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A-STATISTICALLY LOCALIZED SEQUENCES IN n-NORMED SPACES

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Abstract. In 1974, Krivonosov defined the concept of localized sequence that is defined as a generalization of Cauchy sequence in metric spaces. In this present work, the A-statistically localized sequences in n-normed spaces are defined and some main properties of A-statistically localized sequences are given. Also, it is shown that a sequence is A-statistically Cauchy iff its A-statistical barrier is equal to zero. Moreover, we define the uniformly A-statistically localized sequences on n-normed spaces and investigate its relationship with A-statistically Cauchy sequences.

1. Introduction and Background

In 1922, Banach defined normed linear spaces as a set of axioms. Since then, mathematicians keep on trying to find a proper generalization of this concept. The first notable attempt was by Vulich [41]. He introduced K-normed space in 1937. In another process of generalization, Siegfried Gähler [5] introduced 2-metric in 1963. As a continuation of his research, Gähler [6] proposed a mathematical structure, called 2-normed space, as a generalization of normed linear spaces. A.H. Siddiqi delivered a series of lectures on this theme in various conferences in India and Iran. His joint paper with Gähler and Gupta [8] also provide valuable results related to the theme of this paper. Results up to 1977 were summarized in the survey paper by Siddiqi [40]. As a further extension, he introduced n-metric and n-norm in his subsequent works Gähler [7] and regarded normed linear spaces as 1-normed spaces. However, many researchers disagree to consider 2-norm and n-norm as generalization of norm. In spite of this disagreement, several researchers have

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worked on this topic for decades Gürdal and Pehlivan [10,11], Gürdal and Açık [12], Gürdal and Şahiner [13], Gürdal et al. [14], Mohiuddine et al. [23], Mursaleen [24], Savaş and Sezer [37], Savaş and Gürdal [31–33], Savaş et al. [34] and Yegül and Dündar [45,46]. They have found out many interesting properties of this space and lots of fixed point theorems are established.

This paper was inspired by Krivonosov [18], where the concept of a localized sequence was introduced, which can be treated as a generalization of a Cauchy sequence in metric spaces. We will often quote some results from Krivonosov [18], that can be easily transferred to the concept of A-statistically localized sequence and the A-statistical localor of a sequence in n-normed space. Let X is a metric space with a metric $d(\cdot,\cdot)$ and (x_n) is a sequence of points in X. It is an interesting fact that if $F : X \to X$ is a mapping with the condition $d(Fx, Fy) \leq d(x, y)$ for all $x, y \in X$, then for every $x \in X$ the sequence $(F^n x)$ is localized at every fixed point of the mapping F. This means that fixed points of the mapping F is contained in the localor of the sequence $(F^n x)$. Motivating the above facts and the fact that the localor of a sequence can be extended by changing the usual limit to the statistical limit (see Fridy [4]) of a sequence. Recently, the authors in [25] have extended the concepts and results, which were given in [18], by changing the usual limit to the statistical limit in metric spaces. This definition has been extended to statistical localized and ideal localized in metric space Nabiev et al. [25, 26] and in 2-normed spaces Yamancı et al. [43, 44], and they obtained interested results about this concept.

This paper consists of three sections with the new results in sections 2-3. In Section 2 the concept of the A-statistically localized sequence and the A-statistical localor of a sequence in n-normed space is introduced and fundamental properties of A-statistically localized sequences are studied. In Section 3, we prove that a sequence is A-statistically Cauchy sequence if and only if its A-statistical barrier is equal to zero. Moreover, we define the uniformly A-statistically localized sequences on n-normed spaces and investigate its relationship with A-statistically Cauchy sequences and prove that in n-normed linear spaces each A-statistically bounded sequence has everywhere A-statistically localized subsequence.

Throughout this paper, let A be a nonnegative regular matrix and $\mathbb{N}$ will denote the set of all positive integers. Let X and Y be two sequence spaces and $A = (a_{n_k})$ be an infinite matrix. If for each $x \in X$ the series $A_n(x) = \sum_{k=1}^{\infty} a_{n_k} x_k$ converges for each n and the sequence $Ax = \{A_n(x)\} \in Y$, we say that A maps X into Y. By (X, Y) we denote the set of all matrices which maps X into Y. In addition if the limit is preserved, then we denote the class of such matrices by $(X, Y)_{\text{reg}}$. A matrix A is called regular if $A \in (c, c)$ and $\lim_{k\to\infty} A_k(x) = \lim_{k\to\infty} x_k$ for all $x = \{x_k\}_{k\in N} \in c$ when c, as usual, stands for the set of all convergent sequences. It is well known that the necessary and sufficient condition for A to be regular are

i) $\|A\| = \sup_n \sum_k |a_{n_k}| < \infty$;
ii) $\lim a_{n_k} = 0$, for each k;
iii) $\lim_n \sum_k a_{n_k} = 1$.

