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Soft Sets and Soft Topological Notions

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Abstract — In this paper, we extended the notions of operations on soft sets to arbitrary collection of soft sets and introduce the concepts of $F_\sigma -$ soft set and $G_\sigma -$ soft set. Furthermore we give definitions of $\sigma-$ soft locally finite and $\sigma-$ soft closure preserving relative to an arbitrary collection of soft sets and study some of their properties.

Keywords — *Soft symmetric difference, soft perfect, relatively soft discrete, soft closed domain (regularly closed), soft open domain (regularly open)*

1. Introduction

In 1999 D. Molodtsov [1] introduced the concept of soft sets as an additional mathematical tool for modeling and dealing with uncertainties. Shabir and Naz [2] went further and introduced the concept of soft topology. Indeed; the two concepts have received much attention. Researches on properties and applications of soft sets and soft topology have attracted many scholars from various fields. Topological structure of soft sets; concepts of soft open sets, soft closed sets, soft interior point and soft neighborhood of a point have been studied by various authors, for example see [2–7]. Senel and Cagman studied soft topological subspaces and Tantawy et al. [8] studied soft separation axioms. The notions of basic operations on soft sets (soft union and soft intersection) have been defined and studied [2, 3, 6, 9–20] and by several other authors, but the definitions were given in terms of only two soft sets. Ali et al. [9] pointed out by counter example that, several assertions [particularly, Proposition 2.3 (iv)-(vi), Proposition 2.4 and Proposition 2.6(iii),(iv)] in Maji et al. [15] are not true in general.

In this paper we extend the notions of these basic operations to arbitrary collection of soft sets. In section 3 of this paper, we propose a modification of the definition of soft difference of two soft sets [2, 5, 11–13, 16, 21–26]. We further introduce and define some terms relative to arbitrary collection of soft sets in a soft topological space and study some of their properties.

2. Preliminary

Throughout this paper, all soft sets are defined over a common universe X and the collection of all soft sets over X with a set of parameters E is denoted as $SS(X)_E$. We begin with the following well known definition found in the literature as cited in each case.

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Definition 2.1. [1] Let X be an initial universe and E be a set of parameters. Let $\rho(X)$ denote the *power set* of X. A pair (F, E) is called a soft set over X, where $F : E \to \rho(X)$ is a mapping. Thus a soft set over a universe X is a parameterized family of $\rho(X)$. For a given subset A of E, the soft set (F, A) is defined to be $F : A \to \rho(X)$ such that $(F, A) = \{(e, F(e)) : e \in A\}$, where $F(e)$ can be regarded as the set of e-approximate elements of the soft set (F, A).

Definition 2.2. [27] A soft set (F_1, A_1) is said to be a *soft subset* of (F_2, A_2) denoted as $(F_1, A_1)\tilde{\subseteq}(F_2, A_2)$ if $A_1 \subseteq A_2$ and $F_1(\alpha) \subseteq F_2(\alpha), \forall\alpha \in A_1$. Equivalently (F_2, A_2) is said to be a soft superset of (F_1, A_1) denoted as $(F_2, A_2)\tilde{\supseteq}(F_1, A_1)$.

Definition 2.3. [27] Two soft sets (F_1, A_1) and (F_2, A_2) are said to be equal (*soft equal*) denoted as $(F_1, A_1) = (F_2, A_2)$ if $(F_1, A_1)\tilde{\subseteq}(F_2, A_2)$ and $(F_2, A_2)\tilde{\subseteq}(F_1, A_1)$. Equivalently $(F_1, A_1) = (F_2, A_2)$ if $F_1(\alpha) = F_2(\alpha), \forall\alpha \in A_1 = A_2$.

Definition 2.4. [7] The *soft compliment* of a soft set (F, A) denoted as $(F, A)^c$ or (F^c, A) is a mapping $F^c : A \to \rho(X)$ given by $F^c(\alpha) = X \setminus F(\alpha), \forall\alpha \in A$. It is very clear that $(F^c, A)^c = (F, A)$. The mapping $F^c : A \to \rho(X)$ is called the *soft compliment function* of F.

Definition 2.5. [8] A soft set (F, E) is said to be a *null soft set* if $F(\alpha) = \emptyset, \forall\alpha \in E$ and (F,E) is said to be an *absolute soft* set if $F(\alpha) = X, \forall\alpha \in E$. A null soft set is denoted as $\tilde{\emptyset}$ and absolute soft set is denoted as $\tilde{X}$. It is clear that $(\tilde{X})^c = \tilde{\emptyset}$ and $(\tilde{\emptyset})^c = \tilde{X}$

Definition 2.6. [28] Let (F, E) be a soft set and $x \in X$, then

i) x is said to belongs to (F, E) denoted as $x \in (F, E)$ if $\forall\alpha \in E, x \in F(\alpha)$.

ii) (F, E) is called singleton soft set denoted as (x, E) or x_E if $F(\alpha) = \{x\}, \forall\alpha \in E$.

Definition 2.7. [7] A soft set (F, A) is called a *soft point* denoted as F_α if for some $\alpha \in A, F(\alpha) \neq \emptyset$ and $F(\beta) = \emptyset, \forall\beta \in (A \setminus \{\alpha\})$. The soft point F_α is said to belong to another soft set (G, A), denoted as $F_\alpha\tilde{\in}(G, A)$ if $F(\alpha) \subseteq G(\alpha)$.

Definition 2.8. [27] A soft set (F, A) is called a *soft element* if $\exists\alpha \in A$ and $x \in X$ such that $F(\alpha) = \{x\}$ and $F(\beta) = \emptyset, \forall\beta \in (A \setminus \{\alpha\})$. A soft element is denoted as F_α^x. The soft element F_α^x is said to be in a soft set (G, A) denoted as $F_\alpha^x\tilde{\in}(G, A)$ if $x \in G(\alpha)$.

By definition, it is clear that a soft element is a soft point, but the converse may not be true.

Definition 2.9. [20] Let (F_1, A_1) and (F_2, A_2) be soft sets

i) The *soft intersection* of (F_1, A_1) and (F_2, A_2) denoted as $(F_1, A_1)\tilde{\cap}(F_2, A_2)$ is defined to be the soft set (F_3, A_3) where $A_3 = A_1 \cap A_2$ and $\forall\alpha \in A_3, F_3(\alpha) = F_1(\alpha) \cap F_2(\alpha)$;

ii) The *soft union* of (F_1, A_1) and (F_2, A_2) denoted as $(F_1, A)\tilde{\cup}(F_2, A_2)$ is defined to be the soft set (F_3, A_3) where $A_3 = A_1 \cup A_2$ and $\forall\alpha \in A_3$

$$F_3(\alpha) = \begin{cases} F_1(\alpha) \text{ if } \alpha \in (A_1 \setminus A_2) \\ F_2(\alpha) \text{ if } \alpha \in (A_2 \setminus A_1) \\ F_1(\alpha) \cup F_2(\alpha) \text{ if } \alpha \in (A_1 \cap A_2). \end{cases}$$

3. Basic Operations on Soft Sets

An arbitrary indexing set I was used by the authors [4,5,7,8,21–26] to define the soft intersection and soft union over a collection $\{(F_i, A) : i \in I\}$ of soft sets as $(F, A) = \tilde{\bigcap}_{i\in I}(F_i, A)$ and $(F, A) = \tilde{\bigcup}_{i\in I}(F_i, A)$ respectively. We point out here that the two definitions are restrictive and incomplete. It is also worth noting that in $\{(F_i, A) : i \in I\}$, A can also be indexed as well i.e., $\{(F_i, A_i) : i \in I\}$. The question is what is $\tilde{\bigcup}_{i\in I}(F_i, A)$? Clearly the definition in [4,5,7,8,21–26] does not cater for this. In this section we extend the notions of soft intersection and soft union to arbitrary collection of soft sets by the use of (i) and (ii) of definition 2.9 as follows:

Definition 3.1. Let $L = \{(F_\delta, A_\delta) : \delta \in \Delta\}$ be a family of soft sets, then the

i) soft intersection over members of L is defined to be the soft set $(F, A) = \tilde{\bigcap}_{\delta\in\Delta} (F_\delta, A_\delta)$ where $A = \bigcap_{\delta\in\Delta} A_\delta$ and $\forall\alpha \in A, F(\alpha) = \tilde{\bigcap}_{\delta\in\Delta} F_\delta(\alpha), \forall\alpha \in A$;

ii) soft union over members of L is defined to be the soft set $(F, A) = \tilde{\bigcup}_{\delta\in\Delta} (F_\delta, A_\delta)$ where $A = \bigcup_{\delta\in\Delta} A_\delta$ and $\forall\alpha \in A$ and $\forall\Upsilon \subseteq \Delta$ and $F(\alpha) = \begin{cases} \bigcup_{\delta\in\Delta} F_\delta(\alpha) \text{ if } \alpha \in \bigcap_{\delta\in\Delta} A_\delta \\ \bigcup_{\delta\in\Upsilon} F_\delta(\alpha) \text{ if } \alpha \in (\bigcap_{\delta\in\Upsilon} A_\delta \setminus \bigcup_{\delta\in(\Delta\setminus\Upsilon)} A_\delta). \end{cases}$

Example 3.2. Let $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters and $X = \{x_1, x_2, x_3, x_4\}$. $\rho(X) = \{\emptyset, X, \{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_1, x_4\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}\}$, $A_1 = \{e_1, e_3, e_4\}$, $A_2 = \{e_3, e_4\}$ and $A_3 = \{e_1, e_4, e_5\}$.

