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Pseudo-UP Ideals and Pseudo-UP Filters in Pseudo-UP Algebras

Daniel A. Romano*

Abstract
The notion of pseudo-UP algebras is introduced and analyzed in our forthcoming article as a generalization of UP-algebras. In this article, as a continuation of the foregoing, we introduce and analyze concepts of pseudo-UP ideals and pseudo-UP filters in pseudo-UP algebras.

Keywords: *UP-algebra, Pseudo-UP algebra, Pseudo-UP ideal, Pseudo-UP filter.*

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**Corresponding author*

1. Introduction

The concept of pseudo-BCK algebras was introduced in [3] by G. Georgescu and A. Iorgulescu as an extension of BCK algebra. The pseudo-ideals of pseudo-BCK algebras were introduced in 2003 by Y. B. Jun, M. Kondo and K. H. Kim in article [8]. The notion of pseudo-BCI algebras was introduced and analyzed in [1] by W. A. Dudek and Y. B. Jun as a generalization of BCI-algebras. The notion of pseudo-BCI ideals in pseudo-BCI algebras is introduced 2006 in [9] by Y. B. Jun, H. S. Kim and J. Neggers. These substructures in pseudo-BCK and pseudo-BCI algebras have been studied by several authors such as, for example, K. J. Lee and C. H. Park. [11] and G. Dymek [2]. These algebraic structures has been in the focus of many authors (for example, see [4, 6, 7, 19]).

Iampan [5] introduced a new algebraic structure which is called UP-algebras as a generalization of KU-algebras. Somjanta et al. [18] introduced the notion of UP-filters of UP-algebras. The concept of proper UP-filters in such algebras this author introduced 2018 in [12]. In addition, some new types of UP-filters in UP-algebras were introduced by Y. B. Jun and A. Iampan in [10].

In our forthcoming article [17], we introduced the concept of pseudo-UP algebras and some properties of pseudo-UP algebras are studied. In this article, as a continuation of the foregoing, we introduce and analyze the concepts of pseudo-UP ideals and pseudo-UP filters in pseudo-UP algebras.

2. Preliminaries

In this section we will describe some elements of UP-algebras from the literature [5] necessary for our intentions in this text.

Definition 2.1. ([5]) An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a UP-algebra where A is a nonempty set, $' \cdot '$ is a binary operation on A, and 0 is a fixed element of A (i.e. a nullary operation) if it satisfies the following axioms:
(UP-1) $(\forall x, y \in A)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$,
(UP-2) $(\forall x \in A)(0 \cdot x = x)$,
(UP-3) $(\forall x \in A)(x \cdot 0 = 0)$, and
(UP-4) $(\forall x, y \in A)((x \cdot y = 0 \ \wedge \ y \cdot x = 0) \implies x = y)$.

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Definition 2.2. ([17]) A pseudo-UP algebra is a structure $\mathfrak{A} = ((A, \leqslant), \cdot, *, 0)$, where $' \leqslant '$ is a binary relation on a set A, $' \cdot '$ and $' * '$ are internal binary operations on A and $'0'$ is an element of A, verifying the following axioms:
(pUP-1) $(\forall x, yz \in A)(y \cdot z \leqslant (x \cdot y) * (x \cdot z) \ \wedge \ y * z \leqslant (x * y) \cdot (x * z))$;
(pUP-4) $(\forall x, y \in A)((x \leqslant y \ \wedge \ y \leqslant x) \implies x = y)$;
(pUP-5) $(\forall x, y \in A)((y \cdot 0) * x = x \ \wedge \ (y * 0) \cdot x = x)$ and
(pUP-6) $(\forall x, y \in A)((x \leqslant y \iff x \cdot y = 0) \ \wedge \ (x \leqslant y \iff x * y = 0))$.

3. Pseudo-UP ideals and pseudo-UP filters in a pseudo-UP algebra

In the following definition, we introduce the concept of pseudo-UP ideals in pseudo-UP algebras

Definition 3.1. A nonempty subset J of a pseudo-UP algebra $\mathfrak{A}$ is called a pseudo-UP ideal of $\mathfrak{A}$ if it satisfies
(pJ1) $0 \in J$;
(pJ2) $(\forall x, y, z \in A)((x \cdot (y * z) \in J \ \wedge \ y \in J) \implies x \cdot z \in J)$ and
(pJ3) $(\forall x, y, z \in A)((x * (y \cdot z) \in J \ \wedge \ y \in J) \implies x * z \in J)$.

