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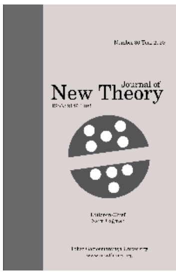

Tritopological Views in Product Spaces

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Abstract − In this paper we define and study the product of tritopological spaces (which we named it δ^*-product). Moreover, to motivate our definition, we show that the product properties for tritopological spaces are not preserved. Further, we provide some necessary and sufficient conditions for these spaces to be preserved under a finite product.

Keywords − *Tritopological spaces, δ^*-product of two tritopological spaces, δ^*-Tychonoff tritopology*

1. Introduction

In mathematics, the Cartesian product of a collection of sets is one of the most important and widely used ideas. The theory of product spaces constitutes a very interesting and complex part of set-theoretic topology. The Cartesian product of arbitrarily topological spaces was defined by Tychonoff in 1930 [1].

Then almost 33 years later in 1963, the idea of bitopological spaces was initiated by Kelly [2], and after that, a large number of papers have been produced in order to generalize the topological concepts to bitopological setting. In 1972, Datta [3] defined the Cartesian product of arbitrarily bitopological spaces. It is also well-known that the Tychonoff Product Theorem plays an important role in a general product.

A tritopological space is simply a set X which is associated with three arbitrary topologies, was initiated by Kovar [4]. In 2004, Hassan introduced the definition of δ^*-open set in tritopological spaces as follows, a subset A of X is said to be δ^*-open set iff $A \subseteq \mathcal{T}\ \mathrm{int}(\mathcal{P}\ \mathrm{cl}(\mathcal{Q}\ \mathrm{int}(A)))$ [5]. And in [6] she defined the δ^*-connectedness in tritopological spaces, also Hassan et al. [7] defined the δ^*-base in tritopological spaces. In [8] and [9] the reader can find a relationship among separation axioms, and relationships among some types of continuous and open functions in topological, bitopological and tritopological spaces, and in 2017, Hassan introduced the new definitions of countability and separability in tritopological spaces namely δ^*-countability and δ^*-separability [10]. In 2017, Hassan presented the concept of soft tritopological spaces [11]. However, no concept of tritopologization in product spaces has been given until now.

In the present paper, the concept of product topological spaces has been generalized to initiate the definition and study of product tritopological spaces. Besides, we introduce and characterize new definitions and theorems in tritopological spaces, and we provide some necessary and sufficient conditions for these spaces to be preserved under the δ^*-product.

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In section 2, some preliminary concepts about tritopological spaces are given. The main section of the manuscript is third which the definition of δ^*-product tritopology of two tritopological spaces with examples and some theorems are given. Section 4 is devoted to the generalization to theorems for tritopological product of spaces. In section 5 the definition of δ^*-Tychonoff tritopology and some theorems are introduced. Finally, in section 6 the conclusions and some future work is suggested

2. Preliminaries

In the following, we will mention some basic definitions and notations in tritopological space which we need in this work.

Definition 2.1. [5] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, a subset A of X is said to be δ^*-open set iff $A \subseteq \mathcal{T}\,\mathrm{int}(\mathcal{P}\,\mathrm{cl}(\mathcal{Q}\,\mathrm{int}(A)))$, and the family of all δ^*-open sets is denoted by $\delta^*.O(X)$. ($\delta^*.O(X)$ not always represent a topology). The complement of δ^*-open set is called a δ^*-closed set.

Definition 2.2. [5] $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is called a discrete tritopological space with respect to δ^*-open if $\delta^*.O(X)$ contains all subsets on X. And $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is called an indiscrete tritopological space with respect to δ^*-open if $\delta^*.O(X) = \{X, \emptyset\}$.

Definition 2.3. [5] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, and let $x \in X$, a subset N of X is said to be a δ^*-nhd of a point x iff there exists a δ^*-open set U such that $x \in U \subset N$. The set of all δ^*-nhds of a point x is denoted by $\delta^* - N(x)$.

Definition 2.4. [7] A collection δ^*-β of a subset of X is said to form a δ^*-base for the tritopology $(\mathcal{T},\mathcal{P},\mathcal{Q})$ iff: δ^*-$\beta \subset \delta^*.O(X)$. for each point $x \in X$ and each δ^*-neighbourhood $\mathcal{N}$ of x there exists some $\mathcal{B} \in \delta^*$-β such that $x \in \mathcal{B} \subset \mathcal{N}$.

Definition 2.5. [5] The function $f: (X,\mathcal{T},\mathcal{P},\mathcal{Q}) \to (Y,\mathcal{T}',\mathcal{P}',\mathcal{Q}')$ is said to be δ^*-continuous at $x \in X$ iff for every δ^*-open set V in Y containing $f(x)$ there exists δ^*-open set U in X containing x such that $f(U) \subset V$. We say f is δ^*-continuous on X iff f is δ^*-continuous at each $x \in X$.

Definition 2.6. [5] The function $f: (X,\mathcal{T},\mathcal{P},\mathcal{Q}) \to (Y,\mathcal{T}',\mathcal{P}',\mathcal{Q}')$ is said to be δ^*-open iff $f(G)$ is δ^*-open in Y for every δ^*-open set G in X.

Definition 2.7. [5] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ and $(Y,\mathcal{T}',\mathcal{P}',\mathcal{Q}')$ are two tritopological spaces and $f: (X,\mathcal{T},\mathcal{P},\mathcal{Q}) \to (Y,\mathcal{T}',\mathcal{P}',\mathcal{Q}')$ be a function, then f is δ^*-homeomorphism if and only if:

i. f is bijective (one to one, onto).
ii. f and f^{-1} are δ^*-continuous (or f is δ^*-continuous and δ^*-open).

