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Semi-Analytical and Finite Element Investigations of the Vibration of a Stepped Beam on an Elastic Foundation

Hakan ERDOĞAN*[1], Safa Bozkurt COŞKUN[2]

Abstract

In this study free vibration behavior of a stepped beam on an elastic foundation is considered. The vibration of uniform beams on an elastic foundation has been previously studied extensively and various solutions are available in the literature. However, the problem considered in current study appears not to have been widely covered in the literature and analytical solutions are strictly limited. To this aim, semi-analytical solutions are obtained first by using Adomian decomposition method, then finite element solutions are computed via structural finite element analysis software (SAP 2000). The free vibration analysis of stepped beam considering the combinations of different support conditions at each end are performed employing semi-analytical and finite element methods. The findings of the analysis are compared and discussed in detail.

Keywords: segmented beam, elastic foundation, vibration, Adomian decomposition method

1. INTRODUCTION

Beam on elastic foundation problems are of great interest for researchers in the fields of civil, mechanical and aeronautical engineering related to the design of structural members of buildings, aircrafts, pipes, railroads, etc.

In the literature, numerous studies were conducted on the vibration analysis of beams [1-3]. Researchers also focused on the special cases such as stepped beams [4] and beams on elastic foundations [5]. Wang [6], Kukla [7] and Belles *et al.* [8] are three interesting contributions to the technical studies about the subject. Thambiratnam and Zhuge [9] developed a simple finite element method and applied to treat the free vibration of

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analysis of Euler-Bernoulli beams on elastic foundations for different cases. Durgun [10] also conducted a study on the analysis of stepped beams on elastic foundation by employing Homotopy Perturbation Method (HPM). Durgun [8] solved the same numerical example with Wang [6] and obtained exactly the same results up to four decimals. The solution proposed by Wang [6] was tedious and is only valid for specific cases and he reported the results only for the beams having free-free and simply supported end conditions. In addition to Wang's work, Durgun [10] also presented the results for fixed-fixed and fixed-hinged end condition cases. However, Durgun [10] has not verified his findings regarding the fixed-fixed and fixed-hinged end conditions with other reliable methods in the literature.

The main aim of this study is to provide semi-analytical and finite element solutions to the same problem in order to verify the results obtained by using HPM [10] and fulfill the lack of analytical solutions provided in the work by Wang [6].

2. VIBRATION OF STEPPED BEAM ON ELASTIC FOUNDATION

In this section, vibration of an Euler beam on Winkler foundation is considered that are the most widely used models for the beams on elastic foundations. Such a beam is shown in Figure 1.

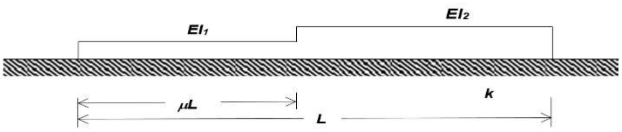

Figure 1 Stepped beam on elastic foundation

Equation of motion for a uniform Euler beam on Winkler foundation is given as:

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y(x,t)}{\partial t^2} + ky(x,t) = 0 \qquad (1)$$

where EI is the flexural rigidity of the beam, ρ is the density per unit volume, A is the cross-sectional area, k is the stiffness of Winkler foundation and y is the transverse displacement.

In order to find the eigenfrequencies ω of the beam one may assume

$$y(x,t) = w(x)\, e^{i\omega t} \qquad (2)$$

Substituting Eq.(2) in Eq.(1) yields

$$EI\frac{d^4 w(x)}{dx^4} - \rho A\omega^2 w(x) + kw(x) = 0 \qquad (3)$$

Eq.(3) can be rearranged as

$$\frac{d^4 w(x)}{dx^4} - \left(\lambda^4 - \frac{k}{EI}\right) w(x) = 0 \qquad (4)$$

where $\lambda^4 = \omega^2\rho A/EI$. Introducing the parameter β, such that, $\beta^4 = \lambda^4 - \frac{k}{EI}$ solution to Eq.(4) is