The following theorem describes the characteristic features of these substructures

Theorem 3.1. *Let J be a pseudo-UP ideal in a pseudo-UP algebra $\mathfrak{A}$. Then:*
(1) $(\forall y, z \in A)((y * z \in J \ \wedge \ y \in J) \implies z \in J)$;
(2) $(\forall x, y \in A)(y \in J \implies x * y \in J)$;
(3) $(\forall y, z \in A)((y \cdot z \in J \ \wedge \ y \in J) \implies z \in J)$ *and*
(4) $(\forall x, y \in A)(y \in J \implies x \cdot y \in J)$.

Proof. Since Equations (3) and (4) can be proved in a similar way to the proofs of equality (1) and (2), we will only show the last two mentioned.
Putting $x = 0$ in (pJ2), we obtain (1) with respect to equality (9) in the article [17].
Putting $z = y$ in (pJ3), we obtain (2) with respect to (pJ1) and to equalities (10) and (8) in the article [17]. □

Corollary 3.1. *Let J be a pseudo-UP ideal in a pseudo-UP algebra $\mathfrak{A}$. Then:*
(5) $(\forall y, z \in A)((y \leqslant z \ \wedge \ y \in J) \implies z \in J)$.

Proof. Let $y, z \in A$ be arbitrary elements such that $y \leqslant z$ and $y \in J$. Then $y \cdot z = 0 \in J$ (and $y * z = 0 \in J$). Thus $z \in J$ by (3) (by (1) respectively). □

If we use the labels

$$\cdot(z, J) = \{y \in A : y \cdot z \in J\} \quad \text{and} \quad *(z, J) = \{y \in A : y * z \in J\},$$

introduced and used in [8] and [9], we can conclude that as a consequence of the preceding theorem, the following is valid

Corollary 3.2. *J be a pseudo-UP ideal in a pseudo-UP algebra $\mathfrak{A}$. Then*
(6) $(\forall z \in A)(z \in J \implies (\cdot(z, J) \subseteq J \ \wedge \ *(z, J) \subseteq J))$.

Based on the orientation expressed in article [18], we introduce the concept of pseudo-UP filters as follows

Definition 3.2. A nonempty subset F of a pseudo-UP algebra $\mathfrak{A}$ is called a pseudo-UP filter of $\mathfrak{A}$, if it satisfies the following properties:
(pF1) $0 \in F$;
(pF2) $(\forall x, y \in A)((x \in F \ \wedge \ x \cdot y \in F \implies y \in F)$; and
(pF3) $(\forall x, y \in A)((x \in F \ \wedge \ x * y \in F \implies y \in F)$.

From this determination, immediately follows:

Proposition 3.1. *Let F be a pseudo-UP filter in a pseudo-UP algebra $\mathfrak{A}$. Then*
$(\forall x, y \in A)((x \in F \ \wedge \ x \leqslant y) \implies y \in F)$.

Proposition 3.2. *A nonempty subset F of a pseudo-UP algebra A is a pseudo-UP filter in $\mathfrak{A}$ if and only if $0 \in F$ and holds*
(F4) $(\forall z \in A)(z \in F \implies (\cdot(z, F) \subseteq F \ \wedge \ *(z, F) \subseteq F))$.

Theorem 3.2. *If F is a pseudo-UP filter of a pseudo-UP algebra $\mathfrak{A}$, then*
(7) $(\forall x, y, z \in A)((x \in F \wedge y \in F \wedge z * y \leqslant x) \implies z \in F)$ *and,*
(8) $(\forall x, y, z \in A)((x \in F \wedge y \in F \wedge z \cdot y \leqslant x) \implies z \in F)$.

Proof. Suppose that F is a pseudo-UP filter of $\mathfrak{A}$ and let $x, y, z \in A$ be arbitrary elements.

Suppose that $z * y \leqslant x$. Then $(z * y) \cdot x = 0 \in F$. Thus $z * y \in \cdot(x, F) \subseteq F$. It follows that $z \in *(y, F) \subseteq F$. Therefore, the condition (8) is proved.

Now let $x \in F$, $y \in F$ and $z \cdot y \leqslant x$ be hold. Then $(z \cdot y) * x = 0 \in F$, and thus $z \cdot y \in *(x, F) \subseteq F$. Hence $z \in \cdot(y, F) \subseteq F$, which shows the condition (8). □

4. Final observation

In the study of algebraic substructures of UP-algebras, this author took part with his texts ([12–16]). And this text should be seen as a continuation of these his efforts.