Definition 2.8. [5] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, a point x is called δ^*-limit point of a subset A of X iff for each δ^*-open set G containing another point different from x in A; that is $(G/\{x\}) \cap A \neq \emptyset$, and the set of all δ^*-limit points of A is denoted by $\delta^* - lm(A)$.

Definition 2.9. [5] A tritopological space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is called δ^*-T_2-space (δ^*-Hausdorff) if and only if for each pair of distinct points x, y of X, there exists two δ^*-open sets G, H such that $x \in G,\ y \in H,\ G \cap H = \emptyset$.

Definition 2.10. [5] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, and let A be any subset of X, then the collection $C = \{G_\lambda : \lambda \in \Lambda\}$ is called δ^*-open cover to A if C is a cover to $A$and $C \subset \delta^*.O(X)$.

Definition 2.11. [5] Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, and let A be any subset of X, then A is called δ^*-compact set iff every δ^*-open cover of A has a finite sub-cover, i.e. for each $\{G_\lambda : \lambda \in \Lambda\}$ of δ^*-open sets for which $A \subset \cup\{G_\lambda : \lambda \in \Lambda\}$, there exist finitely many sets $G_{\lambda 1}, \dots, G_{\lambda n}$ among the G_λ's such that $A \subset G_{\lambda 1} \cup \dots \cup G_{\lambda n}$.

In particular, the space X is called δ^*-compact iff for each collection $\{G_\lambda : \lambda \in \Lambda\}$ of δ^*-open sets for which $X = \cup \{G_\lambda : \lambda \in \Lambda\}$, there exist finitely many sets $G_{\lambda 1}, \ldots, G_{\lambda n}$ among the G_λ's such that $X = G_{\lambda 1} \cup \ldots \cup G_{\lambda n}$.

Definition 2.12. [10] Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ be a tritopological space. Space is said to be a δ^*-second countable (or to satisfy the second axiom of δ^*-countability in tritopology) iff there exists a δ^*-countable base for a tritopology.

Definition 2.13. [12] Let $\{X_\lambda : \lambda \in \Lambda\}$ is an arbitrary collection of sets indexed by Λ, then the Cartesian product of this collection is the set of all mappings x defined by $x: \Lambda \to \cup \{X_\lambda : \lambda \in \Lambda\}$ such that $x(\lambda) \in X_\lambda$ for all $\lambda \in \Lambda$ and is denoted by $\pi\{X_\lambda : \lambda \in \Lambda\}$ or by $\times \{X_\lambda : \lambda \in \Lambda\}$. The set $X_\lambda$is called the λ^{th}coordinate set of the product.

Definition 2.14. [12] Let $X = \times \{X_\lambda : \lambda \in \Lambda\}$, then the mapping $\pi_\lambda : X \to X_\lambda$ defined by $\pi_\lambda(x) = x_\lambda$ for all $x \in$ X is called the λ^{th} projection.

3. Product space of two tritopological spaces

In this section, we shall describe the technique for constructing a tritopology for the Cartesian product $X \times Y$ of two tritopological spaces X and Y with the help of the families of all δ^*-open sets $\delta^*.\mathrm{O(X)}$ and $\delta^*.\mathrm{O(Y)}$ of the two spaces $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ and $(Y, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ shall examine the properties of the tritopology thus obtained in minute details. Subsequent sections will be devoted to the way of tritopologizing the Cartesian product of an arbitrary collection of tritopological spaces.

Because the families of all δ^*-open sets $\delta^*.\mathrm{O(X)}$ and $\delta^*.\mathrm{O(Y)}$ does not always represent a topology [5]. We provide some necessary conditions for these theorems to be valid under a finite product.

Theorem 3.1. Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ and $(Y, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ be two tritopological space and if $\delta^*.\mathrm{O(X)}$ and $\delta^*.\mathrm{O(Y)}$ represent a topology. Then the collection $E = \{G \times H : G \in \delta^*.\mathrm{O(X)} \text{ and } H \in \delta^*.\mathrm{O(Y)}\}$ is a δ^*-base for some tritopology for $X \times Y$.

Proof. Assume that $\delta^*.\mathrm{O(X)}$ and $\delta^*.\mathrm{O(Y)}$ represent a topology. We shall show that E satisfies the conditions [B1] and [B2] of Theorem [7], Since $X \times Y \in E$, we have $X \times Y = \cup \{G \times H : G \in \delta^*.\mathrm{O(X)} \text{ and } H \in \delta^*.\mathrm{O(Y)}\}$. Thus, [B1] is satisfied.

Now let $G_1 \times H_1$ and $G_2 \times H_2$ be any two members of E. We then have

$$(G_1 \times H_1) \cap (G_2 \times H_2) = (G_1 \cap G_2) \times (H_1 \cap H_2) \qquad (1) \text{ [see (2.18) (iii), ch. 1]}$$

Since we assume that $\delta^*.\mathrm{O(X)}$ and $\delta^*.\mathrm{O(Y)}$ represent a topology, we have

$$G_1 \in \delta^*.\mathrm{O(X)},\ G_2 \in \delta^*.\mathrm{O(X)} \to G_1 \cap G_2 \in \delta^*.\mathrm{O(X)}$$

And $H_1 \in \delta^*.\mathrm{O(Y)},\ H_2 \in \delta^*.\mathrm{O(Y)} \to H_1 \cap H_2 \in \delta^*.\mathrm{O(Y)}$.

Hence it follows from (1) that $(G_1 \times H_1) \cap (G_2 \times H_2) \in E$. Thus, we have shown that the intersection of any two members of E is again a member of E and so $[B2]$ is also satisfied. Therefore E is a δ^*-base for some tritopology for $X \times Y$.

