the motion of non-Newtonian fluids in the absence as well as presence of a magnetic field has applications in many areas. A few examples are the flow of nuclear fuel, slurries, flow of liquid metals and alloys, flow of mercury amalgams, handling of biological fluids, flow of blood - a Bingham fluid etc. Recently TURGUT SARPKAYA [1] has studied the flow of a conducting Bingham Plastic fluid between two parallel plates. The author [2] has extended, the problem to include suction and injection also. In the present paper the flow through an annulus with suction and injection has been investigated. It is assumed that the fluid coming from the inner cylinder, moves radially, The shearing stress on the fluid approaching the plug region, approaches the yield stress. This fluid then becomes a part of the plug region. Some amount of the fluid joins the stream of the outer region and this is ejected from the plug. In the steady state we can assume that there is a flow of the fluid with velocity $v_0 L/r$. The external magnetic field is assumed to he $H_0 L/r$. The displacement currents and free charge density are neglected.

2. Basic equations and their integration. Let ϱ, σ, μ_e, μ and P_0 be the density, the electrical conductivity, the magnetic permeability, the viscosity and the yield stress respectively. We use cylindrical coordinates (r, ϑ, z). Again let $\mathbf{V}(v_r, 0, v_z)$, $\mathbf{H}(H_r, 0, H_z)$, $\mathbf{E}(0, E_\vartheta, 0)$, $\mathbf{J}(0, J_\vartheta, 0)$, p and P_{ry} be the velocity, the magnetic field, the electric field, the current density, the pressure and the shearing stress respectively. It is also assumed that the flow is axially symmetric and does not depend upon z. In this case the pressure gradient along the x-axis comes out to be a constant, With the above assumption the simplified equations are:

(1) $$-\frac{v_0^2 L^2}{\bar{r}^3} = -\frac{\partial p}{\partial \bar{r}} - \frac{\mu_e H_z}{4\pi\varrho} \cdot \frac{\partial H_z}{\partial \bar{r}},$$

(2) $$\frac{v_0 L}{\bar{r}} \frac{\partial v_z}{\partial \bar{r}} = -\frac{1}{\varrho}\frac{\partial p}{\partial z} + \frac{1}{\bar{r}}\frac{\partial \bar{r} P_{rz}}{\partial \bar{r}} + \frac{\mu_e}{4\pi\varrho} H_r \frac{\partial H_z}{\partial \bar{r}},$$

(3) $$-\frac{\partial H_z}{\partial \bar{r}} = 4\pi\sigma (E_\vartheta + v_z H_r - v_r H_z),$$

(4) $$P_{rz} = \pm P_0 + \mu \frac{\partial v_z}{\partial \bar{r}}.$$

We now, transform these equations to non-dimensional form with the relations,

$$v_z = u_0 u, \qquad v_0 = m u_0, \qquad \bar{r} = Lr, \qquad H_z = H_0 H, \qquad P = -\frac{L}{\varrho u_0^2}\frac{\partial p}{\partial z},$$

$$P_0 = \varrho u_0^2 \tau_0, \qquad P_{rz} = \varrho u_0^2 \tau, \qquad R = \frac{\varrho u_0 L}{\mu}, \qquad R_m = 4\pi\sigma\mu_e u_0 L,$$

(5) $$M = \mu_e H_0 L \sqrt{\frac{\sigma}{\mu}} \qquad \text{and} \qquad E = \mu_e H_0 u_0 E,$$

where R is the REYNOLD's number, R_m the magnetic REYNOLD's number, M the HARTMANN number, τ_0 the non-dimension yield stress, and $2L$ is the characteristic length. With the above relations the equations (2) to (4) become

(6) $$\frac{m}{r}\frac{du}{dr} = P + \frac{1}{r}\frac{dr\tau}{dr} + \frac{M^2}{RR_m}\frac{1}{r}\frac{dH}{dr},$$

(7) $$u = mH - \frac{r}{R_m}\frac{dH}{dr} - E,$$

(8) $$\tau = \pm \tau_0 + \frac{1}{R}\frac{du}{dr}.$$

Integrating equation (6) once, we get

(9) $$\tau = -\frac{1}{2}Pr + \frac{mu}{r} - \frac{M^2}{RR_m}\frac{H}{r} - \frac{C_1}{r},$$

'here C_1 is an arbitrary constant. Eliminating u and τ from equations (7), (8) and (9) we get

(10) $$r^2 \frac{d^2H}{dr^2} + (1 - mR - m R_m) r \frac{dH}{dr} + (m^2 R R_m - M^2) H = \frac{PR R_m}{2} r^2 + c_1 \pm \tau_0 R R_m r.$$

Solving this equation we have

(11) $$H = Ar^\alpha + Br^\beta + \frac{PR R_m}{2(\alpha-2)(\beta-2)} r^2 \pm \frac{\tau_0 R R_m}{(\alpha-1)(\beta-1)} r + C,$$

where A, B and C are are arbitrary constants to be determined from boundary conditions and

(12) $$\alpha, \beta = \frac{m}{2}(R + R_m) \pm \sqrt{M^2 + \frac{m^2}{4}(R - R_m)^2}.$$

Substituting the value of H in equation (7), we obtain

(13) $$u = A\left(m - \frac{\alpha}{R_m}\right) r^\alpha + B\left(m - \frac{\beta}{R_m}\right) r^\beta + \frac{PR(m R_m - 2)}{2(\alpha-2)(\beta-2)} r^2$$
$$+ mC - E \pm \frac{\tau_0 R R_m}{(\alpha-1)(\beta-1)}\left(m - \frac{1}{R_m}\right) r.$$

Again substituting the value of u in equation (8) we get,

(14) $$\tau = \frac{A\alpha}{R}\left(m - \frac{\alpha}{R_m}\right) r^{\alpha-1} + \frac{B\beta}{R}\left(m - \frac{\beta}{R_m}\right) r^{\beta-1} \pm \frac{\tau_0 (m^2 R R_m - mR - M^2)}{(\alpha-1)(\beta-1)}$$
$$+ \frac{P(m R_m - 2)}{(\alpha-2)(\beta-2)} r.$$

3. Boundary conditions. The continuity of the tangential component of the electric and the magnetic field across a non-conducting boundary gives that E and H must vanish at the outer cylinder as it is assumed that there is no tangential component of the electric and the magnetic field outside the outer cylinder.