The idea of A-statistical convergence was introduced by Kolk [17] using a nonnegative regular matrix A. For a nonnegative regular matrix $A = (a_{n_k})$, a set $K \subset \mathbb{N}$ will be said to have A-density if $\delta_A(K) = \lim_{n\to\infty} \sum_{k\in K} a_{n_k}$ exists. The real number sequence $x = \{x_k\}_{k\in\mathbb{N}}$ is said to be A-statistically convergent to L provided that for every $\varepsilon > 0$ the set $K(\varepsilon) = \{k \in N : |x_k - L| \geqslant \varepsilon\}$ has A-density zero. Note that the idea of A-statistical convergence is an extension of the idea of statistical convergence introduced by Fast [3] using the idea of asymptotic density and later studied by Fridy [4], Connor [1], Salat [29], Gürdal and Yamancı [15], Mohiuddine and Alamri [20], Yamancı and Gürdal [42] and Savaş [30] (also, see [16,19,21,22,35,36,38]). Let $K = \{k(j) : k(1) < k(2) < k(3) < ...\} \subset \mathbb{N}$ and $\{x\}_K = \left\{x_{k(j)}\right\}$ be a subsequence of x. If the set K has A-density zero (i.e. $\delta_A(K) = 0$) the subsequence $\{x\}_K$ of the sequence x is called an A-thin subsequence. If the set K does not have A-density zero, the subsequence $\{x\}_K$ is called an A-nonthin subsequence of x. The statement $\delta_A(K) \neq 0$ means that either $\delta_A(K) > 0$ or $\delta_A(K)$ is not defined (i.e. K does not have A-density).

In [2], Connor and Kline extended the concept of a statistical limit (cluster) point of a number sequence x to a A-statistical limit (cluster) point replacing the matrix C_1 by a nonnegative regular matrix A. Recall that the number λ is a A-statistical limit point of the number sequence x provided that there is a subset $K = \{k(j)\}_{j=1}^{\infty}$ of positive integers with $\delta_A(K) \neq 0$ and $x_{k(j)} \to \lambda$ is $j \to \infty$ (see [2]). The number γ is a A-statistical cluster point of the number sequence $x = (x_k)$ provided that for every $\varepsilon > 0$, $\delta_A(K_\varepsilon) \neq 0$ where $K_\varepsilon := \{k \in \mathbb{N} : |x_k - \gamma| < \varepsilon\}$ (see [2]).

Now we recall the n-normed space which was introduced in [9] and some definitions on n-normed space (see [39]).

Definition 1. *Let $n \in \mathbb{N}$ and X be a real vector space of dimension $d \geq n$. (Here we allow d to be infinite.) A real-valued function $\|.,...,.\|$ on X^n satisfying the following four properties*

(i) $\|x_1, x_2, ..., x_n\| = 0$ *if and only if $x_1, x_2, ..., x_n$ are linearly dependent;*
(ii) $\|x_1, x_2, ..., x_n\|$ *is invariant under permutation;*
(iii) $\|x_1, x_2, ..., x_{n-1}, \alpha x_n\| = |\alpha| \ \|x_1, x_2, ..., x_{n-1}, x_n\|$, *for any $\alpha \in \mathbb{R}$;*
(iv) $\|x_1, x_2, ..., x_{n-1}, y + z\| \leq \|x_1, x_2, ..., x_{n-1}, y\| + \|x_1, x_2, ..., x_{n-1}, z\|$,
is called an n-norm on X and the pair $(X, \|.,...,.\|)$ is called an n-normed space.

It is well-known fact from the following corollary that n-normed spaces are actually normed spaces (see also [7]).

Corollary 1. *([9]) Every n-normed space is an $(n-r)$-normed space for all $r = 1, ..., n-1$. In particular, every n-normed space is a normed space.*

Example 1. *A standard example of an n-normed space is $X = \mathbb{R}^n$ equipped with the n-norm is*

$\|x_1, x_2, ..., x_{n-1}, x_n\| :=$ *the volume of the n-dimensional parallelepiped spanned by $x_1, x_2, ..., x_{n-1}, x_n$ in X.*

Observe that in any n-normed space $(X, \|., ..., .\|)$ we have

$$\|x_1, x_2, ..., x_{n-1}, x_n\| \geq 0$$

and

$$\|x_1, x_2, ..., x_{n-1}, x_n\| = \|x_1, x_2, ..., x_{n-1}, x_n + \alpha_1 x_1 + ... + \alpha_{n-1} x_{n-1}\|$$

for all $x_1, x_2, ..., x_n \in X$ and $\alpha_1, ..., \alpha_{n-1} \in \mathbb{R}$.

Let X be a real inner product space of dimension $d \geq n$. Equip X with the standard n-norm

$$\|x_1, x_2, ..., x_n\|_S := \begin{vmatrix} \langle x_1, x_1 \rangle & \cdots & \langle x_1, x_n \rangle \\ \vdots & \ddots & \vdots \\ \langle x_n, x_1 \rangle & \cdots & \langle x_n, x_n \rangle \end{vmatrix}^{1/2},$$

where $\langle ., . \rangle$ denotes the inner product on X. If $X = \mathbb{R}^n$, then this n-norm is the same as the n-norm in Example 1.

Definition 2. *A sequence $\{x_k\}$ in an n-normed space $(X, \|., ..., .\|)$ is said to convergent to an $l \in X$ if*

$$\lim_{k \to \infty} \|x_k - l, z_1, z_2, ..., z_{n-1}\| = 0$$

for every $z_1, z_2, ..., z_{n-1} \in X$.

Definition 3. *A sequence $\{x_k\}$ in an n-normed space $(X, \|., ..., .\|)$ is called a Cauchy sequence if*

$$\lim_{k,l \to \infty} \|x_k - x_l, z_1, z_2, ..., z_{n-1}\| = 0$$

for every $z_1, z_2, ..., z_{n-1} \in X$.