Remark 3.2. If $\delta^*.\mathrm{O(X)}$ and $\delta^*.\mathrm{O(Y)}$ does not represent a topology; the above theorem is not achieved. Because the intersection of any two members of E is not always a member of E and so $[B2]$ is not satisfied. Therefore E is not a δ^*-base for some tritopology for $X \times Y$. (see example 1.1.4 in [5]).

Definition 3.3. Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ and $(Y, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ be two tritopological space. Then the tritopology (U, V, W) whose δ^*-base is $E = \{G \times H : G \in \delta^*.\mathrm{O(X)} \textit{ and } H \in \delta^*.\mathrm{O(Y)}\}$ is called the δ^*-product tritopology for $X \times Y$ and $(X \times Y, U, V, W)$ is called the δ^*-product space of X and Y.

Observe that in view of theorem (3.1), E is a δ^*-base for some tritopology for $X \times Y$. This is the tritopology (U,V,W) of the above definition.

Theorem 3.4. Let $(\mathrm{X},\mathcal{T},\mathcal{P},\mathcal{Q})$ and $(Y,\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ be two tritopological spaces. And β is a δ^*-base for $(\mathcal{T},\mathcal{P},\mathcal{Q})$ and $\complement$ is a δ^*-base for $(\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$. Then $\wp = \{B \times C : B \in \beta \; and \; C \in \complement\}$ is a δ^*-base for the δ^*-product tritopology (U,V,W) for $X \times Y$.

Proof. *Let* (x,y) be any point of $X \times Y$ and let N be a δ^*-nhd of (x,y) in $X \times Y$. Since $E = \{G \times H : G \in \delta^*.\mathrm{O(X)} \; and \; H \in \delta^*.\mathrm{O(Y)}\}$ is a δ^*-base for (U,V,W), there exists a member $G \times H$ of E such that $(x,y) \in G \times H \subset N$. … (1)

Since G is δ^*-open and β is a δ^*-base for $(\mathcal{T},\mathcal{P},\mathcal{Q})$, there exists some $B \in \beta$ such that $x \in B \subset G$. Similarly, there exists some $C \in \complement$ such that $y \in C \subset H$. It follows that

$(x,y) \in B \times C \subset G \times H$. … (2)

Hence from (1) and (2), we get $(x,y) \in B \times C \subset N$. This implies that $\wp$ is a δ^*-base for $(\mathrm{U,V,W})$.

Example 3.5. Let $X = \{a,b,c\}$, $\mathcal{T} = \{\mathrm{X},\emptyset,\{\mathrm{b}\},\{\mathrm{c}\},\{\mathrm{b,c}\}\}$

, $\mathcal{P} = \{\mathrm{X},\emptyset,\{\mathrm{a}\},\{\mathrm{b}\},\{\mathrm{a,b}\}\}$

, $\mathcal{Q} = \{\mathrm{X},\emptyset,\{\mathrm{a}\},\{\mathrm{c}\},\{\mathrm{a,c}\}\}$

$(\mathrm{X},\mathcal{T})$, $(\mathrm{X},\mathcal{P})$ and $(\mathrm{X},\mathcal{Q})$ are three topological space, then $(\mathrm{X},\mathcal{T},\mathcal{P},\mathcal{Q})$ is a tritopological space, the family of all δ^*-open set of X is: $\delta^*.\mathrm{O(X)} = \{\mathrm{X},\emptyset,\{\mathrm{c}\}\}$

And let $Y = \{p,q,r,s\}$, $\acute{\mathcal{T}} = \{\mathrm{Y},\emptyset\}$

, $\acute{\mathcal{P}} = \{\mathrm{Y},\emptyset\}$

, $\acute{\mathcal{Q}} = \{\mathrm{Y},\emptyset,\{\mathrm{p}\},\{\mathrm{q}\},\{\mathrm{p,q}\},\{\mathrm{r,s}\},\{\mathrm{p,r,s}\},\{\mathrm{q,r,s}\}\}$

$(Y,\acute{\mathcal{T}})$, $(Y,\acute{\mathcal{P}})$ and $(Y,\acute{\mathcal{Q}})$ are three topological space, then $(Y,\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ is a tritopological space, the family of all δ^*-open set of Y is:

$$\delta^*.O(Y) = \{Y,\emptyset,\{p\},\{q\},\{p,q\},\{r,s\},\{p,r,s\},\{q,r,s\}\}$$

Now we will find a δ^*-base for the δ^*-product tritopology of $X \times Y$.

It is easy to see that $\beta = \{\{c\},X\}$ is a δ^*-base for $(\mathcal{T},\mathcal{P},\mathcal{Q})$ and $\complement = \{\{p\},\{q\},\{r,s\}\}$ is a δ^*-base for $(\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$.

Hence by theorem (3.4) above, a δ^*-base for the δ^*-product tritopology is given by

$$\begin{aligned}\wp &= \{\{c\}\times\{p\},\{c\}\times\{q\},\{c\}\times\{r,s\}\},X\times\{p\},X\times\{q\},X\times\{r,s\}\} \\ &= \{\{(c,p)\},\{(c,q)\},\{(c,r),(c,s)\},\{(a,p),(b,p),(c,p)\},\{(a,q),(b,q),(c,q)\}, \\ &\qquad \{(a,r),(a,s),(b,r),(b,s),(c,r),(c,s)\}\}\end{aligned}$$

Definition 3.6. A tritopology $(\mathcal{T},\mathcal{P},\mathcal{Q})$ on a set X is said to be δ^*-weaker (or δ^*-coarser or δ^*-smaller) than another Tritopology $(\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ on X. Or we can say that $(\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ is said to be δ^*-stronger (or δ^*-finer or δ^*-larger) than $(\mathcal{T},\mathcal{P},\mathcal{Q})$) iff $\delta^*.\mathrm{O(X)} \subset \delta^*.\acute{\mathrm{O}}(\mathrm{X})$, (where $\delta^*.\mathrm{O(X)}$ is the family of all δ^*-open sets in$(\mathrm{X},\mathcal{T},\mathcal{P},\mathcal{Q})$ and $\delta^*.\acute{\mathrm{O}}(\mathrm{X})$ is the family of all δ^*-open sets in $(\mathrm{X},\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$).