$$w(x) = C_1\sin(\beta x) + C_2\cos(\beta x)$$
$$+C_3\sinh(\beta x) + C_4\cosh(\beta x) \quad (5)$$

when $\beta^4 \geq 0$ which describes that foundation stiffnes has no effect in the solution. However, if the vibration frequency of the beam is relatively low, *i.e.*, $\beta^4 < 0$ the solution of Eq.(4) becomes

$$w(x) = C_1\sin\left(\frac{\beta x}{\sqrt{2}}\right)\sinh\left(\frac{\beta x}{\sqrt{2}}\right) + C_2\sin\left(\frac{\beta x}{\sqrt{2}}\right)\cosh\left(\frac{\beta x}{\sqrt{2}}\right) + C_3\cos\left(\frac{\beta x}{\sqrt{2}}\right)\sinh\left(\frac{\beta x}{\sqrt{2}}\right) + C_4\cos\left(\frac{\beta x}{\sqrt{2}}\right)\cosh\left(\frac{\beta x}{\sqrt{2}}\right) \qquad (6)$$

which is a different solution when compared to Eq.(5). The coefficients C_1, C_2, C_3, and C_4 in both solutions can be evaluated according to boundary conditions at the supports. These conditions are given as follows:

- For free end $w''(x) = 0$ and $w'''(x) = 0$
- For hinged end $w(x) = 0$ and $w''(x) = 0$
- For clamped end $w(x) = 0$ and $w'(x) = 0$

There are also four boundary conditions due to continuity at the junction of two segments of the stepped beam. These conditions impose the equality of the displacement, the slope, the moment and the shear force at the junction and given as follows:

- $w_1(x) = w_2(x)$

- $w_1'(x) = w_2'(x)$
- $w_1''(x) = \alpha\, w_2''(x)$
- $w_1'''(x) = \alpha\, w_2'''(x)$

where $\alpha = I_2/I_1$.

3. ADOMIAN DECOMPOSITION METHOD

In Adomian decomposition method (ADM) a general form of the following differential equation is assumed.

$$Lu + Nu + Ru = g(x) \tag{7}$$

where $u(x)$ is the unknown solution, $g(x)$ is the source term, L is the linear operator, N is the nonlinear operator and R is the operator for remainder terms. The solution to Eq.(7) is

$$u(x) = f(x) - L^{-1}(Nu) - L^{-1}(Ru) \tag{8}$$

where L^{-1} is the inverse linear operator and $f(x) = L^{-1}(g(x))$. The solution is constructed with an infinite series in the following form

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \tag{9}$$

The nonlinear term Nu is represented by so-called Adomian polynomials given below.

$$Nu = \sum_{n=0}^{\infty} A_n(u_0, u_1, \cdots, u_n) \tag{10}$$

where A_n is the n^{th} Adomian polynomial defined as the following term.

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} N\left(\sum_{n=0}^{\infty} \lambda^k u_k\right) \tag{11}$$

The method leads to successive approximations as follows:

$$u_0(x) = f(x) \tag{12}$$

$$u_n(x) = -L^{-1}(Ru_{n-1} - A_{n-1}) \tag{13}$$

Finally, the solution is calculated by adding the successive approximations given in Eqs.(12-13). An N^{th} order analytical approximation includes the terms up to u_N as given below

$$u(x) = \sum_{n=0}^{N} u_n(x) \tag{14}$$

For further details of the method, the reader may refer to [11].

4. ADM SOLUTION OF THE PROBLEM

An initial approximation based on Eq.(12) may be obtained as

$$w_0(x) = Ax^3 + Bx^2 + Cx + D \tag{15}$$

where $A = y'''(0)/6$, $B = y''(0)/2$, $C = y'(0)$ and $D = y(0)$. Successive approximations for an N^{th} order solution may be computed according to Eq.(13) as

$$w_n(x) = L^{-1}(\beta^4 y_{n-1})\ ,\ \ n > 0 \tag{16}$$

Since there are two segments in the stepped beam, an initial approximation of the form given in Eq.(15) is assumed for both segments of the beam.