Let $a, x_1, ..., x_{n-1} \in X$ and for each $\varepsilon > 0$ define the ε-neighborhood of the points $a, x_1, ..., x_{n-1}$ as the set

$$U_\varepsilon (a, x_1, ..., x_{n-1}) = \{z : \|a - z, x_1 - z, ..., x_{n-1} - z\| < \varepsilon\}.$$

As it is known (see [28]) that the family of all sets

$$W_\Sigma = \bigcap_{i=1}^{n} U_{\varepsilon_i} \left(a, x_{1i}, ..., x_{(n-1)i}\right)$$

with arbitrary pairs $\Sigma = \left\{\left(x_{11}, ..., x_{(n-1)1}, \varepsilon_1\right), ..., \left(x_{1n}, ..., x_{(n-1)n}, \varepsilon_n\right)\right\}$ forms a complete system of neighborhoods of the point $a \in X$. Note that a set M in a linear n-normed space $(X, \|., ..., .\|)$ is said to be bounded if $\beta(M) < \infty$, where

$$\beta(M) = \sup\{\|a - z, x_1 - z, ..., x_{n-1} - z\| : a, x_1, ..., x_{n-1}, z \in M\}.$$

We also suppose that for any $\varepsilon > 0$ there exists a neighborhood U of 0 such that $\|x_1^*, x_2^*, ..., x_n^*\| < \varepsilon$ for all points $x_1^*, x_2^*, ..., x_n^*$ in U.

2. Definitions and notations

In this section, we introduce some basic definitions and notations in n-normed space $(X, \|., ..., .\|)$.

Definition 4. *(a) A sequence (x_n) in n-normed space $(X, \|., ..., .\|)$ is said to be A-statistically localized in the subset $K \subset X$ if the sequence $\|x_n - x, z_1, z_2, ..., z_{n-1}\|$ A-statistically converges for all $x, z_1, z_2, ..., z_{n-1} \in K$.*

(b) the maximal set on which a sequence is A-statistically localized is said to be a A-statistical localor of the sequence. We denote by $\mathrm{loc}^{\mathrm{st}_A}(x_n)$ the A-statistically localor of the sequence (x_n).

(c) A sequence (x_n) in n-normed space $(X, \|., ..., .\|)$ is said to be A-statistically localized everywhere if the A-statistical localor of (x_n) coincides with X.

(d) A sequence (x_n) in n-normed space $(X, \|., ..., .\|)$ is called A-statistically localized in itself if the A-statistically localor contains x_n for almost all n, that is,

$$\delta_A\left(\left\{n : x_n \notin \mathrm{loc}^{\mathrm{st}_A}(x_n)\right\}\right) = 0 \text{ or } \delta_A\left(\left\{n : x_n \in \mathrm{loc}^{\mathrm{st}_A}(x_n)\right\}\right) = 1.$$

(e) A sequence (x_n) is said to be A-statistically localized if the $\mathrm{loc}^{\mathrm{st}_A}(x_n)$ is not empty.

Definition 5. *Let (x_n) be a sequence in an n-normed space $(X, \|., ..., .\|)$. Then the sequence (x_n) is said to be A-statistical convergent to L if for each $\varepsilon > 0$ and any nonzero $z_1, z_2, ..., z_{n-1}$ in X,*

$$\delta_A(\{k \in \mathbb{N} : \|x_n - L, z_1, z_2, ..., z_{n-1}\| \geq \varepsilon\}) = 0.$$

In this case we write $x_n \overset{st_A}{\to} L$ or

$$st_A - \lim_{n\to\infty} \|x_n - L, z\| = 0.$$

Definition 6. *A sequence (x_n) in a linear n-normed space $(X, \|., ..., .\|)$ is said to be a A-statistically Cauchy sequence in X if for every $\varepsilon > 0$ and any nonzero $z_1, z_2, ..., z_{n-1} \in X$ there exists a number $N = N(\varepsilon, z_1, z_2, ..., z_{n-1})$ such that*

$$\delta_A(\{k \in \mathbb{N} : \|x_k - x_m, z_1, z_2, ..., z_{n-1}\| \geq \varepsilon\}) = 0$$

for all $m \geq N$.

We can see from the above definitions that every A-statistically Cauchy sequence in n-normed space $(X, \|.,...,.\|)$ is A-statistically localized everywhere in $(X, \|.,...,.\|)$. Actually, due to

$$|\|x_n - L, z_1, z_2, ..., z_{n-1}\| - \|x - x_m, z_1, z_2, ..., z_{n-1}\|| \leqslant \|x_n - x_m, z_1, z_2, ..., z_{n-1}\|,$$

we get

$$\{n \in \mathbb{N} : \|x_n - x_m, z_1, z_2, ..., z_{n-1}\| \geqslant \varepsilon\}$$
$$\supset \{n \in \mathbb{N} : |\|x_n - L, z_1, z_2, ..., z_{n-1}\| - \|x_m - L, z_1, z_2, ..., z_{n-1}\|| \geqslant \varepsilon\}.$$

Hence, the number sequence $\|x_n - L, z_1, z_2, ..., z_{n-1}\|$ is an A-statistically Cauchy sequence, then $\|x_n - L, z_1, z_2, ..., z_{n-1}\|$ is A-statistically convergent for every $L \in X$ and every nonzero $z \in X$. So, $\|x_n - L, z_1, z_2, ..., z_{n-1}\|$ in n-normed space $(X, \|.,...,.\|)$ is A-statistically localized everywhere.