According to this definition, indiscrete tritopology on any set X with respect to δ^*-open set is the δ^*-weakest whereas the discrete tritopology on any set X with respect to δ^*-open set is the δ^*-strongest. It is easy to see that the collection C off all tritopologies on a set X is a δ^*-partially ordered set with respect to the relation $\leq$ defined by setting $(\mathcal{T},\mathcal{P},\mathcal{Q}) \leq (\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ iff $(\mathcal{T},\mathcal{P},\mathcal{Q})$ is δ^*-weaker than $(\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$, where $(\mathcal{T},\mathcal{P},\mathcal{Q}) \; and \; (\acute{\mathcal{T}},\acute{\mathcal{P}},\acute{\mathcal{Q}})$ are members of C. The indiscrete tritopology on X w.r.t. δ^*-open set is the δ^*-infimum and the discrete tritopology on X w.r.t. δ^*-open set is the δ^*-supremum of $(C,\leq)$.

Theorem 3.7. The δ^*-product tritopology on a non-empty set $X \times Y$ is the δ^*-weak tritopology for $X \times Y$ determined by the projection maps π_x and π_y from the tritopologies on $X\ and\ Y.$ (This theorem is valid when $\delta^*.\mathrm{O}(X)$ and $\delta^*.\mathrm{O}(Y)$ are satisfied a topology)

Proof. The δ^*-weak tritopology has a δ^*-subbase $\{G_\lambda : \mathrm{G}_\lambda = {\pi_x}^{-1}[A_\lambda]\ or\ G_\lambda = {\pi_y}^{-1}[B_\lambda]$, for some A_λ δ^*-open in $\delta^*.\mathrm{O}(X)$ or B_λ δ^*-open in $\delta^*.\mathrm{O}(Y)$ }

The intersection ${\pi_x}^{-1}[A_1] \cap \dots . \cap {\pi_x}^{-1}[A_m] \cap {\pi_y}^{-1}[B_1] \cap \dots \cap \pi_y\left[{B_n}^{-1}\right]$

$$= (A_1 \times Y) \cap \dots \cap (A_m \times Y) \cap (X \times B_1) \dots \cap (X \times B_n)$$

[since $(A \times B) \cap (S \times T) = (A \cap S) \times (B \cap T)$]. Of a finite number of such sets has the form

$(A_1 \cap A_2 \cap \dots A_m) \times (B_1 \cap B_2 \cap \dots \cap B_n) = A^\star \times B^\star$

Where $A^\star$ is δ^*-open in $\delta^*.\mathrm{O}(X)$ and $B^\star$ is δ^*-open in $\delta^*.\mathrm{O}(Y)$. Hence the δ^*-weak tritopology has the same δ^*-base as the δ^*-product tritopology, and so the two tritopologies are the same.

[Note that ${\pi_x}^{-1}[A_1] = A_1 \times Y, {\pi_y}^{-1}[B_1] = X \times B_1\ \ etc.$]

Definition 3.8. Let $(\mathrm{X}, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ and $(Y, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ be two tritopological spaces. Then the mappings

$\pi_x : X \times Y \to X :\ \pi_x\big((x, y)\big) = x \quad \forall (x, y) \in X \times Y$ and

$\pi_y : X \times Y \to Y :\ \pi_y\big((x, y)\big) = y \quad \forall (x, y) \in X \times Y$

are called the projections of the δ^*-product $X \times Y$ on tritopological spaces X and Y respectively.

Theorem 3.9. Let $(\mathrm{X}, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ and $(Y, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ be two tritopological spaces. And let $(X \times Y, U, V, W)$ be the δ^*-product space of the two spaces. then the projections π_x and π_y are δ^*-continuous and δ^*-open mappings. further the δ^*-product tritopology (U, V, W) is the δ^*-coarsest tritopology for which the projections are δ^*-continuous.

Proof. Recall that π_x is a mapping of $X \times Y$ onto X defined by $\pi_x\ ((x, y)) = x$ for every $(x, y) \in X \times Y$. Let G be any δ^*-open set. Then it is evident from the definition of π_x that ${\pi_x}^{-1}[G] = G \times Y$ which is a basic δ^*-open subset of $X \times Y$.

[$\because G \in \delta^*.\mathrm{O}(X),\ Y \in \delta^*.\mathrm{O}(Y) \to G \times Y \in E$ where E is the δ^*-base for (U, V, W)].

Hence π_x is a δ^*-continuous mapping from $(X \times Y, U, V, W)$ to $(\mathrm{X}, \mathcal{T}, \mathcal{P}, \mathcal{Q})$. Similarly, π_y is a δ^*-continuous mapping from $(X \times Y, U, V, W)$ to $(Y, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$. Now let A be any δ^*-open subset of $X \times Y$. Then by the definition of the δ^*-base E for (U, V, W), we have $A = \cup \{G \times H : G \in \delta^*.\mathrm{O}(X), H \in \delta^*.\mathrm{O}(Y)\ and\ G \times H \in E' \subset E\}$.