E	e_1	e_2	e_3	e_4	e_5
F_1	$\{x_1, x_2\}$	$\{x_2, x_3, x_4\}$	$\{x_1, x_4\}$	$\{x_1, x_2, x_4\}$	$\{x_1, x_2, x_3\}$
F_2	$\{x_2, x_3\}$	$\{x_2\}$	$\{x_1, x_2\}$	$\{x_2, x_4\}$	$\{x_2, x_3, x_4\}$
F_3	$\{x_1\}$	$\{x_3\}$	$\{x_4\}$	$\{x_2, x_4\}$	$\{x_1, x_3\}$
F_4	$\{x_3, x_4\}$	$\{x_1\}$	$\{x_2, x_3\}$	$\{x_3\}$	$\{x_4\}$
F_5	$\{x_3\}$	$\{x_3\}$	$\{x_3\}$	$\{x_3\}$	$\{x_3\}$
G	$\emptyset$	$\emptyset$	$\{x_3, x_4\}$	$\emptyset$	$\emptyset$
H	$\emptyset$	$\{x_2\}$	$\emptyset$	$\emptyset$	$\emptyset$

Notice that $A_1 \cap A_2 \cap A_3 = \{e_4\}$ and $A_1 \cup A_2 \cup A_3 = \{e_1, e_3, e_4, e_5\}$ and that

i) $F_1(e_4) \cap F_2(e_4) \cap F_3(e_4) = \{x_1, x_2, x_4\} \cap \{x_2, x_4\} \cap \{x_2, x_4\} = \{x_2, x_4\}$. Therefore,

$$\tilde{\bigcap_{i=1}^{3}}(F_i, A_i) = \{(e_4, \{x_2, x_4\})\}$$

ii) $e_1 \in (A_1 \cap A_3) \setminus A_2$, $e_3 \in (A_1 \cap A_2) \setminus A_3$, $e_4 \in (A_1 \cap A_2 \cap A_3)$ and $e_5 \in A_3 \setminus (A_1 \cup A_2)$

$$F(e_1) = F_1(e_1) \cup F_3(e_1) = \{x_1, x_2\}, F(e_3) = F_1(e_3) \cup F_2(e_3) = \{x_1, x_2, x_4\}$$

$$F(e_4) = F_1(e_4) \cup F_2(e_4) \cup F_3(e_4) = \{x_1, x_2, x_4\} \text{ and } F(e_5) = F_3(e_5) = \{x_1, x_3\}$$

Therefore, $\tilde{\bigcup}_{i=1}^{3} (F_i, A_i) = \{(e_1, \{x_1, x_2\}), (e_3, \{x_1, x_2, x_4\}), (e_4, \{x_1, x_2, x_4\}), (e_5, \{x_1, x_3\})\}$.

It is worth noting that the definition of arbitrary soft union as given in [4, 5, 7, 8, 21–26] has no provision for resolving the soft union $\tilde{\bigcup}_{i=1}^{3} (F_i, A_i)$.
The next result is indeed from the Demogan's Laws in set theory.

Proposition 3.3. (Demogan's) Let $\{(F_\delta, A_\delta) : \delta \in \Delta\}$ be a family of soft sets, then

i) $[\tilde{\bigcup}_{\delta\in\Delta} (F_\delta, A_\delta)]^c = \tilde{\bigcap}_{\delta\in\Delta} (F_\delta, A_\delta)^c$

ii) $[\tilde{\bigcap}_{\delta\in\Delta} (F_\delta, A_\delta)]^c = \tilde{\bigcup}_{\delta\in\Delta} (F_\delta, A_\delta)^c$

Proof. Let F^x_α be any soft element, then

i) $F^x_\alpha \tilde{\in} [\tilde{\bigcup}_{\delta\in\Delta} (F_\delta, A_\delta)]^c \Leftrightarrow F^x_\alpha \tilde{\notin} \tilde{\bigcup}_{\delta\in\Delta} (F_\delta, A_\delta) \Leftrightarrow F^x_\alpha \tilde{\notin} (F_\delta, A_\delta), \forall\delta \in \Delta \Leftrightarrow F^x_\alpha \tilde{\in} (F_\delta, A_\delta)^c, \forall\delta \in \Delta \Leftrightarrow F^x_\alpha \tilde{\in} \tilde{\bigcap}_{\delta\in\Delta} (F_\delta, A_\delta)^c$;

ii) $F_\alpha^x \tilde{\in} [\tilde{\bigcap}_{\delta\in\Delta} (F_\delta, A_\delta)]^c \Leftrightarrow F_\alpha^x \tilde{\notin} \tilde{\bigcap}_{\delta\in\Delta} (F_\delta, A_\delta) \Leftrightarrow F_\alpha^x \tilde{\notin} (F_\delta, A_\delta)$ for some $\delta \in \Delta \Leftrightarrow F_\alpha^x \tilde{\in} (F_\delta, A_\delta)^c$ for some $\delta \in \Delta \Leftrightarrow F_\alpha^x \tilde{\in} \tilde{\bigcup}_{\delta\in\Delta} (F_\delta, A_\delta)^c$.

□

The definition of difference of two soft sets (F, E) and (G, E) over a common universe X was given to be $(F, E) \setminus (G, E) = (H, E)$ where for all $e \in E$, $H(e) = F(e) \setminus G(e)$, [2,5,11–13,16,21–26]. This definition excludes the possibility of taking soft difference of soft sets of the form (F_1, A_1) and (F_2, A_2) where $A_1, A_2 \subseteq E$. Hence, we propose a modification of the definition which follows by some examples:

Definition 3.4. If (F_1, A_1) and (F_2, A_2) are any two soft sets, the soft difference of (F_1, A_1) and (F_2, A_2) denoted as $(F_1, A_1) \setminus (F_2, A_2)$ is defined to be the soft set (F_3, A_1) where

$$F_3(\alpha) = \begin{cases} F_1(\alpha) \text{ if } \alpha \in (A_1 \setminus A_2) \\ F_1(\alpha) \setminus F_2(\alpha) \text{ if } \alpha \in (A_1 \cap A_2) \end{cases}$$

It is worth noting that in $(F_1, A_1) \setminus (F_2, A_2)$ if $A_1 = A_2 = E$ Definition 3.4 reduces to [2,5,11–13, 16,21–26].

Example 3.5. From Example 3.2, $A_1 = \{e_1, e_3, e_4\}$, $A_2 = \{e_3, e_4\}$, $(F_1, A_1) = \{(e_1, \{x_1, x_2\}), (e_3, \{x_1, x_4\}), (e_4, \{x_1, x_2, x_4\})\}$, and $(F_2, A_2) = \{(e_3, \{x_1, x_2\}), (e_4, \{x_2, x_3, x_4\})\}$. Therefore,

i) $(F_1, A_1) \setminus (F_2, A_2) = \{(e_1, \{x_1, x_2\}), (e_3, \{x_4\}), (e_4, \{x_1\})\}$

ii) $(F_2, A_2) \setminus (F_1, A_1) = \{(e_3, \{x_2\}), (e_4, \{x_3\})\}$

Definition 3.6. The *soft symmetric difference* of (F_1, A_1) and (F_2, A_2) denoted as $(F_1, A_1)\tilde{\triangle}(F_2, A_2)$ is defined to be $(F_1, A_1)\tilde{\triangle}(F_2, A_2) = ((F_1, A_1) \setminus (F_2, A_2))\tilde{\cup}((F_2, A_2) \setminus (F_1, A_1))$. From Example 3.5, $(F_1, A_1)\tilde{\triangle}(F_2, A_2) = \{(e_1, \{x_1, x_2\}), (e_3, \{x_2, x_4\}), (e_4, \{x_1, x_3\})\}$.