Lemma 1. *A sequence (x_n) in linear n-normed space $(X, \|.,...,.\|)$ is an A-statistically Cauchy sequence if and only if there exists a subsequence $K = (k_n)$ of $\mathbb{N}$ with $\delta_A(K) = 1$ such that*

$$\lim_{n,m\to\infty} \|x_{k_n} - x_{k_m}, z_1, z_2, ..., z_{n-1}\| = 0$$

for all $z_1, z_2, ..., z_{n-1}$ in X.

Proof. Let (x_n) be an A-statistically Cauchy sequence in $(X, \|.,...,.\|)$. By definition, we can construct a decreasing sequence

$$(K_j) \subset \mathbb{N} \ (K_{j+1} \subset K_j, \ j = 1, 2, ...)$$

such that $\delta_A(K_j) = 1$ and $\|x_{k_1} - x_{k_2}, z_1, z_2, ..., z_{n-1}\| \leq \frac{1}{j}$ for all $z_1, z_2, ..., z_{n-1} \in X$, $k_1, k_2 \in K_j$, $j \in \mathbb{N}$. Further, let $v_1 \in K_1$. Then we can find $v_2 \in K_2$ with $v_2 > v_1$ such that $\frac{|K_2(n)|}{n} > \frac{1}{2}$ for each $n > v_2$. Inductively, we can construct a subsequence $(v_j) \in \mathbb{N}$ such that $v_j \in K_j$ for each $j \in \mathbb{N}$ and

$$\frac{|K_j(n)|}{n} > \frac{j-1}{j}$$

for each $n \geq v_j$. Then, as in [27], it is easy to prove that $\delta_A(K) = 1$ if

$$K = \{k \in \mathbb{N} : 1 \leq k < v_1\} \cup \left[\bigcup_{j\in\mathbb{N}} \{k : v_j \leq k < v_{j+1}\} \bigcap K_j\right].$$

Now, for $\varepsilon > 0$ choose $j \in \mathbb{N}$ such that $j > \frac{1}{\varepsilon}$. If $m, n \in K$ and $m, n > v_j$ we can find $r, s \geq j$ such that $v_r \leq m < v_{r+1}$, $v_s \leq n < v_{s+1}$. Hence, $m \in K_r$ and $n \in K_s$. For definite, suppose that $r \leq s$. Then $K_s \subset K_r$ which implies $m, n \in K_r$. Therefore, for every $z \in X$ we have

$$\|x_m - x_n, z_1, z_2, ..., z_{n-1}\| \leq \frac{1}{r} \leq \frac{1}{j} < \varepsilon.$$

Then we have

$$\lim_{\substack{n,m\to\infty \\ m,n\in K}} \|x_m - x_n, z_1, z_2, ..., z_{n-1}\| = 0.$$

Let us prove the converse. Suppose that $K = (k_n) \subset \mathbb{N}$ is a subsequence of subsets $\mathbb{N}$ such that $\delta_A(K) = 1$ and $\lim_{n,m\to\infty} \|x_{k_n} - x_{k_m}, z_1, z_2, ..., z_{n-1}\| = 0$ for all z in X. Then, for any $\varepsilon > 0$ there exists $p_0 = p_0(\varepsilon, z) \in \mathbb{N}$ such that $\|x_{k_n} - x_{k_m}, z_1, z_2, ..., z_{n-1}\| < \varepsilon$ for all $n, m \geq p_0$. This yields

$$\left\{k \in \mathbb{N} : \left\|x_k - x_{k_{p_0}}, z_1, z_2, ..., z_{n-1}\right\| \geq \varepsilon\right\} \subset \mathbb{N} \backslash \{k_{p_0+1}, k_{p_0+2}, ...\}.$$

Hence

$$\delta_A\left\{k \in \mathbb{N} : \left\|x_k - x_{k_{p_0}}, z_1, z_2, ..., z_{n-1}\right\| \geq \varepsilon\right\} \leq \delta_A\left(\mathbb{N} \backslash \{k_{p_0+1}, k_{p_0+2}, ...\}\right) = 0.$$

So, (x_k) is an A-statistically Cauchy sequence in X. □

Lemma 2. *A sequence (x_k) in $(X, \|., ..., .\|)$ is a A-statistically Cauchy sequence if and only if for every neighborhood U of the origin there is an integer $N(U)$ such that $n, m \geq N(U)$ implies that $x_{k_n} - x_{k_m} \in U$, where $K = (k_n) \subset \mathbb{N}$ and $\delta_A(K) = 1$.*

Proof. Let $z \in X$ and $\varepsilon > 0$. Suppose that there is $K = (k_n) \subset \mathbb{N}$ such that $x_{k_n} - x_{k_m} \in U_\varepsilon(0, z_1, z_2, ..., z_{n-1})$ for $n, m \geq N(U)$, where $U_\varepsilon(0, z_1, z_2, ..., z_{n-1})$ is a neighborhood of zero. This implies $\|x_{k_n} - x_{k_m}, z_1, z_2, ..., z_{n-1}\| < \varepsilon$ for every $n, m \geq N(U)$. Then $\lim_{n,m\to\infty} \|x_{k_n} - x_{k_m}, z_1, z_2, ..., z_{n-1}\| = 0$, i.e., (x_k) is an A-statistically Cauchy sequence in X.