We also assume that cylinders are of radii aL and bL and are at rest. The plug region extends from Lr_1 to Lr_2 and is moving with velocity $u_0 U_0$. As u is decreasing with r in the outer fluid region, du/dy is negative. Therefore we take the sign of τ_0 to be negative for this region. Similarly the sign of τ_0 for the inner region will be positive. Thus the boundary conditions are:

$$\text{at } r = a, \quad u = 0; \qquad \text{at } r = r_1, \quad u = u_0, \quad H = H_1, \quad \tau = \tau_0 \text{ i.e. } \frac{du}{dr} = 0,$$

(15) $$\text{at } r = b, \quad u = 0, \quad H = 0; \quad \text{at } r = r_2, \quad u = u_0, \quad H = H_2, \quad \tau = \tau_0 \text{ i.e. } \frac{du}{dr} = 0.$$

5. Plug region. The equations governing the flow in the plug region are,

$$0 = P + \frac{1}{r}\frac{d r\tau}{dr} + \frac{M^2}{RR_m}\frac{1}{r}\frac{dH}{dr}\,, \tag{16}$$

$$U_0 = mH - \frac{r}{R_m}\frac{dH}{dr}\,. \tag{17}$$

Solving these equations and using the boundary conditions, we obtain

$$\tau = \tfrac{1}{2}P\left(\frac{r_1^2}{r} - r\right) + \tau_0\frac{r_1}{r} + \frac{1}{r}\{\tfrac{1}{2}P(r_2^2 - r_1^2) - \tau_0(r_1 + r_2)\}\frac{r_1^{mRm} - r^{mRm}}{r_1^{mRm} - r_2^{mRm}}\,, \tag{18}$$

$$H = \frac{RR_m}{M^2}\{\tfrac{1}{2}P(r_2^2 - r_1^2) - \tau_0(r_1 + r_2)\}\frac{r^{mRm}}{r_1^{mRm} - r_2^{mRm}} + \frac{U_0}{m}\,, \tag{19}$$

and

$$H_1 = \frac{RR_m}{M^2}\{\tfrac{1}{2}P(r_2^2 - r_1^2) - \tau_0(r_1 + r_2)\}\frac{r^{mRm}}{r_1^{mRm} - r_2^{mRm}} + \frac{U_0}{m}\,, \tag{20}$$

$$H_2 = \frac{RR_m}{M^2}\{\tfrac{1}{2}P(r_2^2 - r_1^2) - \tau_0(r_1 + r_2)\}\frac{r_2^{mRm}}{r_1^{mRm} - r_2^{mRm}} + \frac{U_0}{m}\,. \tag{21}$$

5. Solution for the inner region. Substituting the boundary condition in equations (11), (13) and (14) and using (20) and (21) we can find out the values of all the constants and thus obtain the expression for variables in the inner region as

(22)

$$u = \frac{RR_m}{(\alpha-\beta)(m^2 RR_m - M^2)A_1}\Bigg[\frac{\tfrac{1}{2}P(r_2^2 - r_1^2) - \tau_0(r_1 + r_2)}{r_2^{mRm} - r_1^{mRm}}\, m r_1^{-mR}\, A_1 - P r_1^2\left\{\frac{\alpha}{\beta-2}\left(m - \frac{\beta}{R_m}\right)\right.$$

$$\left.(1 - a^{\beta} r_1^{-\beta}) - \frac{\beta}{\alpha-2}\left(m - \frac{\alpha}{R_m}\right)(1 - a^{\alpha} r_1^{-\alpha})\right\} - \tau_0 r_1\left\{\frac{\alpha}{\beta-1}\left(m - \frac{\beta}{R_m}\right)(1 - a^{\beta} r_1^{-\beta})\right.$$

$$\left.- \frac{\beta}{\alpha-1}\left(m - \frac{\alpha}{R_m}\right)(1 - a^{\alpha} r_1^{-\alpha})\right\}\Bigg]\left[\beta r_1^{\beta}(r^{\alpha} - a^{\alpha}) - a r_1^{\alpha}(r^{\beta} - a^{\beta})\right] + \frac{RR_m}{A_1}\left\{\frac{P r_1^2}{(\alpha-2)(\beta-2)}\right.$$

$$\left.\left(m - \frac{2}{R_m}\right) + \frac{\tau_0 r_1}{(\alpha-1)(\beta-1)}\left(m - \frac{1}{R_m}\right)\right\}\left\{(r_1^{\beta} - a^{\beta})(r^{\alpha} - a^{\alpha}) - (r_1^{\alpha} - a^{\alpha})(r^{\beta} - a^{\beta})\right\}$$

$$+ \frac{PRR_m}{2(\alpha-2)(\beta-2)}\left(m - \frac{2}{R_m}\right)(r^2 - a^2) + \frac{\tau_0 RR_m}{(\alpha-1)(\beta-1)}\left(m - \frac{1}{R_m}\right)(r - a).$$

(23)

$$H=\frac{RR_m}{mM^2(m^2RR_m-M^2)(\alpha-\beta)\Delta_1}\Big[\frac{\frac{1}{2}P(r_2^2-r_1^2)-\tau_0(r_1+r_2)}{r_1^{mRm}-r_2^{mRm}}m\Delta_1 r^{-mR}+Pr_1^2\Big\{\frac{\alpha}{\beta-2}$$

$$\left(m-\frac{\beta}{R_m}\right)(1-a^\beta r_1^{-\beta})-\frac{\beta}{\alpha-2}\left(m-\frac{\alpha}{R_m}\right)(1-a^\alpha r_1^{-\alpha})\Big\}+\tau_0 r_1\Big\{\frac{\alpha}{\beta-1}$$

$$\left(m-\frac{\beta}{R_m}\right)(1-a^\beta r_1^{-\beta})-\frac{\beta}{\alpha-1}\left(m-\frac{\alpha}{R_m}\right)(1-a^\alpha r_1^{-\alpha})\Big\}\Big]\Big[\beta r_1^\beta\{mR_m(mR_m-\beta)r^\alpha$$

$$+M^2a^\alpha\}-\alpha r_1^\alpha\{M^2a^\beta+mR_m(mR_m-\alpha)r^\beta\}\Big]-\frac{RR_m}{mM^2(\Delta_1)}\Big[\frac{Pr_1^2}{(\alpha-2)(\beta-2)}$$

$$\left(m-\frac{2}{R_m}\right)+\frac{\tau_0 r_1}{(\alpha-1)(\beta-1)}\left(m-\frac{1}{R_m}\right)\Big]\Big[\{mR_m(mR_m-\beta)r^\alpha+M^2a^\alpha\}(r_1^\beta-a^\beta)$$

$$-\{mR_m(mR_m-\alpha)r^\beta+M^2a^\beta\}(r_1^\alpha-a^\alpha)+\Big]\frac{PRR_m}{2(\alpha-2)(\beta-2)}\left(r^2-a^2+\frac{2a^2}{mR_m}\right)$$

$$+\frac{\tau_0 RR_m}{(\alpha-1)(\beta-1)}\left(r-a+\frac{a}{mR_m}\right).$$

(24)

$$\tau=\frac{R_m}{(\alpha-\beta)\Delta_1}\Big[\frac{\frac{1}{2}P(r_2^2-r_1^2)-\tau_0(r_1+r_2)}{r_2^{mRm}-r_1^{mRm}}m\Delta_1 r_1^{-mR}-Pr_1^2\Big\{\frac{\alpha}{\beta-2}\left(m-\frac{\beta}{R_m}\right)(1-a^\beta r_1^{-\beta})$$

$$-\frac{\beta}{\alpha-2}\left(m-\frac{\alpha}{R_m}\right)(1-a^\alpha r_1^{-\alpha})\Big\}-\tau_0 r_1\Big\{\frac{\alpha}{\beta-1}\left(m-\frac{\beta}{R_m}\right)(1-a^\beta r_1^{-\beta})-\frac{\beta}{\alpha-1}$$

$$\left(m-\frac{\alpha}{R_m}\right)(1-a^\alpha r_1^{-\alpha})\Big\}\Big](r_1^\beta r^{\alpha-1}-r_1^\alpha r^{\beta-1})+\frac{RR_m}{\Delta_1}\Big\{\frac{Pr_1^2}{(\alpha-2)(\beta-2)}$$

$$\left(m-\frac{2}{R_m}\right)+\frac{\tau_0 r_1}{(\alpha-1)(\beta-1)}\left(m-\frac{1}{R_m}\right)\Big\}\{\alpha(r_1^\beta-a^\beta)r^{\alpha-1}-\beta(r_1^\alpha-a^\alpha)r^{\beta-1}\}$$

$$+\frac{P(mR_m-2)}{(\alpha-2)(\beta-2)}r+\frac{\tau_0(m^2RR_m-mR-M^2)}{(\alpha-1)(\beta-2)}.$$