Hence $\pi_x[A] = \pi_x[\cup \{G \times H : G \in \delta^*.\mathrm{O}(X), H \in \delta^*.\mathrm{O}(Y)\ and\ G \times H \in E']$

$$= \cup \{\pi_x[G \times H] : G \in \delta^*.\mathrm{O}(X), H \in \delta^*.\mathrm{O}(Y)\ and\ G \times H \in E'\}$$

$$= \cup \{G : G \in \delta^*.\mathrm{O}(X)\ and\ G \times H \in E'\} \qquad \text{[By the definition of } \pi_x]$$

$$\in \delta^*.\mathrm{O}(X)$$

It follows that π_x is an δ^*-open mapping. Finally, let (U^*, V^*, W^*) be any tritopology for $X \times Y$ for which the projections are δ^*-continuous and let A be any δ^*-open set of $X \times Y$. Then,

$A = \cup \{G \times H : G \in \delta^*.\mathrm{O}(X), H \in \delta^*.\mathrm{O}(Y)\ and\ G \times H \in E'\}$ where $E' \subset E$

$= \cup \{(G \cap X) \times (Y \cap H) : G \in \delta^*.\mathrm{O}(X), H \in \delta^*.\mathrm{O}(Y)\ and\ G \times H \in E'\}$

$= \cup \{(G \times Y) \cap (X \times H) : G \in \delta^*.\mathrm{O}(X), H \in \delta^*.\mathrm{O}(Y)\ and\ G \times H \in E'\}$

$= \cup \{\pi_x^{-1}[G] \cap \pi_y^{-1}[H]: G \in \delta^*.\mathrm{O}(X), H \in \delta^*.\mathrm{O}(Y) \; and \; G \times H \in E'\} \in \; \delta^*.\mathrm{O}(X \times Y)^*$

[Where $\delta^*.\mathrm{O}(X \times Y)^*$ is the family of all δ^*-open sets of the space$(X \times Y, U^*, V^*, W^*)$]

$\because \pi_x \; is \;\; \delta^*$-continuous $\Rightarrow \pi_x^{-1}[G] \in \delta^*.\mathrm{O}(X \times Y)^*$ and

$\pi_y \; is \;\; \delta^*$-continuous $\Rightarrow \pi_y^{-1}[H] \in \delta^*.\mathrm{O}(X \times Y)^*$ etc.

Thus every δ^*-open set in $\delta^*.\mathrm{O}(X \times Y)$ is δ^*-open in $\delta^*.\mathrm{O}(X \times Y)^*$ and so (U, V, W) is δ^*-coarser than (U^*, V^*, W^*) is any tritopology for $X \times Y$ for which the projections are δ^*-continuous, it follows that (U, V, W) is the δ^*-coarsets tritopology for which the projections are δ^*-continuous.

Theorem 3.10. Let y_o be a fixed element of Y and let $A = X \times \{y_o\}$. Then the restriction of π_x to A is a δ^*-homeomorphism of the subspace A of $X \times Y$ onto X. Similarly, the restriction of π_y to $B = \{x_o\} \times Y, x_o \in X$, is a δ^*-homeomorphism.

Proof. Let g_x denote the restriction of π_x to A, that is, let $g_x: A \to X: g_x((x, y_o)) = x \quad \forall (x, y_o) \in A$. Then $g_x((x_1, y_o)) = g_x((x_2, y_o)) \Longrightarrow x_1 = x_2 \Longrightarrow ((x_1, y_o)) = ((x_2, y_o)) \Longrightarrow g_x$ is one – one,

g_x is evidently onto. Since by the preceding theorem π_x is δ^*-continuous, it follows that g_x is also δ^*-continuous [5]. We now show that g_x is δ^*-open. Let C be any δ^*-open subset of A. Then $C = A \cap B$ for some δ^*-open subset B of $X \times Y$. But

$$B = \cup \{G \times H : G \in \delta^*.O(X), H \in \delta^*.O(Y) \; and \; X \times H \in E'\}$$

Where $E' \subset E$. We then have

$g_x[C] = g_x[A \cap B] = g_x[A \cap [\cup \{G \times H: G \in \delta^*.O(X), H \in \delta^*.O(Y) \; and \; G \times H \in E'\}]$

$= g_x[\cup \{A \cap (G \times H): G \in \delta^*.O(X), H \in \delta^*.O(Y) \; and \; G \times H \in E']$ [Distributive law]

$= \cup \{g_x[(X \times \{y_o\}) \cap (G \times H)]: G \in \delta^*.O(X), H \in \delta^*.O(Y) \; and \; G \times H \in E']$

$= \cup \{g_x[(X \cap G) \times (\{y_o\}) \cap H)]: G \in \delta^*.O(X), H \in \delta^*.O(Y) \; and \; G \times H \in E']$

$= \cup \{g_x[G \times \{y_o\} \cap H)]: G \in \delta^*.O(X), H \in \delta^*.O(Y) \; and \; G \times H \in E'] \quad ...(1)$

If $y_o \notin H$, then it is easy to see from (1) that $g_x[C] = \emptyset$. If $y_o \in H$, then (1) gives

$$g_x[C] = \cup \{g_x[G \times \{y_o\}]: G \in \delta^*.O(X), H \in \delta^*.O(Y) \; and \; G \times H \in E']$$

$$g_x[C] = \cup \{G: G \in \delta^*.O(X), and \; G \times H \in E'\} \in \delta^*.O(X).$$

This implies that g_x is an δ^*-open mapping as well. Thus, we have shown that g_x is one-one, onto, δ^*-continuous and δ^*-open mapping and consequently it is a δ^*-homeomorphism.

4. $\boldsymbol{\delta^*}$-Product invariant properties for finite $\boldsymbol{\delta^*}$-products

We are going to generalize theorems for tritopological product of spaces.

Theorem 4.1. The δ^*-product space $X \times Y$ is δ^*-connected if and only if the tritopological spaces X and Y are δ^*-connected.