$$w_0^{(1)}(x) = A_1x^3 + B_1x^2 + C_1x + D_1 \tag{17}$$

$$w_0^{(2)}(x) = A_2x^3 + B_2x^2 + C_2x + D_2 \tag{18}$$

Eight boundary conditions are required to determine eight unknowns introduced in Eqs.(17) and (18). These conditions are four boundary conditions at the supports and four continuity conditions. Hence eight equations in eight unknowns are produced can be represented in the following matrix form.

$$[K]_{8\times8}\{\Lambda\}_{8\times1} = \{0\}_{8\times1} \tag{19}$$

where $[K]$ includes the term β which is the function of vibration frequency ω and the unknown vector $\{\Lambda\}$ includes unknown coefficients in the initial approximations in Eqs.(17) and (18). The trivial solution of Eq.(19) corresponds the undeformed beam. Hence, a nontrivial solution to the problem can be obtained by equating the determinant of coefficient matrix to zero that lead to free vibration frequencies of the stepped beam on elastic foundation considered.

5. NUMERICAL APPLICATION

Wang [6] calculated analytical solutions for natural frequencies of two stepped beams on elastic foundation, one simply supported and one with free ends. There are no available analytical solutions for the beam with both ends clamped, the beam with one end clamped and one end simply supported and the beam with one end clamped and one end free. For simplicity following abbreviations are used for different combinations of end conditions.

- FF Free – Free
- SS Both ends simply supported
- CC Clamped – Clamped
- CS Clamped – Simply supported

Wang [6] performed the analysis for the following data: $E = 6.50 \times 10^{11}$ Pa, $\rho = 213.60$ kg/m^2, $H_1 = 0.10$ m, $H_2 = 0.15$ m, $B = 0.08$ m, $L = 5.00$ m. Two different foundation modulus were used in the calculations, $\nu = 1/100$ and $\nu = 1/200$ where ν is defined as k/EI. μ denotes the ratio of the length of thinner segment having depth of H1 to total length L.

SAP2000 [12] was employed for obtaining finite element (FE) solutions. Line springs that can be assigned in any of the local axes direction of a frame object is defined in vertical direction to simulate the elastic foundation. SAP2000 [12] distributes the springs associated with the frame object to all of the nodes (Figure 2).

Figure 2 FE model for simply supported stepped beam on elastic foundation (50 beam elements)

Table 1 compares the previous results (analytical [6], HPM [10]) with the results of present study (ADM, FE solution) for natural frequencies of stepped FF beam on elastic foundation. ADM solutions are conducted to 12th order and computations are identical to the analytical [6] and HPM [10] solutions. FE solutions are also in excellent agreement for the first two frequency values.

In Table 2 only first two natural frequencies are available for SS beams [6, 10] and used for comparison. However, first three frequencies are computed for stepped SS beam on elastic foundation using ADM and FEM. ADM results are in excellent agreement with previous results while FE solutions are in very good agreement with the same results for which first two frequencies have the same accuracy of analytical solution.

Analytical solutions for the first three frequencies for stepped CC and CS beams on elastic foundation are not available in the literature. Only HPM solutions [10] exist and the results of this study for these cases are compared with only HPM results.

In Table 3 and Table 4 it seems that ADM solutions are in excellent agreement with HPM results [10]. FE solutions are in very good agreement with both results as in previous cases.