(25)

$$H_1=\frac{RRm}{mM^2(m^2RR_m-M^2)(\alpha-\beta)\Delta_1}\Big[\frac{\frac{1}{2}P(r_2^2-r_1^2)-\tau_0(r_1+r_2)}{r_1^{mRm}-r_2^{mRm}}\cdot m\Delta_1 r_1^{-mR}+Pr_1^2$$

$$\Big\{\frac{\alpha}{\beta-2}\left(m-\frac{\beta}{R_m}\right)(1-a^\beta r_1^{-\beta})-\frac{\beta}{\alpha-2}\left(m-\frac{\alpha}{R_m}\right)(1-a^\alpha r_1^{-\alpha})\Big\}+\tau_0 r_1$$

$$\Big\{\frac{\alpha}{\beta-1}\left(m-\frac{\beta}{R_m}\right)(1-a^\beta r_1^{-\beta})-\frac{\beta}{\alpha-1}\left(m-\frac{\alpha}{R_m}\right)(1-a^\alpha r_1^{-\alpha})\Big\}\Big]\Big[\beta r'^\beta\{mR_m$$

$$(mR_m-\beta)\,r_1{}^{\alpha}+M^2a^{\alpha}\}-\alpha\,r_1{}^{\alpha}\{M^2a^{\beta}+mR_m(mR_m-\alpha)\,r_1{}^{\beta}\}\Big]-\frac{RR_m}{m\,M^2\,\Delta_1}$$

$$\left[\frac{Pr_1^2}{(\alpha-2)(\beta-2)}\left(m-\frac{2}{R_m}\right)+\frac{\tau_0\,r_1}{(\alpha-1)(\beta-1)}\left(m-\frac{1}{R_m}\right)\right]\Big[(r_1{}^{\beta}-a^{\beta})\{mR_m$$

$$(mR_m-\beta)\,r_1{}^{\alpha}+M^2a\}-(r_1{}^{\alpha}-a^{\alpha})\{mR_m-\alpha)\,r_1{}^{\beta}+M^2a^{\beta}\}\Big]+\frac{PR\,R_m}{(\alpha-2)(\beta-2)}$$

$$\left(r_1^2-a^2+\frac{2a^2}{mR_m}\right)+\frac{\tau_0\,RR_m}{(\alpha-1)(\beta-1)}\left(r_1-a+\frac{a}{mR_m}\right).$$

(26)

$$U_0=\frac{PRR_m}{2(\alpha-2)(\beta-2)}\left(m-\frac{2}{R_m}\right)(r_1^2-a^2)-\frac{\tau_0\,RR_m}{(\alpha-1)(\beta-1)}(r_1-a)\left(m-\frac{1}{R_m}\right)$$

$$-\frac{mRR_m\,r_1{}^{-mR}}{(\alpha-\beta)(m^2RR_m-M^2)}\,\frac{\frac{1}{2}P(r_2^2-r_1^2)-\tau_0(r_1+r_2)}{r_1^{mRm}-r_2^{mRm}}-\frac{PRR_m\,r_1^2}{(\alpha-\beta)(m^2RR_m-M^2)}$$

$$\left\{\frac{\alpha}{\beta-2}\left(m-\frac{\beta}{R_m}\right)(1-a^{\beta}\,r_1{}^{-\beta})-\frac{\beta}{\alpha-2}\left(m-\frac{\alpha}{R_m}\right)(1-a^{\alpha}\,r_1{}^{-\alpha})\right\}$$

$$-\frac{\tau_0\,RR_m\,r_1}{(\alpha-\beta)(m^2RR_m-M^2)}\left\{\frac{\alpha}{\beta-1}\left(m-\frac{\beta}{R_m}\right)(1-a^{\beta}r_1{}^{-\beta})-\frac{\beta}{\alpha-1}\left(m-\frac{\alpha}{R_m}\right)(1-a^{\alpha}r_1{}^{-\alpha})\right\}.$$

where

(27) $$\Delta_1=\beta r^{1\beta}(r_1{}^{\alpha}-a^{\alpha})-\alpha r_1{}^{\alpha}(r_1{}^{\beta}-a^{\beta}).$$

6. Solution for the outer fluid region. Similarly we have for the outer region,

(28)

$$u=\frac{RR_m}{(\beta-\alpha)(m^2RR_m-M^2)\,\Delta_2}\left[\frac{\frac{1}{2}P(r_2^2-r_1^2)-\tau_0(r_1+r_2)}{r_1^{mRm}-r_2^{mRm}}\cdot m\Delta_2\,r^{-mR}+Pr_2^2\left\{\frac{\alpha}{\beta-2}\right.\right.$$

$$\left(m-\frac{\beta}{R_m}\right)(1-b^{\beta}\,r_2{}^{-\beta})-\frac{\beta}{\alpha-2}\left(m-\frac{\alpha}{R_m}\right)(1-b^{\alpha}\,r_2{}^{-\alpha})\Big\}-\tau_0r_2\Big\{\frac{\alpha}{\beta-1}\left(m-\frac{\beta}{R_m}\right)$$

$$(1-b^{\beta}\,r_2{}^{-\beta})-\frac{\beta}{\alpha-1}\left(m-\frac{\alpha}{R_m}\right)(1-b^{\alpha}\,r_2{}^{-\alpha})\Big\}\Big]\ [\beta r_2{}^{\beta}(r^{\alpha}-b^{\alpha})-\alpha r_2{}^{\alpha}(r^{\beta}-b^{\beta})]+\frac{1}{\Delta_2}$$

$$\left\{\frac{PR\,(mR_m-2)\,r_2^2}{(\alpha-2)(\beta-2)}-\frac{\tau_0R\,(mR_m-1)\,r_2}{(\alpha-1)(\beta-1)}\right\}\{(r_2{}^{\beta}-b^{\beta})(r^{\alpha}-b^{\alpha})-(r_2{}^{\alpha}-b^{\alpha})(r^{\beta}-b^{\beta})\}$$

$$+\frac{PR\,(mR_m-2)}{2(\alpha-2)(\beta-2)}(r^2-b^2)-\frac{\tau_0R(mR_m-1)}{(\alpha-1)(\beta-1)}(r-b),$$