Proof. Assume that $X \times Y$ is δ^*-connected. Since the projections π_x and π_y are δ^*-continuous and onto mappings, it follows from Theorem in [6] that X and Y are also δ^*-connected spaces. Conversely, let X and Y be δ^*-connected spaces. To show that $X \times Y$ is also δ^*-connected. Let (x_1, y_1) and (x_2, y_2) be any two points of $\delta^*.O(X \times Y)$. Then by theorem (3.10), $\{x_1\} \times Y$ is δ^*-homeomorphic to Y and $X \times \{y_2\}$ is δ^*-homeomorphic to X. Hence $\{x_1\} \times Y$ and $X \times \{y_2\}$ are δ^*-connected by theorem in [6] They intersect in the points (x_1, y_2) and hence there union is a δ^*-connected set by theorem in [6].Since this union contains (x_1, y_1) and (x_2, y_2), it follows from Theorem in [6] that $X \times Y$ is δ^*-connected.

Theorem 4.2. The δ^*-product space $X \times Y$ is δ^*-compact if and only if each of the tritopological spaces X and Y is δ^*-compact.

Proof. Let $X \times Y$ be δ^*-compact. Since the projection maps $\pi_x: X \times Y \to X$ and $\pi_y: X \times Y \to Y$ are δ^*-continuous and onto, it follows from Theorem in [5] that X and Y are also δ^*-compact. Conversely, let X and Y be δ^*-compact spaces. We want to show that $X \times Y$ is δ^*-compact. In view of Theorem in [5], it suffices to show that every basic δ^*-open cover of $X \times Y$ has a finite subcover. Since a basic δ^*-open set in $X \times Y$ is of the form $G \times H$ where G is δ^*-open in X and H is δ^*-open in Y, we may denote a basic δ^*-open cover by $C = \{G_\lambda \times H_\lambda: \lambda \in \Lambda\}$ where G_λ is δ^*-open in X and H_λ is δ^*-open in Y. For a given point $x \in X$, the set $\{x\} \times Y$ is δ^*-homeomorphic to Y by theorem (3.10), and is, therefore, δ^*-compact by theorem in [5]. Since $\{x\} \times Y$, being a subset of $X \times Y$, is covered by C and $\{x\} \times Y$ is δ^*-compact, there exists a finite sub-family of C, say $\{G_{\lambda i} \times H_{\lambda i}: i = 1,2, \dots, n\}$, which covers $\{x\} \times Y$. Let $\cap G_{\lambda i} = G(x)$. Then $G(x)$ is δ^*-open in X and contains x since each $G_{\lambda i}$ contains x. Hence $\{G(x) \times H_{\lambda i}: i = 1,2, \dots, n\}$ is still a finite δ^*-open cover of $\{x\} \times Y$. Proceeding in this manner for each $x \in X$, we construct the collection $\{G(x): x \in X\}$ of δ^*-open sets in X which covers X. By δ^*-compactness of X, there exists a finite subcover $\{G(x_j): j = 1,2, \dots, m\}$ of this cover for X. Since each $G(x_j)$ is an intersection of δ^*-open sets in X which were used to form C, we may select an δ^*-open set $G_{\lambda_{xj}} \in C$ such that $G(x_j) \subset G_{\lambda_{xj}}$ for $j = 1,2, \dots, m$, Therefore $\{G_{\lambda_{xj}}: j = 1,2, \dots, m\}$ is a finite δ^*-open cover of X, and for each j, $1 \le j \le m, \{G_{\lambda_{xj}} \times H_{\lambda_i}: i = 1,2, \dots, n\}$ covers the subset $G(x_j) \times Y$ of $X \times Y$. By its construction the collection $\left\{G_{\lambda_{xj}} \times H_{\lambda_i}: i = 1,2, \dots, n\,;\ j = 1,2, \dots, m\right\}$ is then a finite subcover of C for $X \times Y$ and therefore $X \times Y$ is δ^*-compact by theorem in [5].

Theorem 4.3. The δ^*-product space of two δ^*-second countable tritopological spaces is δ^*-second countable.

Proof. Let X and Y be two δ^*-second countable tritopological spaces. To show that $X \times Y$ is also δ^*-second countable. Let B and C be δ^*-countable bases for X and Y respectively. Consider the collection $D = \{B \times C: B \in \delta^* - \beta,\ C \in \delta^* - \complement\}$. Then D is surely a countable collection [12]. It follows from theorem (3.4), that D is a δ^*-countable bases for $X \times Y$.

Theorem 4.4. The δ^*-product space of two δ^*-Hausdorff tritopological spaces is δ^*-Hausdorff.

Proof. Let X and Y be two δ^*-Hausdorff tritopological spaces. To show that $X \times Y$ is also a δ^*-Hausdorff space. Let (x_1, y_1) and (x_2, y_2) be any two distinct points in $\delta^*.O(X \times Y)$. Then either $x_1 \neq x_2$ or $y_1 \neq y_2$. Take $x_1 \neq x_2$. Since X is δ^*-Hausdorff, there exist δ^*-open sets G_1 and G_2 in $\delta^*.O(X)$ such that $x_1 \in G_1$, $x_2 \in G_2$ and $G_1 \cap G_2 = \emptyset$ [5]. Then $G_1 \times Y$ and $G_2 \times Y$ are δ^*-open subset of $\delta^*.O(X \times Y)$ such that $(x_1, y_1) \in G_1 \times Y$, $(x_2, y_2) \in G_2 \times Y$ and $(G_1 \times Y) \cap (G_2 \times Y) = (G_1 \cap G_2) \times Y = \emptyset \times Y = \emptyset$. It follows that the tritopological space $(X \times Y, U, V, W)$ is δ^*-Hausdorff.

