MODEL OF SPECULATIVE STRATEGY OF CURRENCY DEPARTMENT OF A BANK

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ABSTRACT. In the article, strategy of currency department of bank on the purchase-sale of currency is considered. The basic law of speculative strategy is known long ago: "at first it is necessary to purchase a commodity as cheap as possible, and then sell it as expensive as possible". Such strategy allows to gain a speculative profit which is determined by the margin (that is the difference of courses of sale and purchase of this commodity).

1. INTRODUCTION

The process of speculation is inherent subject of any industry of economy. At the market of equities there is another law of extraction of speculative profit that is known as downplaying: at first sell a commodity, while it stands expensive, and then purchase the same commodity again when its price declines. It is easy to understand that all of possible speculative strategies are taken from different combinations indicated higher than two basic strategies of extraction arrived during their diversification on different markets, variety of financial instruments, by the different set of financial tools between itself and depending on time of activity of financial subject. One of such financial subjects is when there is a bank in position to conduct speculative operations in both directions.

In this paper, as a mathematical model of extraction process of speculative profit, the risk with random bonuses are used. These models were considered by Artyukhov S.V., Bazyukin O.A, Korolev V.Yu., Kudryavcev A.A. [1] and Korolev V.Yu., Minkina P.I., Shorgin C.Ya. [2].

2. PROBLEM

We will denote an initial capital by $X_0 > 0$, which a bank reserves for currency speculations. We will suppose that $X = (X_t)_{t\geq 0}$ is a random process describing the evolution of capital X_0 of the considered bank. The classic process of risk with random bonuses is described by a model [3, p. 783]:

(1) $$X_t = X_0 + ct - S_t,$$

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where S_t is summarized payment of bank at the moment of time t and c is permanent speed of capital receipt per time unit.

We will suppose that a course at the currency market $c_t^0(\omega)$ changes on the considered interval of time and a bank has possibility both to purchase and sell any amount of currency at price $c_t^0(\omega)$. The amount $c_t^0(\omega)$ is named as the cost of exchange at the moment of time t. Here ω is a random event, characterizing a currency market condition. We assume that a part of capital X_0 is exchanged for currency at price of exchange $c_0^0(\omega)$ at the initial moment of time $t=0$ at the currency market. Then the bank assigns the prices for a purchase and sale currencies which also depends on time. We will define them by $c_t^+(\omega)$ (cost of sale) and $c_t^-(\omega)$ (cost of purchase) respectively. Clearly, the price $c_t^+(\omega)$ must be greater than $c_t^0(\omega)$ and price $c_t^-(\omega)$ must be less than $c_t^0(\omega)$. Strategy management works to determine the prices $c_t^+(\omega)$ and $c_t^-(\omega)$ at the moment of time t so that at the end of covered period T it gets a maximum possible profit from the conducted operations.

We will describe the considered model. We suppose that the process of receipt of bonuses in a model (1) is described not as linear function ct but generalized Poisson process so that a risk process looks like:

(2) $$X_T(\omega) = X_0 + \sum_{j=1}^{N^+(T,\omega)} c_j^+(\omega_j) X_j^+(\omega_j) - \sum_{j=1}^{N^-(T,\omega)} c_j^-(\omega_j) X_j^-(\omega_j)$$

and

(3) $$V_T(\omega) = V_0 - \sum_{j=1}^{N^+(T,\omega)} X_j^+(\omega_j) + \sum_{j=1}^{N^-(T,\omega)} X_j^-(\omega_j),$$

where the value $V_t(\omega)$ determines the volume of present currency at the moment of time t, $V_0 \geq 0$ is an initial volume of present currency, $X_j^+(\omega_j)$ is the volume of currency that was sold to j-client, and $X_i^-(\omega_i)$ is the volume of currency that was purchased by i-client. Random processes $N^+(T,\omega)$ and $N^-(T,\omega)$ are accordingly equal to the amount of clients purchasing or selling currency for considered period. Further, we will suppose that all of the considered processes and random variables are independent in an aggregate and each of sequences $\{X_j^+(\omega_j)\}_{j\geq 1}$, $\{X_j^-(\omega_j)\}_{j\geq 1}$, $\{c_j^+(\omega_j)\}_{j\geq 1}$ and $\{c_j^-(\omega_j)\}_{j\geq 1}$ consist of independent, identically distributed random variables. Thus, random processes $X_T(\omega)$ and $V_T(\omega)$ determine the volume of capital and volume of present currency at the end of speculative strategy of bank.