As a consequence of Definition 3.4 we have the following lemma

Lemma 3.7. Let $\{(F_\delta, A_\delta) : \delta \in \Delta\}$ be a family of soft sets and (H, C) be any soft set, then

i) $[(H, C) \setminus \tilde{\bigcup}_{\delta\in\Delta} (F_\delta, A_\delta)] = \tilde{\bigcap}_{\delta\in\Delta} [(H, C) \setminus (F_\delta, A_\delta)]$

ii) $[(H, C) \setminus \tilde{\bigcap}_{\delta\in\Delta} (F_\delta, A_\delta)] = \tilde{\bigcup}_{\delta\in\Delta} [(H, C) \setminus (F_\delta, A_\delta)]$

PROOF. Let F_α^x be any soft element, then

i) $F_\alpha^x \tilde{\in} [(H, C) \setminus \tilde{\bigcup}_{\delta\in\Delta} (F_\delta, A_\delta)] \Leftrightarrow F_\alpha^x \tilde{\in} (H, C)$ and $F_\alpha^x \tilde{\notin} \tilde{\bigcup}_{\delta\in\Delta} (F_\delta, A_\delta) \Leftrightarrow F_\alpha^x \tilde{\in} (H, C)$ and $F_\alpha^x \tilde{\notin} (F_\delta, A_\delta), \forall \delta \in \Delta \Leftrightarrow F_\alpha^x \tilde{\in} (H, C) \setminus (F_\delta, A_\delta), \forall \alpha \in \Delta \Leftrightarrow F_\alpha^x \tilde{\in} \tilde{\bigcap}_{\delta\in\Delta} [(H, C) \setminus (F_\delta, A_\delta)]$

ii) $F_\alpha^x \tilde{\in} [(H, C) \setminus \tilde{\bigcap}_{\delta\in\Delta} (F_\delta, A_\delta)] \Leftrightarrow F_\alpha^x \tilde{\in} (H, C)$ and $F_\alpha^x \tilde{\notin} \tilde{\bigcap}_{\delta\in\Delta} (F_\delta, A_\delta) \Leftrightarrow F_\alpha^x \tilde{\in} (H, C)$ and $F_\alpha^x \tilde{\notin} (F_\delta, A_\delta)$ for some $\delta \in \Delta \Leftrightarrow F_\alpha^x \tilde{\in} (H, C) \setminus (F_\delta, A_\delta)$ for some $\delta \in \Delta \Leftrightarrow F_\alpha^x \tilde{\in} \tilde{\bigcup}_{\delta\in\Delta} [(H, C) \setminus (F_\delta, A_\delta)]$

□

4. Soft Topological Notions

In this section, using examples we discuss basic notions of soft topology and show some important results. We further introduced and defined some terms relative to arbitrary collection of soft sets in a soft topological space and studied some of their properties. We begin our investigation with the following definition.

Definition 4.1. [2] A *soft topology* over the universe X is collection τ of members of $SS(X)_E$ satisfying the following conditions:

1) $\tilde{X}, \tilde{\emptyset} \in \tau$ i.e. $\exists (F_1, E), (F_2, E) \in \tau$ such that $F_1(\alpha) = X, F_2(\alpha) = \emptyset, \forall \alpha \in E$.

2) If $(F_1, A_1), (F_2, A_2) \in \tau$, then $(F_1, A_1) \tilde{\cap} (F_2, A_2) \in \tau$.

3) If $\{(F_\delta, A_\delta) : \delta \in \Delta\}$ is any number of family of members of τ, then $\tilde{\bigcup}_{\delta \in \Delta} (F_\delta, A_\delta) \tilde{\in} \tau$.

The triplet (X, τ, E) is called a *soft topological space*. Members of τ are referred to as *soft open sets* and $(F, A) \in SS(X)_E$ is said to be a *soft closed* in (X, τ, E) if $(F^c, A) \in \tau$. It is clear from the definition that; inductively any finite intersection of members of τ is in τ. Thus if $(F_\delta, A_\delta) \in \tau (\delta = 1, 2, \ldots, n)$ then $\tilde{\bigcap}_{\delta=1}^{n} (F_\delta, A_\delta) \in \tau$ and any number of union of members of τ is in τ. For brevity we will be using the term X_E for (X, τ, E). As a consequent of definition 4.1 we have the following lemma.

Lemma 4.2. Let X_E be a soft topological space, then

i) $\tilde{X}$ and $\tilde{\emptyset} \in \tau$ are closed in X_E. i.e. $(\tilde{X})^c = \tilde{\emptyset}$ and $(\tilde{\emptyset})^c = \tilde{X}$;

ii) If $\{(F_\delta, A_\delta) : \delta \in \Delta\}$ is any number of family of soft closed sets in X_E, then $(\tilde{\bigcap}_{\delta \in \Delta} (F_\delta, A_\delta))^c = \tilde{\bigcup}_{\delta \in \Delta} (F_\delta, A_\delta)^c = \tilde{\bigcup}_{\delta \in \Delta} (F_\delta^c, A_\delta) = (F, A) \in \tau \Rightarrow \tilde{\bigcap}_{\delta \in \Delta} (F_\delta, A_\delta)$ is closed in X_E i.e., intersection of any number of soft closed sets is a soft closed;

iii) If $\{(F_i, A_i) : i = 1, 2, \ldots, n\}$ is a family of soft closed sets in X_E, then $(\tilde{\bigcup}_{i=1}^{n} (F_i, A_i))^c = \tilde{\bigcap}_{i=1}^{n} (F_i, A_i)^c = \tilde{\bigcap}_{i=1}^{n} (F_i^c, A_i) = (F, A) \in \tau \Rightarrow \tilde{\bigcup}_{i=1}^{n} (F_i, A_i)$ is closed in X_E i.e., finite union of soft closed sets is soft closed.

Definition 4.3. $(F, A) \in SS(X)_E$ is said to be *soft clopen* in X_E if (F, A) is both soft closed and soft open in X_E. i.e., $\tilde{X}$ and $\tilde{\emptyset}$ are soft clopen

Definition 4.4. [5] Let X_E be a soft topological space, then $(F, A) \in SS(X)_E$ is said to be a *soft neighborhood* (for brevity: soft nbd) of $(H, C) \in SS(X)_E$ if $\exists (G, B) \in \tau$ such that $(H, C) \tilde{\subseteq} (G, B) \tilde{\subseteq} (F, A)$. Similarly $(F, A) \in SS(X)_E$ is said to be a soft nbd of the soft element F_α^x if $\exists (G, B) \in \tau$ such that $F_\alpha^x \tilde{\in} (G, B) \tilde{\subseteq} (F, A)$. The family of all soft nbd of the soft element F_α^x is called a soft nbd system of F_α^x denoted as $\mathbb{U}_{F_\alpha^x}$.

We now have the following proposition.

Proposition 4.5. $(F, A) \in SS(X)_E$ is soft open in X_E iff (F,A) is a soft nbd of each its soft subsets.

Proof. Let (F, A) be soft open and $(G, B) \tilde{\subseteq} (F, A)$. Then $(G, B) \tilde{\subseteq} (F, A) \tilde{\subseteq} (F, A)$. Hence by definition (F, A) is a nbd of (G, B).
Conversely suppose (F, A) is a soft nbd of each of its soft subsets. This implies $\forall (G_\alpha, B_\alpha) \tilde{\subseteq} (F, A)$ such that $\alpha \in \Delta$ there exist a soft open set (F_α, A_α) such that $(G_\alpha, B_\alpha) \tilde{\subseteq} (F_\alpha, A_\alpha) \tilde{\subseteq} (F, A)$.
Let $(G, B) = \tilde{\bigcup}_{\alpha \in \Delta} (F_\alpha, A_\alpha)$, then (G, B) is soft open and $(G, B) \tilde{\subseteq} (F, A)$. By our hypothesis if (H, C) is any soft open subset of $(F, A), \exists (F_\alpha, A_\alpha) \tilde{\subseteq} (F, A)$ such that $(H, C) \tilde{\subseteq} (F_\alpha, A_\alpha)$.
This implies $(H, C) \tilde{\subseteq} (F_\alpha, A_\alpha) \tilde{\subseteq} \tilde{\bigcup}_{\alpha \in \Delta} (F_\alpha, A_\alpha) = (G, B)$. This implies $(F, A) \tilde{\subseteq} (G, B)$.
Hence, $(F, A) = (G, B)$ which is soft open. □

As consequence of the above proposition, we have the following corollaries.

Corollary 4.6. [5] $(F, A) \in SS(X)_E$ is soft open in X_E iff (F, A) is a nbd of each its soft elements.

Corollary 4.7. [2] $(F, A) \in SS(X)_E$ is soft open in X_E iff (F, A) is a nbd of each its soft points.

Now we give the following propositions.

Proposition 4.8. $(F, A) \in SS(X)_E$ is soft closed in X_E iff given any soft element F^x_α such that $F^x_\alpha \tilde{\notin} (F, A)$ there exists a soft nbd (G, B) of F^x_α such that $(G, B)\tilde{\cap}(F, A) = \tilde{\emptyset}$.

Proof. (F, A) is soft closed and $F^x_\alpha \tilde{\notin}(F, A) \Rightarrow F^x_\alpha \tilde{\in}(F^c, A)$. Now $(F, A)\tilde{\cap}(F^c, A) = \tilde{\emptyset}$ and (F^c, A) is soft open.