5. δ^*- Product tritopology (or δ^*-Tychonoff tritopology)

Definition 5.1. For each λ in an arbitrary index set Λ, let $(X_\lambda, \mathcal{T}_\lambda, \mathcal{P}_\lambda, \mathcal{Q}_\lambda)$ be a tritopological space and let $X = \times\{X_\lambda: \lambda \in \Lambda\}$. Then tritopology $(\mathcal{T}, \mathcal{P}, \mathcal{Q})$ for X which has a δ^*-sub bases the collection $B_* = \{\pi_\lambda{}^{-1}[G_\lambda]: \lambda \in \Lambda, G_\lambda \in \delta^*.O(X)_\lambda\}$ is called the δ^*-product tritopology (or the δ^*-Tychonoff tritopology) for X, and $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is called the δ^*-product space of the given spaces.

Note that here π_λ denotes as usual the λ^{th} projection. The collection B_* is called the defining δ^*-subbase for $(\mathcal{T}, \mathcal{P}, \mathcal{Q})$. the collection β of all finite intersections of elements of B_* would then form a δ^*-base for $(\mathcal{T}, \mathcal{P}, \mathcal{Q})$.

Remark 5.2. Since $\pi_\lambda{}^{-1}[G_\lambda]$ are δ^*-open sets with respect to the δ^*-product tritopology where G_λ is any δ^*-open set in X_λ it follows that the projection π_λ is a δ^*-continuous map for each $\lambda \in \Lambda$.

Theorem 5.3. Let $\{(X_\lambda, \mathcal{T}_\lambda, \mathcal{P}_\lambda, \mathcal{Q}_\lambda): \lambda \in \Lambda\}$. Be an arbitrary collection of tritopological spaces and let $X = \times\{X_\lambda: \lambda \in \Lambda\}$. Let $(\mathcal{T}, \mathcal{P}, \mathcal{Q})$ be a tritopology for X. Then the following statements are equivalent: (when all the families of δ^*-open sets of tritopological spaces represent a topology, this theorem is satisfied)

(a) $(\mathcal{T}, \mathcal{P}, \mathcal{Q})$ is the δ^*-product tritopology for X.

(b) $(\mathcal{T}, \mathcal{P}, \mathcal{Q})$ is the δ^*-smallest tritopology for X for which the projections are δ^*-continuous.

Proof. (a) $\Longrightarrow$ (b): Let π_λ be the λ^{th}projection map and let G_λ be any ${\delta^*}_\lambda$-open subset of X_λ. Then by (a), ${\pi_\lambda}^{-1}[G_\lambda]$ must be ${\delta^*}_\lambda$-open. It follows that π_λ is δ^*-continuous from $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ to $(X_\lambda, \mathcal{T}_\lambda, \mathcal{P}_\lambda, \mathcal{Q}_\lambda)$. Now let $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ be any tritopology on X such that π_λ is δ^*-continuous from $(X, \acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ to $(X_\lambda, \mathcal{T}_\lambda, \mathcal{P}_\lambda, \mathcal{Q}_\lambda)$. for each $\lambda \in \Lambda$. Then ${\pi_\lambda}^{-1}[G_\lambda]$ is δ^*-open in $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ for every $G_\lambda \in \delta^*.O(X)_\lambda$. Since $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ is a tritopology for X, $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ contains all the unions of finite intersections of members of the collection $\{{\pi_\lambda}^{-1}[G_\lambda]: \lambda \in \Lambda \text{ and } G_\lambda \in \delta^*.O(X)_\lambda\}$.

This implies that $\delta^*.\acute{O}(X)$ contains $\delta^*.O(X)(\delta^*.O(X) \subset \delta^*.\acute{O}(X))$, that is $(\mathcal{T}, \mathcal{P}, \mathcal{Q})$ is δ^*-coarser than $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$. It follows $(\mathcal{T}, \mathcal{P}, \mathcal{Q})$ is the δ^*-smallest tritopology for X such that π_λ is δ^*-continuous for each $\lambda \in \Lambda$.

$(b) \Longrightarrow (a)$: Let B_* be the collections of all sets of the form ${\pi_\lambda}^{-1}[G_\lambda]$ where G_λ is an δ^*-open subset of X_λ for $\lambda \in \Lambda$. Then by theorem in [5], a tritopology $(\acute{\mathcal{T}}, \acute{\mathcal{P}}, \acute{\mathcal{Q}})$ for X will make all the projections π_λ δ^*-continuous iff $B_* \subset \delta^*.\acute{O}(X)$. Hence in view of [7], the δ^*-smallest tritopology for X which makes all the projections δ^*-continuous is the tritopology determined by B_* as a δ^*-subbase, that is, it is the δ^*-product tritopology for X [see (5.1)].

Theorem 5.4. Let $\{(X_\lambda, \mathcal{T}_\lambda, \mathcal{P}_\lambda, \mathcal{Q}_\lambda): \lambda \in \Lambda\}$, an arbitrary collection of tritopological spaces and let $X = \times\{X_\lambda: \lambda \in \Lambda\}$. The collection C of all sets of the form $\times\{G_\lambda: \lambda \in \Lambda\}$. Where $G_\lambda \in \delta^*.O(X)_\lambda$ for each $\lambda \in \Lambda$, is a δ^*-base for some tritopology for X. (if $\delta^*.O(X)_\lambda$ satisfy the topology this theorem is valid)

Proof. We shall show that C satisfies the conditions [B1] and [B2] of Theorem in [7].

[B1]: Let $x \in X$ so that $x = \{x_\lambda: \lambda \in \Lambda\}$ where $x_\lambda \in X_\lambda$. Then there exists a ${\delta^*}_\lambda$-open set G_λ (which may be X_λ) such that $x_\lambda \in G_\lambda$. Hence x is an element of a set of the form $\times\{G_\lambda: \lambda \in \Lambda\} = G$ say. Thus, to each $x \in X$, there exists a member G of C such that $x \in G$. It follows that $X = \cup\{G: G \in C\}$.