(29)

$$H = \frac{RR_m}{mM^2(\alpha-\beta)(m^2RR_m - M^2)\Delta_2}\Big[\frac{\frac{1}{2}(Pr_2^2 - r_1^2) - \tau_0(r_1+r_2)}{r_1^{mRm} - r_2^{mRm}} m\Delta_2 r_2^{-mR} + Pr_2^2$$

$$\left\{\frac{\alpha}{\beta-2}\left(m - \frac{\beta}{R_m}\right)(1 - b^\beta r_2^{-\beta}) - \frac{\beta}{\alpha-2}\left(m - \frac{\alpha}{R_m}\right)(1 - b^\alpha r_2^{-\alpha})\right\} - \tau_0 r_2$$

$$\left\{\frac{\alpha}{\beta-1}\left(m - \frac{\beta}{R_m}\right)(1 - b^\beta r_2^{-\beta}) - \frac{\beta}{\alpha-1}\left(m - \frac{\alpha}{R_m}\right)(1 - b^\alpha r_2^{-\alpha})\right\}\Big]$$

$$[\beta r_2^\beta\{mR_m(mR_m-\beta)r^\alpha + M^2 b^\alpha\} - \alpha r_2^\sigma\{M^2 b^\beta + mR_m(mR_m-\alpha)r^\beta\}]$$

$$+ \frac{PRR_m}{2(\alpha-2)(\beta-2)}\left(r^2 - b^2 + \frac{2b^2}{mR_m}\right) - \frac{\tau_0 RR_m}{(\alpha-1)(\beta-1)}\left(r - b + \frac{b}{mR_m}\right),$$

(30)

$$\tau = \frac{R_m}{(\beta-\alpha)\Delta_2}\Big[\frac{\frac{1}{2}P(r_2^2 - r_1^2) - \tau_0(r_1+r_2)}{r_1^{mRm} - r_2^{mRm}} \cdot m\Delta_2 r_2^{-mR} + Pr_2^2\left\{\frac{\alpha}{\beta-2}\left(m - \frac{\beta}{R_m}\right)\right.$$

$$(1 - b^\beta r_2^{-\beta}) - \frac{\beta}{\alpha-2}\left(m - \frac{\alpha}{R_m}\right)(1 - b^\alpha r_2^{-\alpha})\Big\} - \tau_0 \tau_2\left\{\frac{\alpha}{\beta-1}\left(m - \frac{\beta}{R_m}\right)\right.$$

$$(1 - b^\beta r_2^{-\beta}) - \frac{\beta}{\alpha-1}\left(m - \frac{\alpha}{R_m}\right)(1 - b^\alpha r_2^{-\alpha})\Big\}\Big](r_2^\beta r^{\alpha-1} - r_2^\alpha r^{\beta-1}) + \frac{1}{\Delta_2}$$

$$\left\{\frac{P(mR_m-2)}{(\alpha-2)(\beta-2)} r_2^2 - \frac{\tau_0(R_m m-1)}{(\alpha-1)(\beta-1)}\right\}\{\alpha(r_2^\beta - b^\beta)r^{\alpha-1} - \beta(r_2^\alpha - b^\alpha)r^{\beta-1}\}$$

$$+ \frac{P(mR_m-2)}{(\alpha-2)(\beta-2)} r - \frac{\tau_0(m^2RR_m - mR - M^2)}{(\alpha-1)(\beta-1)}.$$

(31)

$$H_2 = \frac{RR_m}{mM^2(\alpha-\beta)(m^2RR_m - M^2)\Delta_2}\Big[\frac{\frac{1}{2}P(r_2^2 - r_1^2) - \tau_0(r_1+r_2)}{r_1^{mRm} - r_2^{mRm}} \cdot m\Delta_2 r_2^{-mR} + Pr_2^2$$

$$\left\{\frac{\alpha}{\beta-2}\left(m - \frac{\beta}{R_m}\right)(1 - b^\beta r_2^{-\beta}) - \frac{\beta}{\alpha-2}\left(m - \frac{\alpha}{R_m}\right)(1 - b^\alpha r_2^{-\alpha})\right\} - \tau_0 r_2$$

$$\left\{\frac{\alpha}{\beta-1}\left(m - \frac{\beta}{R_m}\right)(1 - b^\beta r_2^{-\beta}) - \frac{\beta}{\alpha-1}\left(m - \frac{\alpha}{R_m}\right)(-b^\alpha r_2^{-\alpha} + 1)\right\}\Big]$$

$$\Big[\beta r_2^\beta\{mR_m(mR_m-\beta)r_2^\alpha + M^2 b^\alpha\} - \alpha r_2^\alpha\{M^2 b^\beta + mR_m(mR_m-\alpha)r_2^\beta\}\Big]$$

$$- \frac{R}{\Delta_2}\left\{\frac{Pr_2^2(mR_m-2)}{(\alpha-2)(\beta-2)} - \frac{\tau_0 r_2(mR_m-2)}{(\alpha-1)(\beta-1)}\right\}\{(r_2^\beta - b^\beta)\{mR_m(mR_m-\beta) + M^2 b^\alpha\}$$

$$- (r_2^\alpha - b^\alpha)\{mR_m(mR_m-\alpha)r_2^\beta + M^2 b^\beta\}\} + \frac{PRR_m}{2(\alpha-2)(\beta-2)}\left(r_2^2 - b^2 + \frac{2b^2}{mR_m}\right)$$

$$- \frac{\tau_0 RR_m}{(\alpha-1)(\beta-1}\left(r_2 - b + \frac{b}{mR_m}\right),$$

(32)

$$U_0 = \frac{PR\,(mR_m - 2)}{2\,(\alpha-2)\,(\beta-2)}\,(r_2^2 - b^2) - \frac{\tau_0\, R\,(mR_m - 1)}{(\alpha-1)\,(\beta-1)}\,(r_2 - b) - \frac{RR_m}{(\alpha-\beta)\,(m^2\, RR_m - M^2)}$$

$$\left[\frac{\frac{1}{2}\,(P\,(r_2^2 - r_1^2) - \tau_0 (r_1 + r_2)}{r_1^{mRm} - r_2^{mRm}} \cdot m\, \Delta_2\, r_2^{-mR} + Pr_2^2 \left\{\frac{\alpha}{\beta-2}\left(m - \frac{\beta}{R_m}\right)(1 - b^\beta\, r_2^{-\beta})\right.\right.$$

$$\left. - \frac{\beta}{\alpha-2}\left(m - \frac{\alpha}{R_m}\right)(1 - b^\alpha\, r_2^{-\alpha})\right\} - \tau_0\, r_2 \left\{\frac{\alpha}{\beta-1}\left(m - \frac{\beta}{R_m}\right)(1 - b^\beta\, r_2^{-\beta}) - \frac{\beta}{\alpha-1}\right.$$

$$\left.\left.\left(m - \frac{\alpha}{R_m}\right)(1 - b^\alpha\, r_2^{-\alpha})\right\}\right],$$

where

(33)
$$\Delta_2 = \beta r_2^\beta\,(r_2^\alpha - b^\alpha) - \alpha\, r_2^\alpha\,(r_2^\beta - b^\beta).$$