Table 1
Natural frequencies of FF stepped beam on elastic foundation

v = 1 /100												
	Wang [6], Durgun [10]			ADM			FE (50 Elements)			FE (100 Elements)		
μ	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)
0	5.8531	5.8531	96.4081	5.8531	5.8531	96.4081	5.8531	5.8531	96.2888	5.8531	5.8531	96.3783
0.1	5.8534	6.2201	101.0194	5.8534	6.2201	101.0194	5.8534	6.2199	100.8885	5.8534	6.2201	100.9867
0.2	5.8557	6.5291	100.7912	5.8557	6.5291	100.7912	5.8558	6.5287	100.6555	5.8557	6.5290	100.7573
0.3	5.8635	6.7692	93.4247	5.8635	6.7692	93.4247	5.8636	6.7688	93.2962	5.8636	6.7691	93.3926
0.4	5.8821	6.9403	82.2168	5.8821	6.9403	82.2168	5.8822	6.9399	82.1076	5.8821	6.9402	82.1895
0.5	5.9194	7.0513	72.8477	5.9194	7.0513	72.8477	5.9196	7.0509	72.7549	5.9195	7.0512	72.8245
0.6	5.9876	7.1161	66.7402	5.9876	7.1161	66.7402	5.9876	7.1161	66.6565	5.9876	7.1160	66.7193
0.7	6.1046	7.1494	63.2525	6.1046	7.1494	63.2525	6.1048	7.1492	63.1733	6.1047	7.1494	63.2327
0.8	6.2993	7.1637	61.5574	6.2993	7.1637	61.5574	6.2996	7.1636	61.4814	6.2994	7.1637	61.5384
0.9	6.6214	7.1680	61.4321	6.6214	7.1680	61.4321	6.6216	7.1679	61.3585	6.6214	7.1680	61.4137
1.0	7.1685	7.1685	64.5528	7.1685	7.1685	64.5528	7.1685	7.1685	64.4735	7.1685	7.1685	64.5330

v = 1 /200												
	Wang [6], Durgun [10]			ADM			FE (50 Elements)			FE (100 Elements)		
μ	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)
0	4.1388	4.1388	96.3192	4.1388	4.1388	96.3192	4.1387	4.1387	96.1998	4.1388	4.1388	96.2893
0.1	4.1390	4.3983	100.9251	4.1390	4.3983	100.9251	4.1390	4.3982	100.7942	4.1390	4.3983	100.8924
0.2	4.1406	4.6169	100.6918	4.1406	4.6169	100.6918	4.1407	4.6166	100.556	4.1406	4.6168	100.6579
0.3	4.1462	4.7867	93.3148	4.1462	4.7867	93.3148	4.1462	4.7864	93.1862	4.1462	4.7866	93.2827
0.4	4.1593	4.9076	82.0879	4.1593	4.9076	82.0879	4.1594	4.9074	81.9785	4.1593	4.9076	82.0606
0.5	4.1857	4.9861	72.6959	4.1857	4.9861	72.6959	4.1858	4.9858	72.6029	4.1857	4.9860	72.6727
0.6	4.2340	5.0319	66.5679	4.2340	5.0319	66.5679	4.2341	5.0317	66.4841	4.2340	5.0318	66.5470
0.7	4.3168	5.0554	63.0664	4.3168	5.0554	63.0664	4.3170	5.0553	62.9870	4.3169	5.0554	63.0465
0.8	4.4546	5.0655	61.3656	4.4546	5.0655	61.3656	4.4548	5.0654	61.2893	4.4547	5.0655	61.3465
0.9	4.6823	5.0686	61.2398	4.6823	5.0686	61.2398	4.6824	5.0685	61.1660	4.6823	5.0685	61.2214
1.0	5.0689	5.0689	64.3534	5.0689	5.0689	64.3534	5.0689	5.0689	64.2740	5.0689	5.0689	64.3336