Conversely suppose $\forall, F^x_\alpha \tilde{\notin}(F, A), \exists(G_\delta, B_\delta)$ soft open such that $F^x_\alpha \tilde{\in}(G_\delta, B_\delta)$ and $(G_\delta, B_\delta)\tilde{\cap}(F, A) = \tilde{\emptyset}$. Let $(G, B) = \tilde{\bigcup}_{\delta\in\Delta}(G_\delta, B_\delta)$, then (G, B) is soft open and $(G^c, B) = (F, A)$. Hence, (F, A) is soft closed. □

Proposition 4.9. Let (X, τ, E) be a soft topological space and $\mathbb{U}_{F^x_\alpha}$ be a soft nbd system of a soft element F^x_α, then

i) $\forall(G, A) \in \mathbb{U}_{F^x_\alpha}, F^x_\alpha \tilde{\in}(G, A)$

ii) If $(G, A) \in \mathbb{U}_{F^x_\alpha}$ and $(G, A)\tilde{\subseteq}(H, B)$, then $(H, B) \in \mathbb{U}_{F^x_\alpha}$

iii) If $(G_i, A_i) \in \mathbb{U}_{F^x_\alpha}$, than $\tilde{\bigcap}_{i=1}^{n}(G_i, A_i) \in \mathbb{U}_{F^x_\alpha}$

iv) $\forall(G, A) \in \mathbb{U}_{F^x_\alpha}, \exists(H, B) \in \mathbb{U}_{F^x_\alpha}$ such that $(G, A) \in \mathbb{U}_{H^y_\beta}, \forall H^y_\beta \tilde{\in}(H, B)$

Conversely, if given a collection $\mathbb{U}$ of members of $SS(X)_E$ and for each $F^x_\alpha \in \mathbb{U}$, there exist a nonempty family $\mathbb{U}_{F^x_\alpha}$ satisfying (i-iv), then there exists a unique soft topology on $\mathbb{U}$ such that $\mathbb{U}_{F^x_\alpha}$ is precisely the soft nbd system of F^x_α for each $F^x_\alpha \in \mathbb{U}$.

Proof.

i) Obvious from definition of $\mathbb{U}_{F^x_\alpha}$;

ii) $(G, A) \in \mathbb{U}_{F^x_\alpha}$ and $(G, A)\tilde{\subseteq}(H, B) \Rightarrow F^x_\alpha \tilde{\in}(G, A)\tilde{\subseteq}(H, B) \Rightarrow F^x_\alpha \tilde{\in}(H, B) \Rightarrow (H, B) \in \mathbb{U}_{F^x_\alpha}$;

iii) Let $(G_i, A_i) \in \mathbb{U}_{F^x_\alpha}, i = 1, 2, \ldots, n$. Then for each i, there exists an open soft set (H_i, B_i) such that $F^x_\alpha \tilde{\in}(H_i B_i)\tilde{\subseteq}(G_i, A_i)$. Hence, $F^x_\alpha \in \tilde{\bigcap}_{i=1}^{n}(H_i, B_i)\tilde{\subseteq}\tilde{\bigcap}_{i=1}^{n}(G_i, A_i)$. Since, $\tilde{\bigcap}_{i=1}^{n}(H_i, B_i)$ is soft open, then by definition $\tilde{\bigcap}_{i=1}^{n}(G_i, A_i) \in \mathbb{U}_{F^x_\alpha}$;

iv) $\forall(G, A) \in \mathbb{U}_{F^x_\alpha}, \exists(H, B)$ soft open such $F^x_\alpha \tilde{\in}(H, B)\tilde{\subseteq}(G, A)$. Since (H,B) is soft open, then $(H, B) \in \mathbb{U}_{H^y_\beta}, \forall H^y_\beta \in (H, B)$. Also $(H, B)\tilde{\subseteq}(G, A) \Rightarrow (G, A) \in \mathbb{U}_{H^y_\beta}, \forall H^y_\beta \in (H, B)$.

Conversely, suppose given a collection $\mathbb{U}$ of members of $SS(X)_E$ and for each $F^x_\alpha \in \mathbb{U}$, there exist a nonempty family $\mathbb{U}_{F^x_\alpha}$ satisfying (i-iv). Let $\tau(\mathbb{U}) = \{(G, A) \in \mathbb{U} : (G, A) \in \mathbb{U}_{F^x_\alpha}\}$ together with $\tilde{\emptyset}$

i) By definition of $\tau(\mathbb{U}), \tilde{\emptyset} \in \tau(\mathbb{U})$ and $F^x_\alpha \in \tilde{X} \Rightarrow \tilde{X} \in \tau(\mathbb{U})$;

ii) Let $(G_i, A_i) \in \mathbb{U}_{F^x_\alpha}, i = 1, 2, \ldots, n$ be a family of members of $\tau(\mathbb{U})$, then by definition of $\tau(\mathbb{U})$, $(G_i, A_i) \in \mathbb{U}_{F^x_\alpha}$ whenever $F^x_\alpha \in (G_i, A_i)$. By (iii) $\tilde{\bigcap}_{i=1}^{n}(G_i, A_i) \in \mathbb{U}_{F^x_\alpha}$. Hence, $\tilde{\bigcap}_{i=1}^{n}(G_i, A_i) \in \tau(\mathbb{U})$;

iii) Let $\{(G_\lambda, A_\lambda) : \lambda \in \Lambda\}$ be a family of members of $\tau(\mathbb{U})$, then $\forall\lambda \in \Lambda, (G_\lambda, A_\lambda) \in \mathbb{U}_{F^x_\alpha}$ whenever $F^x_\alpha \in (G_\lambda, A_\lambda)$. Therefore, $F^x_\alpha \in (G_\lambda, A_\lambda)\tilde{\subseteq}\tilde{\bigcup}_{\lambda\in\Lambda}(G_\lambda, A_\lambda) \in \tau(\mathbb{U})$.

□

Definition 4.10. [7] Let X_E be a soft topological space and $(F, A) \in SS(X)_E$. The *soft closure* of (F, A) denoted as $\overline{(F, A)}$ or $cl(F, A)$ is defined to be the soft intersection over all super soft closed sets of (F, A). Thus $\overline{(F, A)} = cl(F, A) = \tilde{\bigcap}\{(G, B) : (F, A)\tilde{\subseteq}(G, B)$ and (G, B) is soft closed$\}$

Thus $\overline{(F, A)} = cl(F, A) = \tilde{\bigcap}_{\alpha\in\Delta}(F_\alpha, A_\alpha)$ such that (F_α, A_α) is a soft closed, $\forall\alpha \in \Delta$. Where the soft intersection is taken over all soft closed supersets (F_α, A_α) of (F, A).

We next introduce the following definition

Definition 4.11. A soft element F_α^x is said to be a *closure soft element* of the soft set (F, A) if $F_\alpha^x \tilde{\in} \overline{(F, A)}$, and a soft set (G, B) is said to be a soft closure subset of (F, A) if $(G, B) \tilde{\subseteq} \overline{(F, A)}$

Thus, we have the following lemma.

Lemma 4.12. Let X_E be a soft topological space and $(F, A) \in SS(X)_E$, then

i) $(F, A) \tilde{\subseteq} \overline{(F, A)}$

ii) $\overline{(F, A)}$ is the smallest soft closed superset of (F, A).

iii) $(F, A) = \overline{(F, A)}$ if and only if (F, A) is a soft closed.

Proof. (i),(ii) and (iii) are trivially obvious from Definition 4.10 □

Lemma 4.13. A soft element F_α^x is a closure soft element of the soft set (F,A) if and only if given any soft open nbd (G,B) of $F_\alpha^x, (G, B) \tilde{\cap} (F, A) \neq \tilde{\emptyset}$

Proof. Let $F_\alpha^x \tilde{\in} \overline{(F, A)}$ and suppose by way of contradiction $G, B) \tilde{\cap} (F, A) = \tilde{\emptyset}$ for some soft open nbd (G, B) of F_α^x. This implies $\underline{(F, A)} \tilde{\subseteq} (G^c, B)$ where (G^c, B) is soft closed superset of (F, A). Now $(F, A) \tilde{\subseteq} (G^c, B) \Rightarrow F_\alpha^x \tilde{\in} \overline{(F, A)} \tilde{\subseteq} (G^c, B) \Rightarrow F_\alpha^x \tilde{\in} G^c, B) \Rightarrow F_\alpha^x \tilde{\notin} (G, B)$. This is a contradiction. Conversely, suppose the condition holds and by way of contradiction $F_\alpha^x \tilde{\notin} \overline{(F, A)}$. This implies that $F_\alpha^x \tilde{\in} (\overline{(F, A)})^c$. Since $(\overline{(F, A)})^c$ is soft open, then by our hypothesis $(\overline{(F, A)})^c \tilde{\cap} (F, A) \neq \tilde{\emptyset}$. This is a contradiction, i.e., $(\overline{(F, A)})^c \tilde{\subseteq} (F^c, A) \Rightarrow [(\overline{(F, A)})^c \tilde{\cap} (F^c, A)] \tilde{\subseteq} (F^c, A) \tilde{\cap} (F, A) = \tilde{\emptyset} \Rightarrow (\overline{(F, A)})^c \tilde{\subseteq} (F^c, A) = \tilde{\emptyset}$. □

We give the following example to demonstrate and make the notions discussed os far clearer.