Conversely, assume that $\lim_{n,m\to\infty} \|x_{k_n} - x_{k_m}, z_1, z_2, ..., z_{n-1}\| = 0$, where $K = (k_n) \subset \mathbb{N}$ and $\delta_A(K) = 1$. Let $W_\Sigma(0)$ be an arbitrary neighborhood of 0 with $\Sigma = \left\{\left(b_{11}, ..., b_{(n-1)1}, \alpha_1\right), ..., \left(b_{1r}, ..., b_{(n-1)r}, \alpha_r\right)\right\}$. By hypothesis, we have

$$\lim_{n,m\to\infty} \left\|x_{k_n} - x_{k_m}, b_{1j}, b_{2j}, ..., b_{(n-1)j}\right\| = 0 \text{ for } j = 1, ..., r.$$

Thus for each α_j there exists an integer N_j such that

$$\left\|x_{k_n} - x_{k_m}, b_{1j}, b_{2j}, ..., b_{(n-1)j}\right\| < \alpha_j$$

for $n, m \geq N_j$. Let $N = \max\{N_1, ..., N_r\}$. Then

$$\begin{aligned} &\left\|x_{k_n} - x_{k_m} - b_{1j}, ..., x_{k_n} - x_{k_m} - b_{(n-1)j}, x_{k_n} - x_{k_m}\right\| \\ &= \left\|x_{k_n} - x_{k_m}, b_{1j}, b_{2j}, ..., b_{(n-1)j}\right\| < \alpha_j \end{aligned}$$

for $n, m \geq N$ implies that $x_{k_n} - x_{k_m} \in W_\Sigma(0)$ for $n, m \geq N$ and thus it follows that (x_k) is an A-statistically Cauchy sequence in X. □

3. Main Results

Proposition 1. *Let (x_n) be an A-statistically localized sequence in linear n-normed space $(X, \|., ..., .\|)$. Then (x_n) is A-statistically bounded in X.*

Proof. Let (x_n) be an A-statistically localized sequence. So, the number sequence $(\|x_n - L, z_1, z_2, ..., z_{n-1}\|)$ A-statistically converges for some $L \in X$ and every $z \in X$. Then the number sequence $(\|x_n - L, z_1, z_2, ..., z_{n-1}\|)$ is A-statistically bounded, i.e., there is $S > 0$ such that

$$\delta_A \left(\{n \in \mathbb{N} : \|x_n - L, z_1, z_2, ..., z_{n-1}\| \geq S\}\right) = 0.$$

This implies that almost all elements of (x_k) are located in the neighborhood $U_S(0, z_1, z_2, ..., z_{n-1})$ of the origin. Then, sequence (x_k) is A-statistically bounded in X. □

Proposition 2. *Let $M = \mathrm{loc}^{\mathrm{st}_A}(x_n)$ and the point $y \in X$ be such that there exists $x \in M$ for any $\varepsilon > 0$ and every nonzero $z_1, z_2, ..., z_{n-1} \in M$ satisfying*

$$\delta_A \left(\{n \in \mathbb{N} : |\|x - x_n, z_1, z_2, ..., z_{n-1}\| - \|y - x_n, z_1, z_2, ..., z_{n-1}\|| \geqslant \varepsilon\}\right) = 0. \quad (1)$$

Then $y \in M$.

Proof. To show that the sequence $\beta_n = \|x_n - y, z_1, z_2, ..., z_{n-1}\|$ satisfies the A-statistically Cauchy criteria is enough. Let $\varepsilon > 0$ and $x \in M = \mathrm{loc}^{\mathrm{st}_A}(x_n)$ is a point that has the property (1). Because the sequence $\|x_n - x, z_1, z_2, ..., z_{n-1}\|$ satisfying the property (1) is A-statistically Cauchy sequence, then there exists a subsequence $K = (k_n)$ of $\mathbb{N}$ with $\delta_A(K) = 1$ such that

$$|\|x - x_{k_n}, z_1, z_2, ..., z_{n-1}\| - \|y - x_{k_n}, z_1, z_2, ..., z_{n-1}\|| \to 0$$

and

$$|\|x_{k_n} - x, z_1, z_2, ..., z_{n-1}\| - \|x_{k_m} - x, z_1, z_2, ..., z_{n-1}\|| \to 0$$

as $m, n \to \infty$. Clearly, there exists $n_0 \in \mathbb{N}$ for any $\varepsilon > 0$ and every nonzero $z_1, z_2, ..., z_{n-1} \in M$ such that for all $n \geq n_0$, $m \geq m_0$, we get

$$|\|x - x_{k_n}, z_1, z_2, ..., z_{n-1}\| - \|y - x_{k_n}, z_1, z_2, ..., z_{n-1}\|| < \frac{\varepsilon}{3} \quad (2)$$

$$|\|x - x_{k_n}, z_1, z_2, ..., z_{n-1}\| - \|x - x_{k_m}, z_1, z_2, ..., z_{n-1}\|| < \frac{\varepsilon}{3}. \quad (3)$$