Table 2
Natural frequencies of SS stepped beam on elastic foundation

v = 1 /100											
	Wang [6], Durgun [10]		ADM			FE (50 Elements)			FE (100 Elements)		
μ	f_1 (Hz)	f_2 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)
0	42.8519	169.9023	42.8519	169.9023	382.0982	42.8518	169.9015	382.0961	42.8519	169.9022	382.0979
0.1	42.5839	166.0556	42.5839	166.0556	366.1519	42.5846	166.0658	366.1715	42.5841	166.0583	366.1570
0.2	41.0205	152.2872	41.0205	152.2872	341.8899	41.0216	152.2827	341.8246	41.0207	152.2861	341.8737
0.3	38.1581	143.7692	38.1581	143.7692	342.1057	38.1587	143.7517	342.0941	38.1583	143.7649	342.1030
0.4	35.0630	143.4669	35.0630	143.4669	323.7582	35.0600	143.4542	323.7748	35.0629	143.4638	323.7625
0.5	32.5477	142.6401	32.5477	142.6401	300.8846	32.5466	142.6422	300.8384	32.5475	142.6407	300.8732
0.6	30.8291	134.7613	30.8291	134.7613	299.4122	30.8275	134.7667	299.3834	30.8287	134.7627	299.4052
0.7	29.8260	124.2508	29.8260	124.2508	291.5502	29.8243	124.2480	291.5687	29.8256	124.2501	291.5550
0.8	29.3576	116.7971	29.3576	116.7971	270.3553	29.3560	116.7862	270.3455	29.3572	116.7945	270.3530
0.9	29.2113	113.7982	29.2113	113.7982	256.8519	29.2102	113.7865	256.8102	29.2110	113.7953	256.8416
1.0	29.1940	113.4278	29.1940	113.4278	254.8031	29.1939	113.4272	254.8017	29.1940	113.4277	254.8029
v = 1 /200											
	Wang [6], Durgun [10]		ADM			FE (50 Elements)			FE (100 Elements)		
μ	f_1 (Hz)	f_2 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)
0	42.6517	169.8519	42.6517	169.8519	382.0758	42.6515	169.8511	382.0737	42.6516	169.8518	382.0755
0.1	42.3838	166.0034	42.3838	166.0034	366.1278	42.3825	166.0136	366.1473	42.3820	166.0060	366.1329
0.2	40.8063	152.2257	40.8063	152.2257	341.8616	40.8074	152.2212	341.7964	40.8066	152.2246	341.8455
0.3	37.9155	143.6983	37.9155	143.6983	342.0765	37.9161	143.6809	342.0649	37.9156	143.6940	342.0738
0.4	34.7788	143.3927	34.7788	143.3927	323.7259	34.7785	143.3800	323.7425	34.7787	143.3896	323.7302
0.5	32.2173	142.5639	32.2173	142.5639	300.8481	32.2161	142.5659	300.8019	32.2170	142.5645	300.8367
0.6	30.4553	134.6785	30.4553	134.6785	299.3743	30.4537	134.6839	299.3454	30.4549	134.6799	299.3672
0.7	29.4174	124.1574	29.4174	124.1574	291.5103	29.4157	124.1546	291.5288	29.4170	124.1567	291.5151
0.8	28.9261	116.6927	28.9261	116.6927	270.3108	28.9245	116.6818	270.3010	28.9257	116.6900	270.3084
0.9	28.7695	113.6864	28.7695	113.6864	256.8028	28.7685	113.6747	256.7611	28.7692	113.6835	256.7925
1.0	28.7506	113.3144	28.7506	113.3144	254.7527	28.7505	113.3139	254.7513	28.7506	113.3143	254.7525

Table 3
Natural frequencies of CC stepped beam on elastic foundation

v = 1 /100												
	Durgun [10]			ADM			FE (50 Elements)			FE (100 Elements)		
μ	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)
0	96.4081	265.3269	520.0532	96.4081	265.3269	520.0532	96.4080	265.3266	520.0519	96.4080	265.3267	520.0528
0.1	83.6134	244.1429	489.1152	83.6134	244.1429	489.1152	83.6146	244.1571	489.1690	83.6136	244.1464	489.1286
0.2	83.6454	237.3657	463.9575	83.6454	237.3657	463.9575	83.6489	237.3735	463.8752	83.6462	237.3675	463.9367
0.3	82.1239	224.3318	461.3383	82.1239	224.3318	461.3383	82.1273	224.3007	461.2855	82.1247	224.3239	461.3249
0.4	78.6631	222.7447	442.1322	78.6631	222.7447	442.1322	78.6628	222.7168	442.1659	78.663	222.7376	442.1406
0.5	76.2090	221.3049	410.8338	76.2090	221.3049	410.8338	76.2048	221.3131	410.7568	76.2079	221.3068	410.8143
0.6	76.4158	208.0189	409.6846	76.4158	208.0189	409.6846	76.4094	208.0273	409.6691	76.4141	208.0210	409.6806
0.7	78.2691	196.9283	391.5603	78.2691	196.9283	391.5603	78.2638	196.9116	391.5853	78.2677	196.9240	391.5664
0.8	77.9781	198.5024	373.5053	77.9781	198.5024	373.5053	77.9761	198.479	373.4486	77.9775	198.4964	373.4910
0.9	72.8548	196.4500	377.8571	72.8548	196.4500	377.8571	72.8546	196.4459	377.8291	72.8547	196.4489	377.8500
1.0	64.5528	176.9868	346.7543	64.5528	176.9868	346.7543	64.5527	176.9866	346.7534	64.5527	176.9867	346.7540
v = 1 /200												
	Durgun [10]			ADM			FE (50 Elements)			FE (100 Elements)		
μ	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)
0	96.3192	265.2946	520.0368	96.3192	265.2946	520.0368	96.311	265.2943	520.0355	96.3191	265.2944	520.0363