7. Expressions for τ_0 and total flux. Using the boundary condition $H(b) = 0$, in equation (31) we obtain the value of τ_0 as

(34)

$$\tau_0 = P\left[\frac{mR_m\,(r_1 + r_2)\, r_2^{-mR}\, \Delta_2}{M^2\,(m^2\, RR_m - M^2)\,(r_1^{mRm} - r_2^{mRm})} - \frac{1}{(\alpha-1)}\,(\alpha - \beta - \beta b^\alpha\, r_2^{-\alpha})\, r_2^{\beta+1}\, b^\alpha + \frac{1}{\beta-1}\right.$$

$$(\alpha - \beta - \alpha\, b^\beta\, r_2^{-\beta})\, r_2^{\alpha+1}\, b^\beta + \frac{r_2\, b^{\alpha+\beta}}{M^2\,(\alpha-1)\,(\beta-1)}\,\{mR_m\,(\alpha-\beta)^2\,(mR_m - 1) - (\alpha-1)$$

$$(mR_m - \beta)\,(m^2\, R_m^2 - mR_m\,\beta + M^2) - (\beta-1)\,(mR_m - \alpha)\,(m^2\, R_m^2 - \alpha\, mR_m + M^2)\}$$

$$\left. + \frac{(\alpha-\beta)\, b\, \Delta_2}{(\alpha-1)\,(\beta-1)}\right]^{-1} \left[\frac{mR_m\,(r_2^2 - r_1^2)\, r_2^{-mR}\, \Delta_2}{2\, M^2\,(m^2\, RR_m - M^2)\, r_1^{mRm} - r_2^{mRm})} - \frac{1}{\alpha-2}\,(\alpha - \beta - \beta r_2^{-\alpha}\, b^\alpha)\right.$$

$$b^\alpha\, r_2^{2+\beta} + \frac{1}{\beta-2}\,(\alpha - \beta - \alpha\, b^\beta\, r_2^{-\beta})\, b^\beta\, r_2^{2+\alpha} + \frac{b^2\,(\alpha-\beta)\,\Delta_2}{(\alpha-2)\,(\beta-2)} + \frac{r_2^2\, b^{\alpha+\beta}}{M^2\,(\alpha-2)\,(\beta-2)}$$

$$\{mR_m\,(\alpha-\beta)^2\,(mR_m - 2) - (\alpha-2)\,(mR_m - \beta)\,(m^2\, R_m^2 - mR_m\,\beta + M^2) - (\beta-2)$$

$$(mR_m - \alpha)\,(m^2\, R_m^2 - m\,\alpha\, R_m + M^2)\}.$$

The total flux is given by

(35)

$$Q = 2\pi \int u\, r\, dr = \frac{\pi R\, R_m}{(\alpha-\beta)\,\Delta_1}\left[\frac{\frac{1}{2}\, P\,(r_2^2 - r_1^2) - \tau_0\,(r_1 + r_2)}{r_1^{mRm} - r_2^{mRm}} \cdot m\Delta_1\, r_1^{-mR} + Pr_1^2 \left\{\frac{\alpha}{\beta-2}\right.\right.$$

$$\left(m - \frac{\beta}{R_m}\right)(1 - a^\beta\, r_1^{-\beta}) - \frac{\beta}{\alpha-2}\left(m - \frac{\alpha}{R_m}\right)(1 - a^\alpha\, r_1^{-\alpha})\right\} + \tau_0\, r_1 \left\{\frac{\alpha}{\beta-1}\left(m - \frac{\beta}{R_m}\right)\right.$$

$$(1-a^\beta r_1^{-\beta})-\frac{\beta}{\alpha-1}\left(m-\frac{\beta}{R_m}\right)(1-a^\alpha r_1^{-\alpha})\Big\}\Big]\Big\{\frac{r_1^\beta}{\alpha+2}(r_1^{\alpha+2}-a^{\alpha+2})-\frac{r_1^\alpha}{\beta+2}$$

$$(r_1^{\beta+2}-a^{\beta+2})\Big\}+\frac{\pi R}{\Delta_1}\Big\{\frac{Pr_1^2(mR_m-2)}{(\alpha-2)(\beta-2)}+\frac{\tau_0 r_1(mR_m-1)}{(\alpha-1)(\beta-1)}\Big\}\Big\{\frac{\alpha}{\alpha+2}(r_1^\beta-a^\beta)$$

$$(r_1^{\alpha+2}-a^{\alpha+2})-\frac{\beta}{\beta+2}(r_1^\alpha-a^\alpha)(r_1^{\beta+2}-a^{\beta+2})\Big\}+\frac{\pi RR_m}{(\alpha-\beta)\Delta_2}$$

$$\Big[\frac{\frac{1}{2}P(r_2^2-r_1^2)-\tau_0(r_1+r_2)}{r_1^{mRm}-r_2^{mRm}}\cdot m\Delta_2 r_1^{-mR}+Pr_2^2\Big\{\frac{\alpha}{\beta-2}\left(m-\frac{\beta}{R_m}\right)(1-b^\beta r_2^{-\beta})$$

$$-\frac{\beta}{\alpha-2}\left(m-\frac{\alpha}{R_m}\right)(1-b^\alpha r_2^{-\alpha})\Big\}-\tau_0 r_2\Big\{\frac{\alpha}{\beta-1}\left(m-\frac{\beta}{R_m}\right)(1-b^\beta r_2^{-\beta})-\frac{\beta}{\alpha-1}$$

$$\left(m-\frac{\alpha}{R_m}\right)(1-b^\alpha r_2^{-\alpha})\Big\}\Big]\Big[\frac{r_2^\beta}{\alpha+2}(b^{\alpha+2}-r_2^{\alpha+2})+\frac{r_2^\alpha}{\beta+2}(r_2^{\beta+2}-b^{\beta+2})\Big]+\frac{\pi R}{\Delta_2}$$

$$\Big[\frac{Pr_2^2(mR_m-2)}{(\alpha-2)(\beta-2)}-\frac{\tau_0 r_2(mR_m-1)}{(\alpha-1)(\beta-1)}\Big]\Big[\frac{\alpha}{\alpha+2}(r_2^\beta-b^\beta)(b^{\alpha+2}-r_2^{\alpha+2})-\frac{\beta}{\beta-2}$$

$$(r_2^\alpha-b^\alpha)(b^{\beta+2}-r_2^{\beta+2})\Big]+\frac{PR(mR_m-2)}{8(\alpha-2)(\beta-2)}(a^4-r_1^4-b^4+r_2^4)$$

$$+\frac{\tau_0 R(mR_m-1)}{6(\alpha-1)(\beta-1)}(a^3+b^3-r_1^3-r_2^3).$$

Equating the values of U_0 from equations (26) and (32) we get an equation in φ_1, φ_2 and τ_0. Again eliminating P and U_0 from equations (20), (21), (25) and (31) we get an other equation in φ_1, φ_2 and τ_0. These two equations will determine the value of φ_1 and φ_2.