In the rest of the paper, we will be interested in the capital of bank and its currency reserve at some fixed moment of time *t*:

$$X_t(\omega) = X_0 + \sum_{j=1}^{N^+(t,\omega)} c_j^+(\omega_j) X_j^+(\omega_j) - \sum_{j=1}^{N^-(t,\omega)} c_j^-(\omega_j) X_j^-(\omega_j) \tag{4}$$

and

$$V_t(\omega) = V_0 - \sum_{j=1}^{N^+(t,\omega)} X_j^+(\omega_j) + \sum_{j=1}^{N^-(t,\omega)} X_j^-(\omega_j)\,, \tag{5}$$

where $N^+(t,\omega)$ is projection (value) of process $N^+(T,\omega)$ and $N^-(t,\omega)$ is projection of process $N^-(T,\omega)$ at the moment of time *t*. We will present numbers $c_t^+(\omega)$ and $c_t^-(\omega)$ as

$$c_t^+(\omega) = c_t^0(\omega) + \delta_t^+(\omega)\,, \quad c_t^-(\omega) = c_t^0(\omega) - \delta_t^-(\omega)\,,$$

where $\delta_t^+(\omega) > 0$ and $\delta_t^-(\omega) > 0$ are values, on which the cost of exchange $c_t^0(\omega)$ differs from the costs of sale and purchase, proposed a bank, giving a possibility to extract a speculative profit. A bank must conduct such strategy that values $\delta_t^+(\omega)$ and $\delta_t^-(\omega)$ give a maximum profit.

It is easy to understand that increase of cost of sale at $\delta_t^+(\omega)$ or diminishing of cost of purchase at $\delta_t^-(\omega)$ conduce for diminishing of amount of clients which are interested in purchasing, selling or exchanging currency in a bank on the average per time unit. On the other hand, obviously as margin $\delta_t^+(\omega) - \delta_t^-(\omega)$ approaching to zero (costs of sale and purchase of currency to $c_t^0(\omega)$) intensity of both streams of clients must be increased.

We suppose that a bank is in a position to exchange currency for UAH (Ukrainian hryvnia) at any moment of time *t* at price $c_t^0(\omega)$. Then by (4) and (5) we get that total assets of bank at the moment of time *t* is equal to

$$c_t^0(\omega)\, V_t(\omega) + X_t(\omega) = X_0 + \sum_{j=1}^{N^+(t,\omega)} \delta_j^+(\omega_j) X_j^+(\omega_j) +$$

$$\sum_{j=1}^{N^-(t,\omega)} \delta_j^-(\omega_j) X_j^-(\omega_j) + c_t^0(\omega)\, V_0 .$$

Our problem is to study a random variable

$$P_t(\omega) = \sum_{j=1}^{N^+(t,\omega)} \delta_j^+(\omega_j) X_j^+(\omega_j) + \sum_{j=1}^{N^-(t,\omega)} \delta_j^-(\omega_j) X_j^-(\omega_j),$$

characterizing profit of the bank where random variables $N^+(t,\omega)$ and $N^-(t,\omega)$ have Poisson distribution with parameters $\lambda_t^+(\omega)$ and $\lambda_t^-(\omega)$, respectively. These parameters are decreasing functions. Unlikely considered in literature (see [4, p. 405]), some complications occurs since the parameters of the Poisson distribution are not constants they are described by some random processes.

3. RESULTS

Strategy of bank consists in achievement of next two aims.

Problem 1. Defining $\delta_t^+(\omega)$ and $\delta_t^-(\omega)$ to maximize the expected profit:

$$\mathrm{M}[P_t(\omega)] \to \max, \tag{6}$$

where $\mathrm{M}[\cdot]$ means the operator of the expected value.

Problem 2. Defining $\delta_t^+(\omega)$ and $\delta_t^-(\omega)$ to get profit not less than the fixed number $P_0 > 0$, with probability more than some significance level, near to 1:

$$\mathrm{P}(P_t(\omega) \ge P_0) \ge \alpha . \tag{7}$$

It is required to find the courses of purchase and sale currencies, providing with large probability to get profit not less than P_0.