0.1	83.5108	244.1077	489.0976	83.5108	244.1077	489.0976	83.512	244.1219	489.1513	83.5110	244.1110	489.1110
0.2	83.5418	237.3276	463.9370	83.5418	237.3276	463.9370	83.5453	237.3354	463.8548	83.5426	237.3294	463.9162
0.3	82.0136	224.2872	461.3165	82.0136	224.2872	461.3165	82.017	224.2562	461.2638	82.0144	224.2794	461.3032
0.4	78.5377	222.6973	442.1082	78.5377	222.6973	442.1082	78.5374	222.6694	442.1422	78.5375	222.6902	442.1169
0.5	76.0668	221.2558	410.8071	76.0668	221.2558	410.8071	76.0625	221.2640	410.7300	76.0657	221.2578	410.7876
0.6	76.2617	207.9649	409.6571	76.2617	207.9649	409.6571	76.2553	207.9734	409.6416	76.2601	207.9669	409.6531
0.7	78.1095	196.8683	391.5305	78.1095	196.8683	391.5305	78.1042	196.8517	391.5555	78.1081	196.8641	391.5367
0.8	77.8138	198.4392	373.4726	77.8138	198.4392	373.4726	77.8118	198.4158	373.4159	77.8133	198.4332	373.4582
0.9	72.6783	196.3846	377.8232	72.6783	196.3846	377.8232	72.6780	196.3805	377.7952	72.6782	196.3835	377.8162
1.0	64.3534	176.9142	346.7173	64.3534	176.9142	346.7173	64.3534	176.9140	346.7164	64.3534	176.9141	346.7169

Table 4
Natural frequencies of CS stepped beam on elastic foundation

v = 1 /100												
	Durgun [10]			ADM			FE (50 Elements)			FE (100 Elements)		
μ	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)
0	66.5734	214.9847	448.4203	66.5734	214.9847	448.4203	66.5731	214.9836	448.4177	66.5731	214.9837	448.4182
0.1	56.5047	195.8623	420.3049	56.5047	195.8623	420.3049	56.5049	195.8703	420.3450	56.5046	195.8636	420.3136
0.2	56.2305	192.4146	399.6111	56.2305	192.4146	399.6111	56.2318	192.4243	399.5628	56.2307	192.4164	399.5977
0.3	55.8497	181.3330	393.1700	55.8497	181.3330	393.1700	55.8516	181.3177	393.1022	55.8500	181.3286	393.1518
0.4	53.7662	176.7855	386.3098	53.7662	176.7855	386.3098	53.7671	176.7596	386.3394	53.7662	176.7784	386.3160
0.5	50.7342	177.9354	355.5730	50.7342	177.9354	355.5730	50.7338	177.9263	335.5419	50.7338	177.9325	355.5641
0.6	47.9739	172.5648	346.3939	47.9739	172.5648	346.3939	47.9714	172.5730	346.3369	47.9731	172.5662	346.3785
0.7	46.0879	159.7651	342.7486	46.0879	159.7651	342.7486	46.0845	159.7659	342.7674	46.0869	159.7647	342.7522
0.8	45.1264	148.8735	319.4041	45.1264	148.8735	319.4041	45.1230	148.8606	319.4003	45.1254	148.8697	319.4021
0.9	44.8212	144.0672	301.8586	44.8212	144.0672	301.8586	44.8188	144.0499	301.8069	44.8204	144.0624	301.8447
1.0	44.7878	143.4492	299.0073	44.7878	143.4492	299.0073	44.7876	143.4485	299.0056	44.7876	143.4486	299.0059