[B2] Let $G \in C$ and $G' \in C$. Then $\times\{G_\lambda: \lambda \in \Lambda\} = G$ And $\times\{{G'}_\lambda: \lambda \in \Lambda\} = G'$

Where $G_\lambda \in \delta^*.O(X)_\lambda$ and ${G'}_\lambda \in \delta^*.O(X)_\lambda$ for every $\lambda \in \Lambda$. Now

$$G \cap G' = (\times\{G_\lambda: \lambda \in \Lambda\}) \cap \times\{{G_\lambda}': \lambda \in \Lambda\})$$
$$= \times\{G \cap G': \lambda \in \Lambda\} \quad \ldots (1)$$

Since $(\mathcal{T}_\lambda, \mathcal{P}_\lambda, \mathcal{Q}_\lambda)$ is a tritopology for X_λ, we have $G_\lambda \in \delta^*.O(X)_\lambda$ and ${G'}_\lambda \in \delta^*.O(X)_\lambda \rightarrow G \cap G' \in \delta^*.O(X)_\lambda$. (that is if $\delta^*.O(X)_\lambda$ represent a topology)

It follows from (1) that $G \cap G' \in C$. Thus [B2] is also satisfied.

Theorem 5.5 Let f be a mapping of a tritopological space Y into a δ^*-product space $X = \times\{X_\lambda: \lambda \in \Lambda\}$. Then f is δ^*-continuous iff the composition $\pi_\lambda \, o \, f : Y \rightarrow X_\lambda$ is δ^*-continuous.

Proof. Let f be δ^*-continuous. Since all projection is δ^*-continuous, it follows from Theorem in [5], that π_λ of is also δ^*-continuous.

Conversely, let each composition map π_λ of be δ^*-continuous and let U be any member of the defining δ^*-subbase B_* of the δ^*-product space X. then ${\pi_\lambda}^{-1}[G] = U$ for some $\lambda \in \Lambda$ and some $G \in \delta^*.O(X)_\lambda$. Also $f^{-1}[U] = f^{-1}[{\pi_\lambda}^{-1}[G]] = (\pi_\lambda o f)^{-1}[G]$.

Since $\pi_\lambda of$ is δ^*-continuous, it follows that $(\pi_\lambda of)^{-1}[G] = f^{-1}[U]$ is δ^*-open in Y. Thus, we have shown that the inverse image under f of every sub basic δ^*-open set in the δ^*-product space X is δ^*-open in Y. It follows from Theorem in [5] that f is δ^*-continuous.

Theorem 5.6. Each projection map is an δ^*-open map.

Proof. The proof left to the reader.

Theorem 5.7. Let X be the non-empty δ^*-product space $\times \{X_\lambda : \lambda \in \Lambda\}$. Then a non-empty δ^*-product subset $F = \times \{F_\lambda : \lambda \in \Lambda\}$ is δ^*-closed in X if and only if each F_λ is δ^*-closed in X_λ.

Proof. Let F_λ is δ^*-closed in X_λ for every $\lambda \in \Lambda$ Since the projection $\pi_\lambda$is δ^*-continuous, for each $\lambda \in \Lambda$. ${\pi_\lambda}^{-1}[F_\lambda]$ is δ^*-closed in X, it easy to see that $F = \cap \{{\pi_\lambda}^{-1}[F_\lambda] : \lambda \in \Lambda\}$.

It follows that F is δ^*-closed in X, being an intersection of δ^*-closed sets [5].

Conversely, let $F = \times \{F_\lambda : \lambda \in \Lambda\}$ be δ^*-closed in X. To show that each F_λ is δ^*-closed in X_λ. Let $\mu \in \Lambda$ be arbitrary. we shall show that F_μ is δ^*-closed in X_λ. Let μ be any δ^*-limit point F_μ in X_μ. Consider the point z where $\pi_\lambda(z) = z_\mu$ and $\pi_\lambda(z)$ is an element of F_μ for $\lambda \neq \mu$ Let G be any basic δ^*-open set for the δ^*-product topology containing z. Then $\pi_\mu(G)$ is δ^*-open by theorem (5.6) and contains z_μ. Since z_μ is a δ^*-limit point of $\pi_\mu(G)$ must contain a point x_μ of F_μ different from z_μ Therefore G contains the point x where $\pi_\lambda(x) = \pi_\lambda(z)$ for $\lambda \neq \mu$ and $\pi_\lambda(x) = x_\mu$. Evidently, $x \in F$, Also since x and z differ in μ^{th} coordinate, we have $x \neq z$ Thus we have shown that every basic δ^*-open set containing z contains a point of F different from z. Hence z is a δ^*-limit point of F. Since F is δ^*-closed in X, $z \in F$ which implies that $\pi_\mu(z) \in \pi_\mu(F)$. Thus F_μ contains all its δ^*-limit points and so F_μ is δ^*-closed. Since μ was arbitrary, we see that F_μ is δ^*-closed for every $\lambda \in \Lambda$.

6. Conclusion

The purpose of this article is to introduce the concept of the product in tritopological spaces namely δ^*-product spaces. Several properties of δ^*- product spaces concept is established. Moreover, we obtain a characterization and preserving theorems with the help of some necessary conditions and interesting examples. And we generalise theorems in δ^*-connectedness, δ^*-compactness, δ^*-second countability and δ^*-Hausdorff for tritopological product of spaces. Furthermore, the uses of tritopological results in this paper and some other papers are worthy for possible applications in areas of science and social science for the future.

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