From (2), (3) and (4)

$$\begin{aligned}
&|\|y - x_{k_n}, z_1, z_2, ..., z_{n-1}\| - \|y - x_{k_m}, z_1, z_2, ..., z_{n-1}\|| \\
&\leq |\|y - x_{k_n}, z_1, z_2, ..., z_{n-1}\| - \|x - x_{k_n}, z_1, z_2, ..., z_{n-1}\|| \\
&+ |\|x - x_{k_n}, z_1, z_2, ..., z_{n-1}\| - \|x - x_{k_m}, z_1, z_2, ..., z_{n-1}\|| \\
&+ |\|x - x_{k_m}, z_1, z_2, ..., z_{n-1}\| - \|y - x_{k_n}, z_1, z_2, ..., z_{n-1}\|| \quad (4)
\end{aligned}$$

we have that

$$|\|y - x_{k_n}, z_1, z_2, ..., z_{n-1}\| - \|y - x_{k_m}, z_1, z_2, ..., z_{n-1}\|| < \varepsilon \quad (5)$$

for all $n \geq n_0$, $m \geq n_0$, i.e.,

$$|\|y - x_{k_n}, z_1, z_2, ..., z_{n-1}\| - \|y - x_{k_m}, z_1, z_2, ..., z_{n-1}\|| \to 0 \text{ as } m, n \to \infty$$

for the subset $K = (k_n) \subset N$ with $\delta_A(K) = 1$. This means that the sequence $\|y - x_n, z_1, z_2, ..., z_{n-1}\|$ is an A-statistically Cauchy sequence, which finishes the proof. □

Definition 7. *A point a in a n-normed space $(X, \|., ..., .\|)$ is called a limit point of a set M in X if for an arbitrary $\Sigma = \{(x_{11}, ..., x_{(n-1)1}, \varepsilon_1), ..., (x_{1n}, ..., x_{(n-1)n}, \varepsilon_n)\}$ there is a point $a_\Sigma \in M$, $a_\Sigma \neq a$ such that $a_\Sigma \in W_\Sigma(a)$.*

Moreover, a subset $Y \subset X$ is called a closed subset of X if Y contains every its limit point. If Y^0 is the set of all points of a subset $Y \subset X$, then the set $\overline{Y} = Y \cup Y^0$ is called the closure of the set Y.

Proposition 3. *A-statistically localor of any sequence is a closed subset of the n-normed space $(X, \|., ..., .\|)$.*

Proof. Let $y \in \overline{\text{loc}^{\text{st}_A}}(x_n)$. Then, for arbitrary

$$\Sigma = \{(x_{11}, ..., x_{(n-1)1}, \varepsilon_1), ..., (x_{1n}, ..., x_{(n-1)n}, \varepsilon_n)\}$$

there is a point $x \in \text{loc}^{\text{st}_A}(x_n)$ such that $x \neq y$ and $x \in W_\Sigma(y)$. Hence

$$\delta_A(\{n \in \mathbb{N} : |\|x - x_n, z_1, z_2, ..., z_{n-1}\| - \|y - x_n, z_1, z_2, ..., z_{n-1}\|| \geqslant \varepsilon\}) = 0$$

for any $\varepsilon > 0$ and every $z_1, z_2, ..., z_{n-1} \in \text{loc}^{\text{st}_A}(x_n)$, because we get

$$\begin{aligned} &|\|x - x_n, z_1, z_2, ..., z_{n-1}\| - \|y - x_n, z_1, z_2, ..., z_{n-1}\|| \\ &\leq \|y - x_n, z_1, z_2, ..., z_{n-1}\| < \varepsilon \end{aligned}$$

for almost all n. As a result, the hypothesis of Proposition 2 is satisfied. So, $y \in \text{loc}^{\text{st}_A}(x_n)$, that is, $\text{loc}^{\text{st}_A}(x_n)$ is closed. □

Recall that the point y is an A-statistical limit point of the sequence (x_n) in n-normed space $(X, \|., ..., .\|)$ if there is a set $K = \{k_1 < k_2 < ...\} \subset \mathbb{N}$ such that $\delta_A(K) \neq 0$ and $\lim_{n\to\infty} \|x_{k_n} - y, z_1, z_2, ..., z_{n-1}\| = 0$. A point ξ is called an A-statistical cluster point if

$$\delta_A(\{n \in \mathbb{N} : \|x_n - \xi, z_1, z_2, ..., z_{n-1}\| < \varepsilon\}) \neq 0$$

for each $\varepsilon > 0$ and every $z_1, z_2, ..., z_{n-1} \in X$.

We can give the following results because of the inequality

$$|\|x_n - y, z_1, z_2, ..., z_{n-1}\| - \|x - y, z_1, z_2, ..., z_{n-1}\|| \leq \|x_n - x, z_1, z_2, ..., z_{n-1}\|.$$

Proposition 4. *Let $y \in X$ be an A-statistical limit point (an A-statistical cluster point) of a sequence (x_n) in n-normed space $(X, \|., ..., .\|)$. Then the number $\|y - x, z_1, z_2, ..., z_{n-1}\|$ is an A-statistical limit point (an A-statistical cluster point) of the sequence $\{\|x_n - x, z_1, z_2, ..., z_{n-1}\|\}$ for each $x \in X$ and every nonzero $z_1, z_2, ..., z_{n-1} \in X$.*

Proposition 5. *All A-statistical limit points (A-statistical cluster points) of the A-statistically localized sequence (x_n) in n-normed space $(X, \|.,...,.\|)$ have the same distance from each point x of the A-statistical localor* $\mathrm{loc}^{\mathrm{st}_A}(x_n)$.