Looking at the [8–10], some types of pseudo-UP filters one can be introduced in pseudo-UP algebras. For example: one type of pseudo-UP filters can be introduced by requiring that the set F satisfies $0 \in F$ and the following conditions

$(\forall x, y, z \in A)((((x \cdot y) * y) \cdot z \in F \wedge z \in F) \implies x * y \in F)$ and
$(\forall x, y, z \in A)((((x * y) \cdot y) * z \in F \wedge z \in F) \implies x \cdot y \in F)$.

Another type of pseudo-UP filters could be a subset of F of a pseudo-UP algebra A if $0 \in F$ and the following holds
$(\forall y \in A)(y \in F \implies \cdot(y, F) \cap *(y, F) \subseteq F)$.

Further, drawing on the ideas in article [10], some other similar conditions could be analyzed.

References

[1] Dudek, W. A., Jun, Y. B.: *Pseudo-BCI algebras*. East Asian Math. J., **24**(2), 187-–190 (2008).

[2] Dymek, G.: *Atoms and ideals of pseudo-BCI-algebras*. Comment. Math., **52**(1), 73-–90 (2012).

[3] Georgescu, G., Iorgulescu, A.: *Pseudo-BCK algebras: An extension of BCK-algebras*. Combinatorics, computability and logic (Constanta, 2001), (pp. 97-–114), Springer Ser. Discrete Math. Theor. Comput. Sci. Springer, London, 2001.

[4] Emanovský, P., Kühr, J.: *Some properties of pseudo-BCK- and pseudo-BCI-algebras*. Fuzzy Sets and Systems, **339**, 1–16 (2018).

[5] Iampan, A.: *A new branch of the logical algebra: UP-algebras*. J. Algebra Relat Top., **5**(1), 35-–54 (2017).

[6] Iorgulescu, A.: *On pseudo-BCK algebras and porims*. Sci. Math. Japonicae Online, **10**, 293-–305 (2004).

[7] Iorgulescu, A.: *Classes of pseudo-BCK algebras – Part I*. J. of Mult.-Valued Logic Soft Comput., **12**, 71-–130 (2006).

[8] Jun, Y. B., Kondo, M., Kim, K. H.: *Pseudo-ideals of pseudo-BCK algebras*. Sci. Math. Japonicae Online, **8**, 87–91 (2003).

[9] Jun, Y. B., Kim, H. S., Neggers, J.: *On pseudo-BCI ideals of pseudo-BCI algebras*. Mat. Vesnik, **58**(1), 39-–46 (2006).

[10] Jun, Y. B., Iampan, A.: *Shift UP-filters and decompositions of UP-filters in UP-Algebras*. Missouri J. Math. Sci., **31**(1), 36–45 (2019).

[11] Lee, K. J., Park, C. H.: *Some ideals of pseudo-BCI-algebras*. J. Appl. Math. Inform., **27**(1-2), 217-–231 (2009).

[12] Romano, D. A.: *Proper UP-filters in UP-algebra*. Universal J. Math. Appl., **1**(2), 98–100 (2018).

[13] Romano, D. A.: *Notes on UP-ideals in UP-algebras*. Comm. Advan. Math. Sci., **1**(1), 35–38 (2018).

[14] Romano, D. A.: *Some properties of proper UP-filters of UP-algebras*. Fund. J. Math. Appl., **1**(2), 109–111 (2018).

[15] Romano, D. A.: *Some decompositions of UP-ideals and proper UP-filters*. Math. Advan. Pure Appl. Sci., **2**(1), 16–18 (2019).

[16] Romano, D. A.: *Pseudo-valuations on UP-algebras*. Universal J. Math. Appl., **2**(3), 138–140 (2019).

[17] Romano, D. A.: *Pseudo-UP algebras, An introduction*. *Bull. Int. Math. Virtual Inst.*, **10**(2), 349–355 (2020).

[18] Somjanta, J., Thuekaew, N. Kumpeangkeaw, P., Iampan, A.: *Fuzzy sets in UP-algebras*. Ann. Fuzzy Math. Inform., **12**(6), 739–756 (2016).

[19] Walendziak, A.: *On axiom systems of pseudo-BCK algebras*. Bull. Malays. Math. Sci. Soc. (2), **34**(2), 287-–293 (2011).

Affiliations

DANIEL A. ROMANO
ADDRESS: International Mathematical Virtual Institute 6,
Kordunaşka Street, 78000 Banja Luka, Bosnia and Herzegovina.
E-MAIL: bato49@hotmail.com
ORCID ID: 0000-0003-1148-3258