To simplify the decision of (6) we divide the period [0, *T*] on the equal intervals of time (equal, for example, to one day) and we suppose that on this interval, parameters $\delta_t^+(\omega)$, $\delta_t^-(\omega)$, $\lambda_t^+(\omega)$

and $\lambda_t^-(\omega)$ do not depend on time *t*. They are random variables but not random processes. Then on "short" interval the expected profit is equal to [4, p. 407]:

(8) $$\mathrm{M}[P_t(\omega)] = \lambda^+\delta^+ \mathrm{M}[X_1^+] + \lambda^-\delta^- \mathrm{M}[X_1^-].$$

It is clear that parameters δ^+ and δ^- have influence on intensity of bank's clients, therefore decision of (6), even with the shown simplification, it is impossible in general case. Additional assumptions are needed about the concrete type of functional dependences $\lambda^+(\delta)$ and $\lambda^-(\delta)$. These types of dependences can be received in practice, coming from the statistical considering. We will consider problem 1 on the set based on intercommunication of intensity of clients and margin. We suppose that at large critical values $\delta^+ > \delta_0^+$ and $\delta^- > \delta_0^-$ the streams of clients dry up. Values δ_0^+ and δ_0^- can be certain both by the clean mathematical consideration and by economic, conditioned elasticity of currency. Further we will suppose that $\delta^+ = \delta^- = \delta$ which means that a situation at the currency market during one day is stable enough.

We will suppose that $\lambda^+(\delta)$ and $\lambda^-(\delta)$ quadratically depend on δ :

(9) $$\lambda^+(\delta) = a_0^+ - a_1^+\delta - a_2^+\delta^2 \text{ and } \lambda^-(\delta) = a_0^- - a_1^-\delta - a_2^-\delta^2,$$

where $a_i^{\pm}$ are some positive constants (*i* = 0, 1, 2). Equality (9) is analytical expression of a classical demand curve: the change of demand depends only on the change of margin and does not depend on the current value of service. In other words, elasticity of demand at price in any point of the examined demand curve is constant and does not depend on a point. Set $\alpha_1^+ = \mathrm{M}[X_1^+]$, $\alpha_1^- = \mathrm{M}[X_1^-]$. After the substitution of correlations (9) in (8), we get

$$[\mathrm{M}P_t(\omega)] = \delta\left(a_0^+\alpha_1^+ + a_0^-\alpha_1^- - \delta(a_1^+\alpha_1^+ + a_1^-\alpha_1^-) - \delta^2(a_2^+\alpha_1^+ + a_2^-\alpha_1^-)\right) = F(\delta).$$

Right part of the last equality is a cubic function $F(\delta)$. $F(\delta)$ has two critical points (zeros of derivate $F'(\delta)$**),** one of which negative (on it the expected value takes minimum value), and in the second point

(10) $$\delta_1 = \frac{a_1^+\alpha_1^+ + a_1^-\alpha_1^- + \sqrt{\left(a_1^+\alpha_1^+ + a_1^-\alpha_1^-\right)^2 + \left(a_0^+\alpha_1^+ + a_0^-\alpha_1^-\right)^2}}{3(a_2^+\alpha_1^+ + a_2^-\alpha_1^-)}$$

the expected value takes maximum value. A critical value δ can be defined by

(11) $$\left(a_0^+\alpha_1^+ + a_0^-\alpha_1^- - \delta(a_1^+\alpha_1^+ + a_1^-\alpha_1^-) - \delta^2(a_2^+\alpha_1^+ + a_2^-\alpha_1^-)\right) = 0.$$

We suppose that a bank conducts currency operations in the conditions of crisis in principle: “how much currency is purchased so much and sold”. The amount of the purchased and sold currency expected in this case are equal to $\alpha_1^+ = \alpha_1^-$. Then the formula (10) becomes

(12) $$\delta_1 = \frac{a_1^+ + a_1^- + \sqrt{\left(a_1^+ + a_1^-\right)^2 + \left(a_0^+ + a_0^-\right)^2}}{3(a_2^+ + a_2^-)}.$$

Therefore, a margin does not depend on the expected influx of clients. Moreover, because of the balance of the sold and purchased currency in one day it is possible to consider it as zero, i.e. $\lambda^+(\delta) = \lambda^-(\delta)$, then the formula (12) becomes

(13) $$\delta_1 = \frac{a_1^+ + \sqrt{\left(a_1^+\right)^2 + \left(a_0^+\right)^2}}{3a_2^+}.$$

For example, on the basis of operations for a few previous days, regressive dependence of intensity of buyers was built:

$$\lambda^+(\delta) = 3{,}6 - 6\delta - 130\delta^2.$$

Then by (13) we get the optimum value of margin: δ_1=0,1 UAH.

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