8. Solution for the flow between two cylinders without suction or injection under a radial magnetic field. If $m\to 0$, we have $\alpha\to M$ and $\beta\to -M$. Substituting these values in the equations (18) to (35) and simplifying we have for the inner region,

(36)

$$u=\frac{\{\frac{1}{2}P(r_1^2-r_1^2)-\tau_0(r_1+r_2)\}R}{2M^2\log r_2/r_1}\{r_1^{-M}(r^M-a^M)+r_1^M)(r^{-m}-a^{-m})\}$$

$$-\frac{PRr_1^2}{2M(m^2-4)}\{(m+2)r_1^{-m}(r^M-a^M)+(M-2)r_1^M(r^{-m}-a^{-m})\}-\frac{\tau_0 Rr_1}{2M(m^2-1)}$$

$$\{(M+1)r_1^{-m}(r^M-a^M)+(M-1)r_1^M(r^{-m}-a^{-M})\}+\frac{PR(r^2-a^2)}{M^2-4}-\frac{\tau_0 R(r-a)}{M^2-1},$$

(37)

$$H = H_2 + \frac{RR_m}{M^2}\{\tfrac{1}{2}P(r_2^2 - r_1^2) - \tau_0(r_1 + r_2)\}\Big\{1 + \frac{M}{2\log r_2/r_1}(r_1^{-m}r^M - r_1^M r^{-m})$$

$$+ \frac{PRR_m r_1^2}{2M^2(m^2-4)}\{m(r^M r_1^{-m} - r_1^M r^{-m}) + 2(a_1^{-m}r^M + r_1^M r^{-m}) - 4\} + \frac{\tau_0 RR_m r_1}{2M(M^2-1)}$$

$$\{M(r^M r_1^{-m} - \tau_1^M r^{-M}) + r_1^{-m}r^M + r_1^M r^{-m} - 2\} - \frac{PRR_m}{2(M^2-4)}(r^2 - r_1^2)$$

$$- \frac{\tau_0 RR_m}{M^2-1}(r - r_1),$$

(38)

$$\tau = \frac{\tfrac{1}{2}P(r_2^2 - r_1^2) - \tau_0(r_1 + r_2)}{2M\log r_2/r_1}\{r_1^{-m}r^M - r_1^M r^{-m}\} - \frac{Pr_1^2}{2(M^2-4)}\{(M+2)r_1^{-m}r^{m-1}$$

$$- (m-2)r_1^M r^{-m-1}\} - \frac{\tau_0 r_1}{2(m^2-1)}\{(M+1)r_1^{-m}r^{m-1} - (M-1)r_1^M r^{-M-1}\}$$

$$+ \frac{2Pr}{m^2-4} + \frac{\tau_0 M^2}{m^2-1},$$

(39)

$$U_0 = \frac{1}{a^M r_1^{-m} + a^{-m}r_1^M}\Big[\Big(\frac{PR_1^2}{m^2-4} + \frac{\tau_0 Rr_1}{m^2-1}\Big)(a^M r_1^{-m} + a^{-m}r^M - 2) + \Big(\frac{2PRr_1^2}{M(m^2-4)}$$

$$+ \frac{\tau_0 Rr_1}{m(m^2-4)}\Big)(a^m r_1^{-m} - a^{-m}r_1^m) + \frac{2PR(r_1^2 - a^2)}{m^2-4} + \frac{2\tau_0 R(r_1 - a)}{m^2-1}\Big],$$

Similarly we have for the outer region,

(40)

$$u = \frac{\{\tfrac{1}{2}P(r_2^2 - r_1^2) - \tau_0(r_1 + r_2)\}R}{2M^2\log r_2/r_1}\{r_2^{-m}(r^M - b^M) + r_2^M(r^{-m} - b^{-m})\} - \frac{PRr_2^2}{2M(M^2-4)}$$

$$\{(m+2)r_2^{-m}(r^M - b^m) + (m-2)(r^{-m}b^{-m})r_2^M\} + \frac{\tau_0 Rr_2}{2M(m^2-1)}$$

$$\{(M+1)(r^M - b^m)r_2^{-m} + (m-1)r_2^m(r^{-m} - b^{-m})\}\frac{PR(r^2 - b^2)}{m^2-4} - \frac{\tau_0 R}{m^2-1)}(r - b),$$

(41)

$$H = H_2 + \frac{RR_m[\tfrac{1}{2}P(r_2^2 - r_1^2) - \tau_0(r_1 + r_2)]}{2M^2\log r_2/r_1}(r_2^{-m}r^m - r_2^M r^{-m}) + \frac{PRR_m r_2^2}{2m^2(m^2-4)}$$

$$\{M(r^M r_2^{-m} - r^{-M}r_2^M) + 2(r_2^{-M}r^m + r^{-m}r_2^m - 2)\} - \frac{\tau_0 RR_m r_2}{2m^2(m^2-4)}$$

$$\{m(r_2^{-m}r^m r^{m} - r_2^M) + (r_2^{-m}r^m + r_2^m - 2)\} - \frac{PRR_m}{2(m^2-4)}(r^2 - r_2^m) + \frac{\tau_0 RR_m}{m^2-1}(r - r_2),$$

(42)

$$\tau = \frac{\frac{1}{2} P(r_2^2 - r_1^2) - \tau_0 (r_1 + r_2)}{2 M \log r_2/r_1} (r_2^{-m} r^{-m-1} - r_2^m r^{-M-1}) - \frac{P r_2^2}{2(m^2-4)} \{ (M+2) r_2^{-m} r^{M-1}$$

$$- (M-2) r_2^m r^{-m-1} \} + \frac{\tau_0 r_2}{2(m^2-1)} \{ (m+1) r_2^{-m} r^{m-1} - (m-1) r_2^M r^{-m-1} \}$$

$$+ \frac{2 Pr}{m^2-4} - \frac{\tau_0 M^2}{m^2-1},$$

(43)

$$U_0 = \frac{1}{b^M r_2^{-m} + b^{-m} r_2^m} \Big[\Big(\frac{P R r_2^2}{m^2-4} - \frac{\tau_0 R r_2}{m^2-1} \Big) (b^m r_2^{-m} + b^{-m} r_2^m - 2) + \Big(\frac{2 P R r_2^2}{M(m^2-4)}$$

$$- \frac{\tau_0 R r_2}{m(m^2-1)} \Big) (b^m r_2^{-m} - b^{-m} r_2^m) + \frac{2 P R (r_2^2 - b^2)}{m^2-4} - \frac{2 \tau_0 R (r_2 - b)}{m^2-1} \Big],$$

where

$$H_2 = \frac{R R_m [\frac{1}{2} P (r_2^2 - r_2^2) - \tau_0 (r_1 + r_2)]}{2 m^2 \log r_2/r_1} (r_2^m b^{-m} - r_2^{-m} b^m) + \frac{P R R_m r_2^2}{2m^2 (m^2-4)}$$

$$\{ m (r_2^m b^{-m} - r_2^{-m} bM) - 2 (r_2^{-m} b^m + r_2^m b^{-m} - 2) \} + \frac{P R R_m}{2(m^2-4)} (b^2 - r_2^2)$$

$$- \frac{\tau_0 R R_m}{m^2-1} (b - r_2).$$

For the plug region, we have

(44) $$\tau = - \tau_0 \frac{r_2}{r} - \frac{1}{2} P \Big(r - \frac{r_2^2}{r} \Big) + \frac{1}{r} \{ \tfrac{1}{2} P (r_2^2 - r_1^2) - \tau_0 (r_1 + r_2) \} \frac{\log r}{\log r_2/r_1},$$