Example 4.14. Let $E = \{e_1, e_2, e_3, e_4\}$, $X = \{x_1, x_2, x_3, x_4, x_5\}$, $A_1 = E$, $A_2 = \{e_1, e_2, e_3\}$, $A_3 = \{e_1, e_2, e_4\}$, $A_4 = \{e_1, e_3, e_4\}$, $A_5 = \{e_2, e_3, e_4\}$, $A_6 = \{e_1, e_4\}$, $A_7 = \{e_2, e_3\}$, $A_8 = \{e_1, e_2\}$, $A_9 = \{e_3, e_4\}$, $A_{10} = \{e_1, e_3\}$, $A_{11} = \{e_2, e_4\}$, $A_{12} = \{e_1\}$, $A_{13} = \{e_2\}, A_{14} = \{e_3\}$, $A_{15} = \{e_4\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, (F_i, E), (F_i, A_i) : i = 1, 2, \ldots, 15\}$. As indicated on the table below, all the soft closed sets are $\tilde{\emptyset}, \tilde{X}, (H_i, E), (H_i, A_i) : i = 1, 2, \ldots, 15$ where $(F_i^c, E) = (H_i, E)$ and $(F_i^c, A_i) = (H_i, A_i)$.

E	e_1	e_2	e_3	e_4	E	e_1	e_2	e_3	e_4
$\tilde{X}$	X	X	X	X	$\tilde{\emptyset}$	$\emptyset$	$\emptyset$	$\emptyset$	$\tilde{\emptyset}$
F_1	$\{x_1\}$	$\{x_3,x_4\}$	$\{x_1,x_3,x_4\}$	$\{x_2,x_3,x_4,x_5\}$	H_1	$\{x_2,x_3,x_4,x_5\}$	$\{x_1,x_2,x_5\}$	$\{x_2,x_5\}$	$\{x_1\}$
F_2	$\{x_1\}$	$\{x_3,x_4\}$	$\{x_1,x_3,x_4\}$	$\emptyset$	H_2	$\{x_2,x_3,x_4,x_5\}$	$\{x_1,x_2,x_5\}$	$\{x_2,x_5\}$	X
F_3	$\{x_1\}$	$\{x_3,x_4\}$	$\emptyset$	$\{x_2,x_3,x_4,x_5\}$	H_3	$\{x_2,x_3,x_4,x_5\}$	$\{x_1,x_2,x_5\}$	X	$\{x_1\}$
F_4	$\{x_1\}$	$\emptyset$	$\{x_1,x_3,x_4\}$	$\{x_2,x_3,x_4,x_5\}$	H_4	$\{x_2,x_3,x_4,x_5\}$	X	$\{x_2,x_5\}$	$\{x_1\}$
F_5	$\emptyset$	$\{x_3,x_4\}$	$\{x_1,x_3,x_4\}$	$\{x_2,x_3,x_4,x_5\}$	H_5	X	$\{x_1,x_2,x_5\}$	$\{x_2,x_5\}$	$\{x_1\}$
F_6	$\{x_1\}$	$\emptyset$	$\emptyset$	$\{x_2,x_3,x_4,x_5\}$	H_6	$\{x_2,x_3,x_4,x_5\}$	X	X	$\{x_1\}$
F_7	$\emptyset$	$\{x_3,x_4\}$	$\{x_1,x_3,x_4\}$	$\emptyset$	H_7	X	$\{x_1,x_2,x_5\}$	$\{x_2,x_5\}$	X
F_8	$\{x_1\}$	$\{x_3,x_4\}$	$\emptyset$	$\emptyset$	H_8	$\{x_2,x_3,x_4,x_5\}$	$\{x_1,x_2,x_5\}$	X	X
F_9	$\emptyset$	$\emptyset$	$\{x_1,x_3,x_4\}$	$\{x_2,x_3,x_4,x_5\}$	H_9	X	X	$\{x_2,x_5\}$	$\{x_1\}$
F_{10}	$\{x_1\}$	$\emptyset$	$\{x_1,x_3,x_4\}$	$\emptyset$	H_{10}	$\{x_2,x_3,x_4,x_5\}$	X	$\{x_2,x_5\}$	X
F_{11}	$\emptyset$	$\{x_3,x_4\}$	$\emptyset$	$\{x_2,x_3,x_4,x_5\}$	H_{11}	X	$\{x_1,x_2,x_5\}$	X	$\{x_1\}$
F_{12}	$\{x_1\}$	$\emptyset$	$\emptyset$	$\emptyset$	H_{12}	$\{x_2,x_3,x_4,x_5\}$	X	X	X
F_{13}	$\emptyset$	$\{x_3,x_4\}$	$\emptyset$	$\emptyset$	H_{13}	X	$\{x_1,x_2,x_5\}$	X	X
F_{14}	$\emptyset$	$\emptyset$	$\{x_1,x_3,x_4\}$	$\emptyset$	H_{14}	X	X	$\{x_2,x_5\}$	X
F_{15}	$\emptyset$	$\emptyset$	$\emptyset$	$\{x_2,x_3,x_4,x_5\}$	H_{15}	X	X	X	$\{x_1\}$
$\tilde{\emptyset}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\tilde{X}$	X	X	X	X

i) If $(G_1, E) = \{(e_1, \{x_1, x_3, x_5\}), (e_2, \{x_1, x_2, x_3, x_4\}), (e_3, \emptyset), (e_4, X)\}$ and $(G_2, E) = \{(e_1, \emptyset), (e_2, \{x_3, x_4\}), (e_3, \emptyset), (e_4, \{x_3\})\}$ then (G_1, E) is a soft nbd of (G_2, E) i.e., $(G_2, E) \tilde{\subseteq} (F_{11}, E) \tilde{\subseteq} (G_1, E)$ where (F_{11}, E) is soft open;

ii) If $(G_3, E) = \{(e_1, \{x_1\}), (e_2, \{x_2\}), (e_3, \{x_5\}), (e_4, \{x_5\})\}$, then $\overline{(G_3, E)} = (H_7, E) \tilde{\cap} (H_{13}, E) \tilde{\cap} (H_{14}, E) \tilde{\cap} \tilde{X}$ $= \{(e_1, X\}), (e_2, \{x_1, x_2, x_5\}), (e_3, \{x_2, x_5\}), (e_4, X)\} = (H_7, E)$ which is soft closed;

iii) $F_{e_1}^{x_5}\tilde{\notin}(G_3, E)$ but $F_{e_1}^{x_5}$ is a closure soft element of (G_3, E) i.e., $F_{e_1}^{x_5}\tilde{\in}\overline{(G_3, E)}$.

Definition 4.15. [7] Let X_E be a soft topological space and $(F, A) \in SS(X)_E$. The *soft interior* of (F, A) denoted as $int(F, A)$ is defined to be the soft union of all soft open subsets of (F, A).
$int(F, A) = \tilde{\bigcup}\{(G, B) : (G, B)\tilde{\subseteq}(F, A)$ and (G, B) is soft open $\}$. Thus $int(F, A) = \tilde{\bigcup}_{\alpha\in\Delta} (F_\alpha, A_\alpha)$ such that (F_α, A_α) is soft open,$\forall\alpha \in \Delta$. Where the soft union is taken over all soft open subsets (F_α, A_α) of (F, A).

Definition 4.16. A soft element F_α^x is said to be an *interior soft element* of the soft set (F, A) if $F_\alpha^x\tilde{\in}int(F, A)$.

Lemma 4.17. Let X_E be a soft topological space and $(F, A) \in SS(X)_E$, then

i) $int(F, A)$ is soft open and $int(F, A)\tilde{\subseteq}(F, A)$.

ii) $int(F, A) = (F, A)$ if and only if (F, A) is soft open.

iii) For any soft open subset (G, B) of (F, A), $(G, B)\tilde{\subseteq}int(F, A)\tilde{\subseteq}(F, A)$ i.e., $int(F, A)$ is the largest soft open subset of (F, A).

Proof. (i),(ii) and (iii) are trivially obvious from Definition 4.15 □

Example 4.18. In example 4.14. If $(G_4, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_4\}), (e_3, \emptyset), (e_4, \{x_1, x_2, x_3\})\}$. Then, $int(G_4, A_3) = \{(e_1, \{x_1\}), (e_2, \emptyset), (e_3, \emptyset), (e_4, \emptyset)\} = (F_{12}, E)$, which is soft open.

Definition 4.19. Let X_E be a soft topological space and $(F, A) \in SS(X)_E$. The *soft exterior* of (F, A) denoted as $ext(F, A)$ is defined to be the soft union over all soft open sets disjoint from (F, A). That is $ext(F, A) = \tilde{\bigcup}\{(G, B) : (G, B)$ is soft open and $(G, B)\tilde{\cap}(F, A) = \tilde{\emptyset}\}$.
Thus $ext(F, A) = \tilde{\bigcup}_{\alpha\in\Delta} (F_\alpha, A_\alpha)$ such that (F_α, A_α), is soft open $\forall\alpha \in \Delta$ and the soft union is taken over all soft open sets (F_α, A_α) such that $(F_\alpha, A_\alpha)\tilde{\cap}(F, A) = \tilde{\emptyset}$.

Lemma 4.20. Let X_E be a soft topological space and $(F, A) \in SS(X)_E$, then

i) $ext(F, A)$ is soft open and $ext(F, A)\tilde{\cap}int(F, A) = \tilde{\emptyset}$.

ii) $ext(F, A) = int(F^c, A)\tilde{\subseteq}(F^c, A)$

iii) If (G, B) is soft open and $(G, B)\tilde{\cap}(F, A) = \tilde{\emptyset}$, then $(G, B)\tilde{\subseteq}ext(F, A)$ i.e. $ext(F, A)$ is the largest soft open set disjoint from (F, A).