Proof. Actually, if y_1, y_2 are two A-statistical limit points (A-statistical cluster points) of the sequence (x_n) in n-normed space $(X, \|.,...,.\|)$, then the numbers $\|y_1 - x, z_1, z_2, ..., z_{n-1}\|$ and $\|y_2 - x, z_1, z_2, ..., z_{n-1}\|$ are A-statistical limit points of the A-statistically convergent sequence $\|x - x_n, z_1, z_2, ..., z_{n-1}\|$. As a result, $\|y_1 - x, z_1, z_2, ..., z_{n-1}\| = \|y_2 - x, z_1, z_2, ..., z_{n-1}\|$. □

Proposition 6. $\mathrm{loc}^{\mathrm{st}_A}(x_n)$ *only contains one A-statistical limit (cluster) point of the sequence (x_n) in n-normed space $(X, \|.,...,.\|)$. In particular, everywhere localized sequence only has one A-statistical limit (cluster) point.*

Proof. Let $x, y \in \mathrm{loc}^{\mathrm{st}_A}(x_n)$ be two A-statistical limit or cluster points of the sequence (x_n) in n-normed space $(X, \|.,...,.\|)$. Then, we have that

$$\|x - x, z_1, z_2, ..., z_{n-1}\| = \|x - y, z_1, z_2, ..., z_{n-1}\|$$

from the Proposition 5. But $\|x - x, z_1, z_2, ..., z_{n-1}\| = 0$. This means $\|x - y, z_1, z_2, ..., z_{n-1}\| = 0$ for $x \neq y$. This is a contradiction. □

Proposition 7. *Let $y \in \mathrm{loc}^{\mathrm{st}_A}(x_n)$ be an A-statistical limit point of the sequence (x_n). Then $x_n \overset{st_A}{\to} y$.*

Proof. The sequence $\{\|x_n - y, z_1, z_2, ..., z_{n-1}\|\}$ A-statistically converges and some subsequence of this sequence converges to zero, i.e., $x_n \overset{st_A}{\to} y$. □

Definition 8. *Let (x_n) be the A-statistically localized sequence with the A-statistically localor $M = \mathrm{loc}^{\mathrm{st}_A}(x_n)$. The number*

$$\mu = \inf_{x \in M} \left(st_A\text{-} \lim_{n\to\infty} \|x - x_n, z_1, z_2, ..., z_{n-1}\| \right)$$

is said to be the A-statistical barrier of (x_n).

Theorem 1. *Let (x_n) be the A-statistically localized sequence in n-normed space $(X, \|.,...,.\|)$. Then (x_n) is A-statistically Cauchy sequence if and only if its A-statistical barrier is equal to zero.*

Proof. Let (x_n) be an A-statistically Cauchy sequence in n-normed space $(X, \|.,...,.\|)$. Then, there exists the set $K = \{k_1 < k_2 < ... < k_n < ...\} \subset \mathbb{N}$ such that $\delta_A(K) = 1$ and $\lim_{n,m\to\infty} \|x_{k_n} - x_{k_m}, z_1, z_2, ..., z_{n-1}\| = 0$. Hence, there exists $n_0 \in \mathbb{N}$ for each $\varepsilon > 0$ and every nonzero $z_1, z_2, ..., z_{n-1} \in X$ such that

$$\left\| x_{k_n} - x_{k_{n_0}}, z_1, z_2, ..., z_{n-1} \right\| < \varepsilon$$

for all $n \geq n_0$. Because an A-statistically Cauchy sequence is A-statistically localized everywhere, we get $st_A\text{-}\lim_{n\to\infty} \left\| x_n - x_{k_{n_0}}, z_1, z_2, ..., z_{n-1} \right\| \leq \varepsilon$, that is, $\mu \leq \varepsilon$. Since $\varepsilon > 0$ is arbitrary, we have $\mu = 0$.

In contrast, if $\mu = 0$ then there is $x \in M = \mathrm{loc}^{\mathrm{st}_A}(x_n)$ for each $\varepsilon > 0$ such that $\|x, z_1, z_2, ..., z_{n-1}\| = st_A\text{-}\lim_{n\to\infty} \|x - x_n, z_1, z_2, ..., z_{n-1}\| < \frac{\varepsilon}{2}$ for every nonzero $z_1, z_2, ..., z_{n-1} \in M$. At this stage,

$$\delta_A\left(\left[n \in \mathbb{N} : |\|x, z_1, z_2, ..., z_{n-1}\| - \|x - x_n, z_1, z_2, ..., z_{n-1}\|| \geq \frac{\varepsilon}{2} - \|x, z_1, z_2, ..., z_{n-1}\|\right]\right) = 0.$$

So,

$$\delta_A\left(\left\{n \in \mathbb{N} : \|x - x_n, z_1, z_2, ..., z_{n-1}\| \geq \frac{\varepsilon}{2}\right\}\right) = 0,$$

that is, $st_A\text{-}\lim_{n\to\infty} \|x - x_n, z_1, z_2, ..., z_{n-1}\| = 0$. Therefore, (x_n) is an A-statistically Cauchy sequence. □

Theorem 2. *Let (x_n) be A-statistically localized in itself and let (x_n) contain a A-nonthin Cauchy subsequence. Then (x_n) is an A-statistically Cauchy sequence in itself.*