v = 1 /200												
	Durgun [10]			ADM			FE (50 Elements)			FE (100 Elements)		
μ	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)
0	66.4446	214.9448	448.4012	66.4446	214.9448	448.4012	66.4443	214.9438	448.3986	66.4443	214.9438	448.3991
0.1	56.3529	195.8184	420.2844	56.3529	195.8184	420.2844	56.3530	195.8264	420.3245	56.3527	195.8198	420.2931
0.2	56.0771	192.3682	399.5875	56.0771	192.3682	399.5875	56.0784	192.3779	399.5392	56.0773	192.3700	399.5741
0.3	55.6914	181.2788	393.1444	55.6914	181.2788	393.1444	55.6932	181.2634	393.0766	55.6917	181.2744	393.1262
0.4	53.5919	176.7255	386.2827	53.5919	176.7255	386.2827	53.5928	176.6997	386.3124	53.5919	176.7185	386.2889
0.5	50.5341	177.8737	355.5421	50.5341	177.8737	355.5421	50.5333	177.8646	355.5111	50.5337	177.8708	355.5332
0.6	47.7443	172.4997	346.3611	47.7443	172.4997	346.3611	47.7419	172.5079	346.3041	47.7435	172.5011	346.3457
0.7	45.8312	159.6924	342.7145	45.8312	159.6924	342.7145	45.8279	159.6932	342.7333	45.8302	159.6921	342.7181
0.8	44.8500	148.7919	319.3664	44.8500	148.7919	319.3664	44.8466	148.7790	319.3626	44.8490	148.7882	319.3644
0.9	44.5350	143.9791	301.8169	44.5350	143.9791	301.8169	44.5327	143.9618	301.7652	44.5343	143.9743	301.8030
1.0	44.5000	143.3596	298.9644	44.5000	143.3596	298.9644	44.4998	143.3589	298.9626	44.4998	143.3590	298.9630

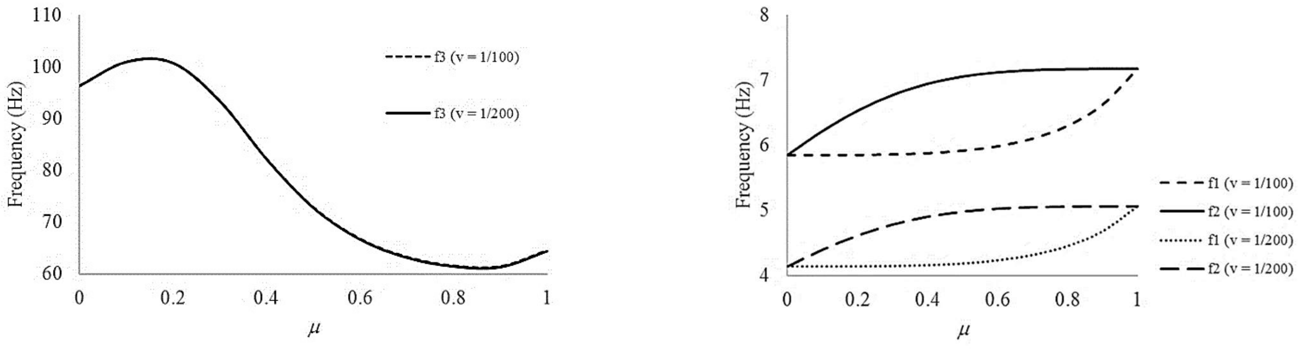

Figure 3 Effect of foundation stiffness on natural frequencies of stepped FF beam

Effect of foundation stiffness on first two frequencies is clearly illustrated in Fig.3 However, foundation effect for the third natural frequency is indistinguishable.