Proof. (i),(ii) and (iii) are trivially obvious from Definition 2.4 and 4.19. □

Example 4.21. In Example 4.14. If $(G_4, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_4\}), (e_3, \emptyset), (e_4, \{x_1, x_2, x_3\})\}$.
Then $ext(G_4, E) = \{(e_1, \emptyset), (e_2, \emptyset), (e_3, \{x_1, x_3, x_4\}), (e_4, \emptyset)\} = (F_{14}, E)$ which is soft open.
By (i) of lemma 4.20 $ext(G_4, E)\tilde{\cap}int(G_4, E) = (F_{14}, E)\tilde{\cap}(F_{12}, E) = \tilde{\emptyset}$.

We further introduce the following definition.

Definition 4.22. Let X_E be a soft topological space. A soft element F_α^x in X_E is said to be a *boundary soft element* of $(F, A) \in SS(X)_E$ if $F_\alpha^x\tilde{\notin}int(F, A)$ and $F_\alpha^x\tilde{\notin}ext(F, A)$. The soft union over of all soft boundary elements of (F, A) is called the *soft boundary* of (F, A) which we denote as $Fr(F, A)$.

Now as consequence of Definitions 4.10, 4.15, ref5b and 4.22 we provide the following two lemmas.

Lemma 4.23. By Definition 4.22 $int(F, A)\tilde{\cup}Fr(F, A)\tilde{\cup}(F, A) = \tilde{X}$

Lemma 4.24. Let X_E be a soft topological space and $(F, A) \in SS(X)_E$, then

i) $(\overline{(F, A)})^c = ext(F, A)$;

ii) $\overline{(F, A)} = int(F, A)\tilde{\cup}Fr(F, A)$;

iii) $\overline{(F, A)} = (int[(F^c, A)])^c$;

iv) $Fr(F,A) = \overline{(F,A)}\tilde{\cap}\overline{(F^c,A)}$;

v) $Fr(F,A) = ((int(F,A)\tilde{\cup}ext(F,A))^c$.

PROOF.

i) $F_\alpha^x\tilde{\in}(\overline{(F,A)})^c \Leftrightarrow F_\alpha^x\tilde{\notin}\overline{(F,A)} \Leftrightarrow$ there exists soft open nbd (G,B) of F_α^x such that $(F,A)\tilde{\cap}(G,B) = \tilde{\emptyset} \Leftrightarrow F_\alpha^x\tilde{\in}ext(F,A)$. Lemma 4.13 and Definition 4.19;

ii) By Lemma 4.23 $ext(F,A) = (int(F,A)\tilde{\cup}Fr(F,A))^c$. By (i) $(\overline{(F,A)})^c = (int(F,A)\tilde{\cup}Fr(F,A))^c$. Hence, $\overline{(F,A)} = int(F,A)\tilde{\cup}Fr(F,A)$.

iii) $F_\alpha^x\tilde{\in}\overline{(F,A)} \Leftrightarrow F_\alpha^x\tilde{\in}(int(F,A)\tilde{\cup}Fr(F,A)) \Leftrightarrow F_\alpha^x\tilde{\in}int(F,A)$ or $F_\alpha^x\tilde{\in}Fr(F,A) \Leftrightarrow F_\alpha^x\tilde{\notin}ext(F,A) \Leftrightarrow F_\alpha^x\tilde{\notin}int(F^c,A) \Leftrightarrow F_\alpha^x\tilde{\in}(int[(F^c,A)])^c$

iv) $F_\alpha^x\tilde{\in}\overline{(F,A)}\tilde{\cap}\overline{(F^c,A)} \Leftrightarrow F_\alpha^x\tilde{\in}\overline{(F,A)}$ and $F_\alpha^x\tilde{\in}\overline{(F^c,A)}$
$\Leftrightarrow$ given any open nbd (G,B) of $F_\alpha^x, (G,B)\tilde{\cap}(F,A) \neq \tilde{\emptyset}$ and $(G,B)\tilde{\cap}(F^c,A) \neq \tilde{\emptyset}$
$\Leftrightarrow F_\alpha^x\tilde{\notin}int(F,A)$ and $F_\alpha^x\tilde{\notin}int(F^c,A) \Leftrightarrow F_\alpha^x\tilde{\notin}int(F,A)$ and $F_\alpha^x\tilde{\notin}ext(F,A) \Leftrightarrow F_\alpha^x\tilde{\in}Fr(F,A)$

v) $F_\alpha^x\tilde{\in}Fr(F,A) \Leftrightarrow F_\alpha^x\tilde{\notin}int(F,A)$ and $F_\alpha^x\tilde{\notin}ext(F,A) \Leftrightarrow F_\alpha^x\tilde{\notin}(int(F,A)\tilde{\cup}ext(F,A))$
$\Leftrightarrow F_\alpha^x\tilde{\in}(int(F,A)\tilde{\cup}ext(F,A))^c$

□

Remark 4.25. It is obvious from (iv) and (v) that, boundary of (F,A) is soft closed in X_E

Example 4.26. In Example 4.14. If $(G_4,E) = \{(e_1,\{x_1,x_2\}),(e_2,\{x_4\}),(e_3,\emptyset),(e_4,\{x_1,x_2,x_3\})\}$.

i) By Definition4.10, $\overline{(G_4,E)} = \tilde{X}\tilde{\cap}(H_{14},E) = (H_{14},E) = \{(e_1,X\}),(e_2,X),(e_3,\{x_2,x_5\}),(e_4,X)\}$ which is soft closed and $(G_4,E)\tilde{\subseteq}(H_{14},E)$.

ii) By (i) of Lemma 4.24, $(\overline{(G_4,E)})^c = \{(e_1,X),(e_2,X),(e_3,\{x_2,x_5\}),(e_4,X)\}^c = \{(e_1,\emptyset),(e_2,\emptyset),(e_3,\{x_1,x_3,x_4\}),(e_4,\emptyset)\} = ext(G_4,E)$

iii) By (ii) of Lemma 4.24, $\overline{(G_4,E)} = int(G_4,E)\tilde{\cup}Fr(G_4,E) = (F_{12},E)\tilde{\cup}(H_{10},E) = (H_{14},E)$.

iv) By (iii) of Lemma 4.24, $\overline{(G,E)} = (int[(G_4^c,E)])^c = (int[\{(e_1,\{x_3,x_4,x_5\}),(e_2,\{x_1,x_2,x_3,x_5\}),(e_3,X),(e_4,\{x_4,x_5\})\}])^c = (F_{14},E)^c = \{(e_1,X),(e_2,X),(e_3,\{x_2,x_5\}),(e_4,\{x_4,X)\} = (H_{14},E)$ which is soft closed.

v) By (iv) of Lemma 4.24, $Fr(G_4,E) = \overline{(G_4,E)}\tilde{\cap}\overline{(G_4^c,E)} = (H_{10},E)$ which is soft closed.

vi) By (v) Lemma of 4.24, $Fr(G_4,E) = ((int(G_4,E)\tilde{\cup}ext(G_4,E))^c = [(F_{12},E)\tilde{\cup}(F_{14},E)]^c = \{(e_1,\{x_2,x_3,x_4,x_5\}),(e_2,X),(e_3,\{x_2,x_5\}),(e_4,X)\} = (H_{10},E)$.

vii) By Definition4.10, The soft elements

$$F_{e_1}^{x_2},F_{e_1}^{x_3},F_{e_1}^{x_4},F_{e_1}^{x_5},F_{e_2}^{x_1},F_{e_2}^{x_2},F_{e_2}^{x_3},F_{e_2}^{x_4},F_{e_2}^{x_5},F_{e_3}^{x_2},F_{e_3}^{x_5},F_{e_4}^{x_1},F_{e_4}^{x_2},F_{e_4}^{x_3},F_{e_4}^{x_4}\tilde{\notin}Int(G_4,E)$$

and

$$F_{e_1}^{x_2},F_{e_1}^{x_3},F_{e_1}^{x_4},F_{e_1}^{x_5},F_{e_2}^{x_1},F_{e_2}^{x_2},F_{e_2}^{x_3},F_{e_2}^{x_4},F_{e_2}^{x_5},F_{e_3}^{x_2},F_{e_3}^{x_5},F_{e_4}^{x_1},F_{e_4}^{x_2},F_{e_4}^{x_3},F_{e_4}^{x_4}\tilde{\notin}ext(G_4,E)$$

$$\begin{aligned} Fr(G_4,E) &= F_{e_1}^{x2}\tilde{\cup}F_{e_1}^{x3}\tilde{\cup}F_{e_1}^{x4}\tilde{\cup}F_{e_1}^{x5}\tilde{\cup}F_{e_2}^{x1}\tilde{\cup}F_{e_2}^{x2}\tilde{\cup}F_{e_2}^{x3}\tilde{\cup}F_{e_2}^{x4}\tilde{\cup}F_{e_2}^{x5}\tilde{\cup}F_{e_3}^{x2}\tilde{\cup}F_{e_3}^{x5}\tilde{\cup}F_{e_4}^{x1}\tilde{\cup}F_{e_4}^{x2}\tilde{\cup}F_{e_4}^{x3}\tilde{\cup}F_{e_4}^{x4} \\ &= \{(e_1,\{x_2,x_3,x_4,x_5\}),(e_2,X),(e_3,\{x_2,x_5\}),(e_4,X)\} = (H_{10},E) \end{aligned}$$

The following definition which describe the derived soft set of a soft set is found in [19] and is given as

Definition 4.27. [5] A soft element F_α^x in X_E is said to be a limiting soft element of (F,A) if given any soft open nbd (G,A) of $F_\alpha^x, ((G,B)\setminus F_\alpha^x)\tilde{\cap}(F,A) \neq \tilde{\emptyset}$. The set of all limiting soft elements of (F,A) denoted as $(F,A)'$ is called the derived soft set of (F,A).