Proof. Let (x'_n) be a A-nonthin Cauchy subsequence of (x_n). Without loss of generality we can suppose that all elements of (x'_n) are in $\mathrm{loc}^{\mathrm{st}_A}(x_n)$. Because (x'_n) is a Cauchy sequence by Theorem 1,

$$\inf_{x'_n} \lim_{m\to\infty} \|x'_m - x'_n, z_1, z_2, ..., z_{n-1}\| = 0.$$

In other hand, because (x_n) is A-statistically localized in itself, then

$$st_A\text{-}\lim_{m\to\infty} \|x_m - x'_n, z_1, z_2, ..., z_{n-1}\| = st_A - \lim_{m\to\infty} \|x'_m - x'_n, z_1, z_2, ..., z_{n-1}\| = 0.$$

This means

$$\mu = \inf_{x\in M}\left(st_A\text{-}\lim_{m\to\infty} \|x_m - x, z_1, z_2, ..., z_{n-1}\|\right) = 0,$$

that is, (x_n) is an A-statistically Cauchy sequence in itself. □

Let $x \in X$ and $\delta > 0$. Recall that the sequence (x_n) in n-normed space $(X, \|., ..., .\|)$ is said to be A-statistically bounded if there is a subset $K = \{k_1 < k_2 < ... < k_n \subset ...\}$ of $\mathbb{N}$ such that $\delta_A(K) = 1$ and $(x_{k_n}) \subset U_\delta(0, z_1, z_2, ..., z_{n-1})$, where $U_\delta(0, z_1, z_2, ..., z_{n-1})$ is some neighborhood of the origin. It is obvious that (x_{k_n}) is a bounded sequence in X and it has a localized in itself subsequence. As a result, the following statement is correct:

Theorem 3. *Each A-statistically bounded sequence in n-normed space $(X, \|., ..., .\|)$ has an A-statistically localized in itself subsequence.*

Definition 9. *An infinite subset $L \subset (X, \|., .\|)$ is called thick relatively to a non-empty subset $Y \subset X$ if for each $\varepsilon > 0$ there is the a point $y \in Y$ such that the neighborhood $U_\varepsilon(0, z_1, z_2, ..., z_{n-1})$ has infinitely many points of L. In particular, if the set L is thick relatively to its subset $Y \subset L$ then L is said to be thick in itself.*

Theorem 4. *The following statements are equivalent to each other in n-normed space $(X, \|.,...,.\|)$:*

(i) Each bounded subset of X is totally bounded.

(ii) Each bounded infinite set of X is thick in itself.

(iii) Each A-statistically localized in itself sequence in X is an A-statistically Cauchy sequence.

Proof. It is obvious that (i) implies (ii). Now, we prove that (ii) implies (iii). Let $(x_n) \subset X$ be an A-statistically localized in itself. Then (x_n) is A-statistically bounded sequence in X. Then here is an infinite set L of points of (x_n) such that L is a bounded subset of X. By the supposition, the set L is thick in itself. So, we can choose $x_k \in L$ for every $\varepsilon > 0$ such that the neighborhood $U_\varepsilon (0, z_1, z_2, ..., z_{n-1})$ contains infinitely many points of X, say $x'_1, ..., x'_n, ...$. The sequence $(\|x'_n - x_k, z_1, z_2, ..., z_{n-1}\|)$ A-statistically converges and

$$st_A - \lim_{n\to\infty} \|x'_n - x_K, z_1, z_2, ..., z_{n-1}\| \leq \varepsilon$$

for the sequence (x'_n). Therefore, the A-statistically barrier of (x_n) is equal to zero. Then (x_n) is a Cauchy sequence.

Suppose that (iii) is satisfied, but (i) is not. Then, there is a subset $L \subset X$ such that L is not totally bounded. This means that there exists $\varepsilon > 0$ and a sequence $(x_n) \subset L$ such that $\|x_n - x_m, z_1, z_2, ..., z_{n-1}\| > \varepsilon$ for any $n \neq m$ and every nonzero $z_1, z_2, ..., z_{n-1} \in L$.

Because (x_n) is A-statistically bounded by Theorem 3, it has an A-statistically localized in itself sequence (x'_n). Due to $\|x'_n - x'_m, z_1, z_2, ..., z_{n-1}\| > \varepsilon$ for any $n \neq m$, the subsequence is not an A-statistically Cauchy sequence. This contradicts (iii). Therefore, (iii) implies (ii), which finish the proof. □

From Theorem 2 and 3, we get the property (iii) is equivalent to

(iv) each A-statistically bounded sequence has an A-statistically Cauchy subsequence.

Definition 10. *A sequence (x_n) in n-normed space $(X, \|.,...,.\|)$ is said to be uniformly A-statistically localized on the subset L of X if the sequence $\{\|x - x_n, z_1, z_2, ..., z_{n-1}\|\}$ uniformly A-statistically converges for all $x \in L$ and every nonzero $z_1, z_2, \ldots, z_{n-1}$ in L.*

Proposition 8. *Let (x_n) be uniformly A-statistically localized on the set $L \subset X$ and $w \in Y$ is such that for every $\varepsilon > 0$ and every nonzero $z_1, z_2, ..., z_{n-1}$ in L, there is $y \in L$ satisfying the property*

$$\delta_A (\{n \in \mathbb{N} : |\|w - x_n, z_1, z_2, ..., z_{n-1}\| - \|y - x_n, z_1, z_2, ..., z_{n-1}\|| \geqslant \varepsilon\}) = 0.$$

Then $w \in \mathrm{loc}^{\mathrm{st}_A} (x_n)$ and (x_n) is uniformly A-statistically localized on a set that contains such points w.

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