For stepped SS, CC and CS beams there are no significant difference in the natural frequencies for $\nu = 1/100$ and $\nu = 1/200$. Hence, between Figs. 4 and 6 only the variation of first three frequencies is plotted for $\nu = 1/100$.

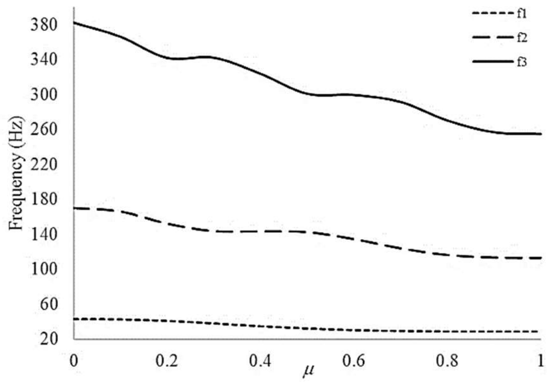

Figure 4 Variation of natural frequencies of stepped SS beam

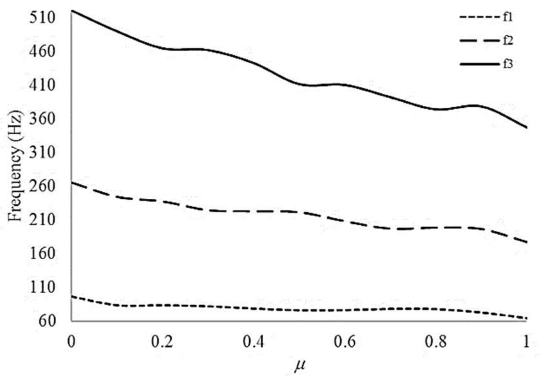

Figure 5 Variation of natural frequencies of stepped CC beam

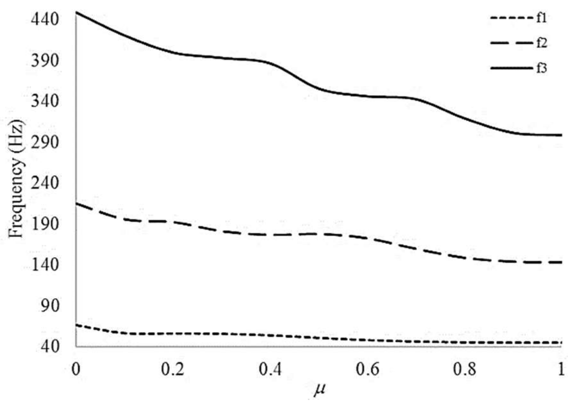

Figure 6 Variation of natural frequencies of stepped CS beam

It can be mentioned that natural frequencies of the stepped SS, CC and SS beams decreases with increasing μ values considering the variations shown in figures 4-6.

6. CONCLUSIONS

In this study, natural frequencies of stepped beams on elastic foundations are investigated via ADM and FEM. Analytical solutions for this problem are available for the beam with free ends and for simply supported beam. There are no other available analytical solutions for the beam with both ends clamped and for the beam with one end clamped and one end simply supported. All four cases previously were solved using HPM. ADM solutions of this study are in perfect agreement with analytical and HPM solutions. FE solutions are computed employing SAP 2000 software by using 50-element and 100-element models. Both FE models produced reliable results when compared ADM and previously available solutions.

Effect of foundation modulus is found to be distinguishable only for stepped FF beam considering the two different foundation modulus investigated in the scope of study. Variation of frequencies with the position of the intersection point of the two segments is also depicted graphically for stepped SS, CC and CS beams for which it is observed that natural frequencies decreases while μ increases. This result makes sense; As μ value increases, the frequency is mostly dominated with segment one that is having relatively low moment of inertia.

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The Declaration of Conflict of Interest/ Common Interest

No conflict of interest or common interest has been declared by the authors.

Authors' Contribution

In this study, the contributions of the authors during the research, analysis, submission, review and editing stages are equal.

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The authors declare that this document does not require an ethics committee approval or any special permission.

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