Thus, we give the following lemmas and their proofs.

Lemma 4.28. F^x_α is a limiting soft element of $(F,A) \in SS(X)_E$ if and only if $F^x_\alpha \tilde{\in} \overline{((F,A) \setminus F^x_\alpha)}$.

PROOF. $F^x_\alpha \tilde{\in} (F,A)' \Leftrightarrow \forall$soft open nbd (G,B) of $F^x_\alpha, ((G,B) \setminus F^x_\alpha)) \tilde{\cap} (F,A) \neq \tilde{\emptyset}$
$\Leftrightarrow \forall$soft open nbd (G,B) of $F^x_\alpha, (G,B) \tilde{\cap} ((F,A) \setminus F^x_\alpha)) \neq \tilde{\emptyset}$. Applying Lemma 4.13 $F^x_\alpha \tilde{\in} \overline{((F,A) \setminus F^x_\alpha)}$. □

Lemma 4.29. If $\{(F_\delta, A_\delta) : \delta \in \Delta\}$ is a family of members of $SS(X)_E$, then

$$\tilde{\bigcup_{\delta \in \Delta}} (F_\delta, A_\delta)' \tilde{\subseteq} (\tilde{\bigcup_{\delta \in \Delta}} (F_\delta, A_\delta))'$$

PROOF. $F^x_\alpha \tilde{\in} \tilde{\bigcup}_{\delta \in \Delta} (F_\delta, A_\delta)' \Rightarrow$ for some $\delta_o \in \Delta, F^x_\alpha \tilde{\in} (F_{\delta_o}, A_{\delta_o})'$
$\Rightarrow$ given any soft open nbd (G,B) of $F^x_\alpha, ((G,B) \setminus F^x_\alpha) \tilde{\cap} (F_{\delta_o}, A_{\delta_o}) \neq \tilde{\emptyset}$
$\Rightarrow$ given any soft open nbd (G,B) of $F^x_\alpha, ((G,B) \setminus F^x_\alpha) \tilde{\cap} [\tilde{\bigcup}_{\delta \in \Delta} (F_\delta, A_\delta)] \neq \tilde{\emptyset} \Rightarrow F^x_\alpha \tilde{\in} (\tilde{\bigcup}_{\delta \in \Delta} (F_\delta, A_\delta))'$ □

Lemma 4.30. $\tilde{\bigcup}_{\delta \in \Delta} \overline{(F_\delta, A_\delta)} \tilde{\subseteq} \overline{(\tilde{\bigcup}_{\delta \in \Delta} (F_\delta, A_\delta))}$ for any family $\{(F_\delta, A_\delta) : \delta \in \Delta\}$ of $SS(X)_E$.

PROOF. $F^x_\alpha \tilde{\in} \tilde{\bigcup}_{\delta \in \Delta} \overline{(F_\delta, A_\delta)} \Rightarrow$ for some $\delta_o \in \Delta, F^x_\alpha \tilde{\in} \overline{(F_{\delta_o}, A_{\delta_o})} \Rightarrow$ given any soft open nbd (G,B) of $F^x_\alpha, (G,B) \tilde{\cap} (F_{\delta_o}, A_{\delta_o}) \neq \tilde{\emptyset} \Rightarrow$ given any soft open nbd (G,B) of

$$F^x_\alpha, (G,B) \tilde{\cap} [\tilde{\bigcup_{\delta \in \Delta}} (F_\delta, A_\delta)] \neq \tilde{\emptyset} \Rightarrow F^x_\alpha \tilde{\in} \overline{(\tilde{\bigcup_{\delta \in \Delta}} (F_\delta, A_\delta))}$$

□

Lemma 4.31. F^x_α is a boundary soft element of (F,A) if and only if given soft open nbd $(G.B)$ of $F^x_\alpha, (G,B) \tilde{\cap} (F,A) \neq \tilde{\emptyset}$. and $(G,B) \tilde{\cap} (F^c, A) \neq \tilde{\emptyset}$.

PROOF. $F^x_\alpha \tilde{\in} Fr(F,A) \Leftrightarrow F^x_\alpha \tilde{\in} \overline{(F,A)} \tilde{\cap} \overline{(F^c,A)} \Leftrightarrow F^x_\alpha \tilde{\in} \overline{(F,A)}$ and $F^x_\alpha \tilde{\in} \overline{(F^c,A)}$. Applying Lemma 4.13 given any soft open nbd $(G.B)$ of $F^x_\alpha, (G,B) \tilde{\cap} (F,A) \neq \tilde{\emptyset}$ and $(G,B) \tilde{\cap} (F^c,A) \neq \tilde{\emptyset}$. □

Definition 4.32. [29] Let X_E be a soft topological space and $(F,A), (G,B) \in SS(X)_E$, then (F,A) is said to be *soft dense* in (G,B) if $(F,A) \tilde{\subseteq} (G,B)$ and $(G,B) \tilde{\subseteq} \overline{(F,A)}$. A soft set $(F,A) \in SS(X)_E$ is said to be soft dense in X_E if $\overline{(F,A)} = \tilde{X}$.
By (i) of Example 4.26 (G_4, E) is soft dense in (H_{14}, E).

We now introduce the following definitions.

Definition 4.33. Let X_E be a soft topological space, $(F,A) \in SS(X)_E$ is said to be

i) *boundary soft set* in X_E if $int(F,A) = \tilde{\emptyset}$;

ii) *nowhere soft dense* in X_E if $int\overline{(F,A)} = \tilde{\emptyset}$;

iii) *relatively soft discrete* in X_E if for every soft element F^x_α in (F,A) there exist a soft nbd (G,B) of F^x_α such that $(G,B) \tilde{\cap} (F,A) = F^x_\alpha$;

iv) *a soft closed domain (or regularly soft closed)* if $(F,A) = \overline{int(F,A)}$;

v) a soft open domain (or regularly soft open) if $(F,A) = int\overline{(F,A)}$;

vi) soft perfect if $\overline{(F,A)} = (F,A) = (F,A)'$.

As a consequence of definitions 4.32 and 4.33, we provide and prove the following proposition.

Proposition 4.34. Let X_E be a soft topological space and $(F,A) \in SS(X)_E$, then the following are equivalent

i) (F,A) is soft dense in X_E.

ii) $\tilde{X}$ is the only soft closed superset of (F, A).

iii) for every non empty soft open set (G, B), $(G, B)\tilde{\cap}(F, A) \neq \tilde{\emptyset}$.

iv) (F^c, A) is a boundary soft set.

PROOF.

i) (i $\Rightarrow$ ii) (F, A) is soft dense in X_E and (G, B) is soft closed superset of (F, A), implies $\tilde{X} = \overline{(F, A)}\tilde{\subseteq}(G, B)\tilde{\subseteq}\tilde{X} = \overline{(F, A)} \Rightarrow (G, B) = \tilde{X}$

ii) (ii$\Rightarrow$iii) Suppose (ii) holds and suppose by way of contradiction there exist a soft open set $(G, B) \neq \tilde{\emptyset}$ such that $(G, B)\tilde{\cap}(F, A) = \tilde{\emptyset}$. This implies $(F, A)\tilde{\subseteq}(G^c, B)$ where (G^c, B) is soft closed. Therefore, by our hypothesis we have $(G^c, B) = \tilde{X}$, and this implies $(G, B) = \tilde{\emptyset}$. This is a contradiction.

iii) (iii $\Rightarrow$ iv) Suppose (iii) holds and suppose by way of contradiction $int(F^c, A) \neq \tilde{\emptyset}$. By our hypothesis, $int(F^c, A)\tilde{\cap}(F, A) \neq \tilde{\emptyset}$. But $int(F^c, A)\tilde{\subseteq}(F^c, A)$ and $(F^c, A)\tilde{\cap}(F, A) = \tilde{\emptyset}$ implies $int(F^c, A)\tilde{\cap}(F, A) = \tilde{\emptyset}$. This is a contradiction.

iv) (iv $\Rightarrow$ i) $int(F^c, A) = \tilde{\emptyset} \Rightarrow$ every non null soft open set contains a soft element of (F, A). Hence, given any soft element F^x_α, if (G, B) is soft open nbd of F^x_α, then $(G, B)\tilde{\cap}(F, A) \neq \tilde{\emptyset}$. This implies $\forall F^x_\alpha \tilde{\in} \tilde{X}, F^x_\alpha \tilde{\in} \overline{(F, A)} \Rightarrow \tilde{X}\tilde{\subseteq}\overline{(F, A)}$. Since $\overline{(F, A)}\tilde{\subseteq}\tilde{X}$, then $\overline{(F, A)} = \tilde{X}$. Therefore, (F, A) is soft dense in X_E.