(45) $$H = H_2 + \frac{R R_m [\frac{1}{2} P (r_2^2 - r_1^2) - \tau_0 (r_1 + r_2)]}{m^2 \log r_2/r_1)} \log r_2/r,$$

(46) $$U_0 = \frac{\{ \frac{1}{2} P (r_2^2 - r_1^2) - \tau_0 (r_1 + r_2) \} R}{M^2 \log r_2/r_1}.$$

Now equating U_0 from equations (39), (43) and (46), we obtain two equations

(47)

$$\tau_0 = P \Big[\frac{r_2}{m^2-1} \{ b^m r_2^{-m} + b^{-m} r_2^m - 2 + \frac{1}{M} (b^m r_2^{-m} - b^{-m} r_2^m) + 2 (r_2 - b) \}$$

$$-\frac{r_1+r_2}{2M^2\log r_2/r_1}(r_2^{-m}b^m+b^{-m}r_2^M)\Big]^{-1}\Big[\frac{r_2^2}{m^2-4}\{b^m r_2^{-m}+b^{-m}r_2^m-2+\frac{2}{M}$$

$$(b^m r_2^{-m}-b^{-m}r_2^m)+2(r_2^2-b^2)-\frac{r_2^2-r_1^2}{4M^2\log r_2/r_1}(b^m r_2^{-m}+b^{-m}r_2^m)\Big],$$

and

(48)

$$\tau_0=P\Big[\frac{r}{m^2-1}\{a^m r_1^{-m}+a^{-m}r_1^M-2+\frac{1}{M}(aM r_1^{-m}-a^{-m}r_1^m)+2(r_1-a)\}$$

$$+\frac{r_1+r_2}{2M^2\log r_2/r_1}(a^m r_1^{-m}+a^{-m}r_1^m)\Big]^{-1}\Big[\frac{r_2^2-{}_1^2}{4m^2\log r_2/r_1}(a^m r_1^{-m}+a^{-m}r_1^M)-\frac{r_1^2}{m^2-4}$$

$$\{\tau^m r_1^{-m}+a^{-m}r_1^m-2+\frac{2}{m}(a^m r_1^{-m}-a^{-m}r_1^m)+2(r_1^2-a^2)\}\Big].$$

These two equations will determine the value of r_1 and r_2.

The total flux is given by

(49)

$$Q=\frac{\pi R[P(r_2^2-r_1^2)-2\tau_0(r_1+r_2)]}{4M^2(m^2-4)\log r_1/r_2}\{M(2r_1^2-2r_2^2-a^{2+m}r_1^{-m}-a^{2-m}r_1^m+b^{2+m}$$

$$r_2^{-m}+b^{2-m}r_2^m)+2(a^{2+m}r_1^{-m}-a^{2-m}r_1^m+b^{2-m}r_2^m-b^{2+m}r_2^{-m})\}$$

$$+\frac{\pi R}{2(2-a^M r_1^{-m}-a^{-m}r_1^m)}\Big[\frac{Pr_1^2}{m^2-4}\{(m+2)(1-a^{-m}r_1^m)+(m+2)(1-a^m r_1^{-m})\}$$

$$+\frac{\tau_0 r_1}{m^2-1}\{m-1)(1-a^{-m}r_1^M)+(m+1)(1-a^m r_1^{-m})\}\Big]\Big[\frac{r_1^{-m}}{m+2}(r_1^{m+2}-a^{m+2})$$

$$+\frac{r_1^m}{m-2}(r_1^{2-m}-a^{2-m})\Big]\frac{\pi R}{(2-a^m r_1^{-m}-a^{-m}r_1^m)(m^2-4)}\Big\{\frac{2Pr_1^2}{m^2-4}+\frac{\tau_0 r_1}{M^2-1}\Big\}$$

$$\{(m+2)(r_1^m-a^m)(r_1^{2-m}-a^{2-m})-(m-2)(r_1^{-m}-a^{-m})(r_1^{2+m}-a^{2+M})\}$$

$$\frac{\pi R}{2(2-b^m r_2^{-m}-b^{-m}r_2^m)}\Big[\frac{Pr_2^2}{m^2-4}\{m+2)(1-b^m r_2^{-m})+(m-2)(1-b^{-m}r_2^m)\}$$

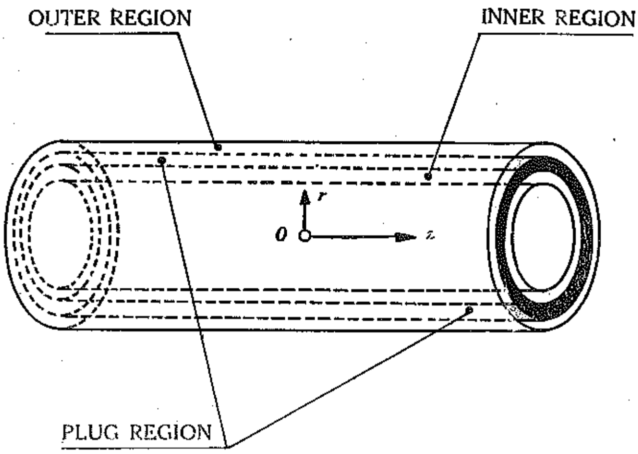
$$-\frac{\tau_0 r_2}{m^2-1}\{(m-1)(1-b^m r_2^{-m})+(m-1)(1-b^m r_2^{-m})\}\Big]\Big[\frac{r_2^{-m}}{m+2}(b^{m+2}-r_2^{m+2})$$

$$\frac{r_2^m}{m+2}(b^{2-m}-r_2^{2-m})\Big]+\frac{\pi RM}{(m^2-4)(2-b^m r_2^{-m}-b^{-m}r_2^m)}\Big[\frac{2Pr_2^2}{m^2-4}-\frac{\tau_0 r_2}{m^2-1}\Big]$$

$$[(m-2)(r_2^{-m}-b^{-m})(b^{m+2}-r_2^{m+2})-(m+2)(r_2^m-b^m)(b^{2-m}-r_2^{2-m})]$$

$$+\frac{\pi PR}{8(M^2-4)}(a^4-r_1^4+r_2^4-b^4)-\frac{\pi\tau_0 R}{6(m^2-1)}(a^3+b^3-r_1^3-{}_2^3).$$

9. Hydrodynamic flow of Bingham plastics between two cylinders with suction and injection. When $M \to 0$, we have the purely hydrodynamic flow. For this case $\alpha \to mR\,(=\nu)$ and $\beta \to mR_m$ $(=0)$. Substituting these values in the obtained solution and simplifying we get for the inner region