□

4.1. $F_\sigma-$ Soft Set , $G_\sigma-$ Soft Set, $\sigma-$ Soft Locally Finite and σ- soft Discrete Collections

In this section we introduce the concepts of $F_\sigma-$ Soft Set , $G_\sigma-$ Soft Set, $\sigma-$ Soft Locally Finite and σ- soft Discrete Collections and prove some important results. We first introduce the following definitions.

Definition 4.35. : Let X_E be a soft topological space. The soft union of a countable number of soft closed sets is called an $F_\sigma-$ soft set and the soft intersection of a countable number of soft open sets is called a $G_\sigma-$ soft set. The soft compliment of an $F_\sigma-$ soft set is a $G_\sigma-$ soft set and conversely. The soft intersection of two $F_\sigma-$ soft sets is an $F_\sigma-$ soft set. Thus, if $(F, A) = \tilde{\bigcup}_{i=1}^{\infty}(F_i, A_i)$ and $(G, B) = \tilde{\bigcup}_{i=1}^{\infty}(G_i, B_i)$ where (F_i, A_i) and (G_i, B_i) are soft closed, then evidently the soft intersection of (F, A) and (G, B) is given to be $(F, A)\tilde{\cap}(G, B) = (\tilde{\bigcup}_{i=1}^{\infty}(F_i, A_i))\tilde{\cap}(\tilde{\bigcup}_{i=1}^{\infty}(G_i, B_i)) = \tilde{\bigcup}_{i=1}^{\infty}[(F_i, A_i\tilde{\cap}(G_i, B_i)]$, thus $(F, A)\tilde{\cap}(G, B)$ is an $F_\sigma-$ soft set. Similarly the soft union of two $G_\sigma-$ soft sets is a $G_\sigma-$ soft set, thus if $(F, A) = \tilde{\bigcap}_{i=1}^{\infty}(F_i, A_i)$ and $(G, B) = \tilde{\bigcap}_{i=1}^{\infty}(G_i, B_i)$ where (F_i, A_i) and (G_i, B_i) are soft open, then $(F, A)\tilde{\cup}(G, B) = (\tilde{\bigcap}_{i=1}^{\infty}(F_i, A_i))\tilde{\cup}(\tilde{\bigcap}_{i=1}^{\infty}(G_i, B_i)) = \tilde{\bigcap}_{i=1}^{\infty}[(F_i, A_i)\tilde{\cup}(G_i, B_i)]$ thus $(F, A)\tilde{\cup}(G, B)$ is a $G_\sigma-$ soft set. The soft union of a countable number of $F_\sigma-$ soft sets is an F_σ soft set and the soft intersection of a countable number of $(G_\sigma-)$ soft sets is a $(G_\sigma-$ soft set.

Definition 4.36. [30] A collection $\mathcal{F} = \{(F_\lambda, A_\lambda) : \lambda \in \Lambda\}$ of members of $SS(X)_E$ in a soft topological X_E is said to be soft *locally finite* if and only f for every soft element F^x_α in X_E there exists a soft open nbd (F, A) of F^x_α such that (F, A) intersects only finitely many members of $\mathcal{F}$.

Definition 4.37. A collection $\mathcal{F} = \{(F_\lambda, A_\lambda) : \lambda \in \Lambda\}$ of members of $SS(X)_E$ in a soft topological X_E is said to be soft:-

(i) *soft discrete* if and only if for every soft element F^x_α in X_E there exists a soft open nbd (F, A) of F^x_α such that (F, A) intersects at most one member of $\mathcal{F}$;

(ii) $\sigma-$ s*oft locally finite* (σ- *soft discrete*) if and only if $\mathcal{F} = \tilde{\bigcup}_{i=1}^{\infty} \mathcal{F}_i$ with each $\mathcal{F}_i$ soft locally finite (soft discrete) collection;

(iii) *soft point-finite* if and only iff every soft element F_α^x in X_E is contained only in finitely many members of $\mathcal{F}$;

(v) *soft closure preserving* if and only if every subcollection $\mathcal{B}$ of $\mathcal{F}$ is soft closure preserving. i.e., $\overline{\tilde{\bigcup}\{B; B \in \mathcal{B}\}} = \tilde{\bigcup}\{\overline{B} : B \in \mathcal{B}\}$;

(vi) σ- *soft closure preserving* if it is the soft union of a sequence of soft closure preserving subcollection.

We now give and prove the following lemmas.

Lemma 4.38. If $\mathcal{F} = \{(F_\delta, A_\delta) : \delta \in \Delta\}$ is a soft locally finite (soft discrete) collection of members of $SS(X)_E$, then $\{\overline{(F_\delta, A_\delta)} : \delta \in \Delta\}$ is soft locally finite (soft discrete)

Proof. Pick a soft element F_α^x and soft open nbd (G, B) of F_α^x such that $(G, B)\tilde{\cap}(F_\delta, A_\delta) = \tilde{\emptyset}$ except for finitely (discretely) many δ. Then $(G, B)\tilde{\cap}(F_\delta, A_\delta) = \tilde{\emptyset}$ except for these same δ. □

Lemma 4.39. If $\mathcal{F} = \{(F_\delta, A_\delta) : \delta \in \Delta\}$ is a soft locally finite collection of members of $SS(X)_E$, then $\tilde{\bigcup}\overline{(F_\delta, A_\delta)} = \overline{\tilde{\bigcup}(F_\delta, A_\delta)}$. In particular, the union of a soft locally finite collection of soft closed sets is soft closed.

Proof. $\tilde{\bigcup}\overline{(F_\delta, A_\delta)} \tilde{\subseteq} \overline{\tilde{\bigcup}(F_\delta, A_\delta)}$ follows from Lemma 4.30. Suppose $F_\alpha^x \tilde{\in} \overline{\tilde{\bigcup}(F_\delta, A_\delta)}$. Now some soft nbd (G, B) of F_α^x meets only finitely many members of $\mathcal{F}$, say $(F_{\delta_1}, A_{\delta_1}), (F_{\delta_2}, A_{\delta_2}) \ldots, (F_{\delta_n}, A_{\delta_n})$. Since every soft nbd of F_α^x meets $\tilde{\bigcup}(F_\delta, A_\delta)$, then every soft nbd of F_α^x must also meet $\tilde{\bigcup}_{i=1}^{n}(F_{\delta_i}, A_{\delta_i})$. Therefore, it follows that $F_\alpha^x \tilde{\in} \overline{(F_{\delta_1}, A_{\delta_1})\tilde{\cup}(F_{\delta_2}, A_{\delta_2})\tilde{\cup} \ldots \tilde{\cup}(F_{\delta_n}, A_{\delta_n})} = \tilde{\bigcup}_{i=1}^{n}\overline{(F_{\delta_i}, A_{\delta_i})}$ so that, for some k, $F_\alpha^x \tilde{\in} \overline{(F_{\delta_k}, A_{\delta_k})}$. Thus $\overline{\tilde{\bigcup}(F_\delta, A_\delta)} \tilde{\subseteq} \tilde{\bigcup}\overline{(F_\delta, A_\delta)}$, and the result follows. □

Lemma 4.40. Let $\mathcal{F} = \{(F_\delta, A_\delta) : \delta \in \delta\}$ be a soft locally finite collection of members of $SS(X)_E$ and $F = \bigcup_{\delta \in \delta}(F_\delta, A_\delta)$.

i) If all sets of the family $\mathcal{F}$ are soft closed, then F is soft closed;

ii) If the family $\mathcal{F}$ consists of soft clopen sets, then F is soft clopen.

Proof. i) If all the family of $\mathcal{F}$ are soft closed, then $(F_\delta, A_\delta) = \overline{(F_\delta, A_\delta)}, \forall \delta \in \delta$, Hence, by Lemma 4.39, $F = \tilde{\bigcup}_{\delta \in \Delta}(F_\delta, A_\delta) = \tilde{\bigcup}_{\delta \in \Delta}\overline{(F_\delta, A_\delta)} = \overline{\tilde{\bigcup}_{\delta \in \Delta}(F_\delta, A_\delta)}$ is soft closed;

ii) If the the family $\mathcal{F}$ consist of soft clopen sets, then (F_δ, A_δ) is soft closed $\forall \delta \in \delta$ and (F_δ, A_δ) is soft open $\forall \delta \in \delta$. Since, $F = \tilde{\bigcup}_{\delta \in \Delta}(F_\delta, A_\delta)$, then F is soft open (i.e. union of any number of soft open sets is soft open) and by (i) F is soft closed. Hence F is soft clopen.

□

5. Conclusion

In this paper, we have extended the notions of operation on soft sets to arbitrary collection of soft sets and introduced the concepts of $F_\sigma-$ soft Set and $G_\sigma-$ soft Set. Using examples, we have discussed basic notions of soft topology and showed some important results. We have further introduced some terms relative to arbitrary collection of soft sets in a soft topological space and studied some of their properties.

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