(50) $$u = \Big(\frac{P r_1^{2-\nu}}{2-\nu} + \frac{\tau_0 r_1^{1-\nu}}{1-\nu} (r^\nu - a^\nu) - \frac{P\nu}{2(2-\nu)} (r^2 - a^2) - \frac{\tau_0 \nu_a}{1-\nu} (r-a),$$

(51) $$U_0 = \Big(\frac{P r_1^{2-\nu}}{2-\nu} + \frac{\nu_0 r_1^{1-\nu}}{1-\nu} (r_1^\nu - a^\nu) - \frac{P\nu}{2(2-\nu)} r_1^2 - a^2 \Big) - \frac{\tau_0 \nu}{1-\nu} (r_1 - a),$$

(52) $$\tau = \Big(\frac{P r_1^{2-\nu}}{2-\nu} + \frac{\tau_0 r_1^{1-\nu}}{1-\nu} \Big) r^{\nu-1} - \frac{P\nu}{2-\nu} r + \frac{\tau_0}{1-\nu}\,.$$

For the outer region we get

(53) $$u = \Big(\frac{P r_2^{2-\nu}}{2-\nu} - \frac{\tau_0 r_2^{1-\nu}}{1-\nu} \Big) (r^\nu - b^\nu) - \frac{P\nu}{2(2-\nu)} (r^2 - b^2) + \frac{\tau_0 \nu}{1-\nu} (r-b),$$

(54) $$U_0 = \Big(\frac{P r_2^{2-\nu}}{2-\nu} - \frac{\tau_0 r_2^{1-\nu}}{1-\nu} \Big) (r_2^\nu - b^\nu) - \frac{P\nu}{2(2-\nu)} (r_2^2 - b^2) + \frac{\tau_0 \nu}{1-\nu} (r_2 - b),$$

(55) $$\tau = \Big(\frac{P r_2^{2-\nu}}{2-\nu} - \frac{\tau_0 a_2^{1-\nu}}{1-\nu} \Big) r^{\nu-1} - \frac{P\nu}{2-\nu} r - \frac{\tau_0}{1-\nu},$$

(56) $$\tau_0 = \tfrac{1}{2} P (r_2 - r_1).$$

The total flux is given by

(57)
$$Q = \frac{\pi\nu}{2+\nu} \Big(\frac{P r_1^{2-\nu}}{2-\nu} + \frac{\tau_0 r_1^{1-\nu}}{1-\nu} \Big) (a^{\nu+2} - r_1^{2+\nu}) + \frac{\pi\nu}{\nu+2} \Big(\frac{P r_2^{2-\nu}}{2-\nu} - \frac{\tau_0 r_2^{1-\nu}}{1-\nu} \Big)$$
$$(r_2^{2+\nu} - b^{2+\nu}) + \frac{\pi P \nu}{8(2-\nu)} (b^4 - a^4 + r_1^4 - r_2^4) - \frac{\pi \tau_0 \nu}{6(1-\nu)} (a^3 + b^3 - r_1^3 - r_2^3)$$

10. Flow of a newtonian fluid between two cylinders with suction and injection under a radial magnetic field. If $\tau_0 \to 0$ we have $r_1 = r_2$. Substituting this in the solution and eliminating r_1 we obtain

(58)

$$u = \frac{P\,RR_m}{2(\alpha-2)(\beta-2)\,\Delta_3}\Big[\frac{2b^2M^2}{R_m^2}\{(b^\alpha-a^\alpha)(r^\beta-b^\beta)-(b^\beta-a^\beta)(r^\alpha-b^\alpha)\}$$

$$-\left(m-\frac{2}{R_m}\right)(b^2-a^2)\left\{\beta\left(m-\frac{\alpha}{R_m}\right)b^\beta(r^\alpha-b^\alpha)-\alpha\left(m-\frac{\beta}{R_m}\right)b^\alpha(r^\beta-b^\beta)\right\}\Big]$$

$$\frac{PR\,(mR_m-2)}{2(\alpha-2)(\beta-2)}(r^2-b^2)$$

(59)

$$H = \frac{PR\,R_m}{2(\alpha-2)(\beta-2)\,\Delta_3}\Big[ab^2\Big\{\left(m-\frac{\beta}{R_m}\right)(b^\beta-a^\beta)(r^\alpha-b^\alpha)-\left(m-\frac{\alpha}{R_m}\right)(b^\alpha-a^\alpha)$$

$$(r^\beta-b^\beta)\Big\}-\left(m-\frac{2}{R_m}\right)(b^2-a^2)\Big\{\beta\left(m-\frac{\alpha}{R_m}\right)b^\beta(r^\alpha-b^\alpha)+\alpha\left(m-\frac{\beta}{R_m}\right)$$

$$b^\alpha(b^\beta-r^\beta)\Big\}\Big]+\frac{PR\,R_m}{2(\alpha-2)(\beta-2)}(r^2-b^2),$$

where

$$\Delta_3 = \beta\left(m-\frac{\alpha}{R_m}\right)b^\beta(a^\alpha-b^\alpha)+\alpha\left(m-\frac{\beta}{R_m}\right)b^\alpha(b^\beta-a^\beta). \tag{60}$$

The total flux is given by

(61)

$$Q = \frac{\pi\,P\,RR_m}{2(\alpha-2)(\beta-2)\,\Delta_3}\Big[\frac{2M^2b^2}{R_m^2}\Big\{(b^\alpha-a^\alpha)\left[\frac{1}{\beta+2}(b^{\beta+2}-a^{\beta+2})-\frac{b^\beta}{2}(b^2-a^2)\right]$$

$$-(b^\beta-a^\beta)\left[\frac{1}{\alpha+2}(b^{\alpha+2}-a^{\alpha+2})-\frac{b^\alpha}{2}(b^2-a^2)\right]\Big\}-\left(m-\frac{2}{R_m}\right)(b^2-a^2)$$

$$\Big\{\beta\left(m-\frac{\alpha}{R_m}\right)b^\beta\left[\frac{1}{\alpha+2}(b^{\alpha+2}-a^{\alpha+2})-\frac{b^\alpha}{2}(b^2-a^2)\right]-\alpha\left(m-\frac{\beta}{R_m}\right)b^\alpha$$

$$\left[\frac{1}{\beta+2}(b^{\beta+2}-a^{\beta+2})-\frac{b^\beta}{2}(b^2-a^2)\right]\Big\}\Big]-\frac{\pi\,P\,RR_m}{8(\alpha-2)(\beta-2)}\left(m-\frac{2}{R_m}\right)(b^2-a^2)^2.$$

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ÖZET

Emme ve püskürtülmeye tâbi tutulan, iletken bir BINGHAM plâstik cisminin iletken olmayan aynı eksenli iki silindrik yüzey arasındaki akımı tetkik edilmektedir: yarıçap doğrultusunda bir magnetik alan, cisim üzerinde etki yapmaktadır. Tam çözüm elde edilmiş ve birkaç özel hal incelenmiştir.