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# A New Subclass of Meromorphic Starlike Functions Defined by Certain Integral Operator

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**Abstract** − The aim of this paper is to introduce a new class $\sum\limits_p^*(\alpha,\beta,\sigma)$ of meromorphically starlike functions defined by certain integral operator in the unit disc $E=\{z \mid 0<|z|<1\}$ and investigate coefficients, distortion properties and radius of convexity for the class. Furthermore it is shown that the class $\sum\limits_p^*(\alpha,\beta,\sigma)$ is closed under convex linear combinations and integral transforms.

***Keywords*** − *Meromorphic, distortion, radius of convexity, integral transforms*

## 1. Introduction

Let $\Sigma$ be denote the class of all functions $f(z)$ of the form

$$f(z)=\frac{1}{z}+\sum_{n=1}^{\infty}a_n z^n \tag{1}$$

which are regular in $E=\{z: 0<|z|<1\}$, with a simple pole at the origin. Let $\sum\limits_s$, $\sum\limits^*(\alpha)$ and $\sum\limits_k(\alpha),(0\le\alpha<1)$ denote the subclasses of $\sum$ that are univalent, moromorphically starlike of order $\alpha$ and meromorphically convex of order $\alpha$ respectively. Analytically $f(z)$ of the form (1) is in $\sum\limits^*(\alpha)$ if and only if

$$Re\left\{-\frac{zf'(z)}{f(z)}\right\}>\alpha,(z\in E) \tag{2}$$

Similarly, $f\in\sum\limits_k(\alpha)$ if and only if $f(z)$ is of the form (1) and satisfies

$$Re\left\{-\left(1+\frac{zf''(z)}{f'(z)}\right)\right\}>\alpha,(z\in E) \tag{3}$$

It being understood that if $\alpha=1$ then $f(z)=\frac{1}{z}$ is the only function which is $\sum\limits^*(1)$ and $\sum\limits_k(1)$.

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The classes $\overset{*}{\sum}(\alpha)$ and $\sum\limits_{k}(\alpha)$ have been extensively studied by Pommerenke [1], Clunie [2], Royster [3] and others. Recently the integral operator of $f(z)$ in $\sum\limits_{s}$ for $\sigma > 0$ is denoted by $I^{\sigma}$ and defined as following

$$I^{\sigma} f(z) = \frac{1}{z^2\Gamma(\sigma)} \int\limits_0^z \left(\log \frac{z}{t}\right)^{\sigma-1} tf(t)dt \tag{4}$$

That is defined by Jung et al. [4]. It is easy to verify that if $f(z)$ is of the form (1), then

$$I^{\sigma} f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \left(\frac{1}{n+2}\right)^{\sigma} a_n z^n \tag{5}$$

The aim of the present paper is to introduce the class of meromorphically starlike functions which we denote by $\overset{*}{\sum}(\alpha,\beta,\sigma)$ for some $\alpha(0 \le \alpha < 1)$, $\beta(0 < \beta \le 1)$ and $\sigma > 0$. We then consider the class $\overset{*}{\sum\limits_{p}}(\alpha,\beta,\sigma) = \sum\limits_{p} \cap \overset{*}{\sum}(\alpha,\beta,\sigma)$ and extend some of the results of Juneja et al. [5] to this class. We obtain coefficient estimates, distortion properties and radius of convexity for the class. Furthermore it is shown that the class $\overset{*}{\sum\limits_{p}}(\alpha,\beta,\sigma)$ is closed under convex linear combinations and integral transforms.

**Definition 1.1.** Let the function $f(z)$ be defined by (1). Then $f(z) \in \overset{*}{\sum}(\alpha,\beta,\sigma)$ if and only if

$$\left|\frac{z[I^{\sigma} f(z)]'}{I^{\sigma} f(z)} + 1\right| < \beta \left|\frac{z[I^{\sigma} f(z)]'}{I^{\sigma} f(z)} + 2\alpha - 1\right|,$$

for some $\alpha(0 \le \alpha < 1)$, $\beta(0 < \beta \le 1)$, $\sigma > 0$ and for all $z \in E$.

## 2. Coefficient estimates

In this section we obtain a sufficient condition for a function to be in $\overset{*}{\sum}(\alpha,\beta,\sigma)$.

**Theorem 2.1.** Let $f(z) = \frac{1}{z} + \sum\limits_{n=1}^{\infty} a_n z^n$ be regular in $E$. If

$$\sum_{n=1}^{\infty} [(1+\beta)n + (2\alpha-1)\beta + 1] \left[\frac{1}{n+2}\right]^{\sigma} |a_n| \le 2\beta(1-\alpha) \tag{6}$$

for some $0 \le \alpha < 1, 0 < \beta \le 1$ and $\sigma > 0$ then $f(z) \in \overset{*}{\sum}(\alpha,\beta,\sigma)$.

Proof. Suppose (6) holds for all admissible values of $\alpha$ and $\beta$. Consider the expression

$$H(f,f') = \left|z[I^{\sigma} f(z)]' + [I^{\sigma} f(z)]\right| - \beta \left|z[I^{\sigma} f(z)]' + (2\alpha-1)[I^{\sigma} f(z)]\right| \tag{7}$$

The we have

$$H(f,f') \le \left|\sum_{n=1}^{\infty} (n+1) \left[\frac{1}{n+2}\right]^{\sigma} a_n z^n\right| - \beta \left|2(\ \alpha-1)\frac{1}{z} + \sum_{n=1}^{\infty} (n+2\alpha-1) \left[\frac{1}{n+2}\right]^{\sigma} a_n z^n\right|$$

$$\Rightarrow rH(f,f') = \sum_{n=1}^{\infty} (n+1) \left[\frac{1}{n+2}\right]^{\sigma} |a_n| r^{n+1} - \beta \left\{2(\alpha-1) - \sum_{n=1}^{\infty} (n+2\alpha-1) \left[\frac{1}{n+2}\right]^{\sigma} |a_n| r^{n+1}\right\}$$

$$= \sum_{n=1}^{\infty} [(1+\beta)n + (2\alpha-1)\beta + 1] \left[\frac{1}{n+2}\right]^{\sigma} |a_n| r^{n+1} - 2\beta(1-\alpha).$$

Since the above inequality holds for all $r, 0 < r < 1$, letting $r \to 1$, we have

$$H(f,f') \le \sum_{n=1}^{\infty} [(1+\beta)n + (2\alpha-1)\beta + 1] \left[\frac{1}{n+2}\right]^{\sigma} |a_n| - 2\beta(1-\alpha)$$

$$\le 0, \text{ by (6)}.$$

Hence it follows that $\left|\frac{z[I^\sigma f(z)]'}{I^\sigma f(z)}+1\right| < \beta\left|\frac{z[I^\sigma f(z)]'}{I^\sigma f(z)}+2\alpha-1\right|$.
So that $f(z) \in \sum^*(\alpha,\beta,\sigma)$. Hence the theorem. □

**Theorem 2.2.** Let the function $f(z)=\frac{1}{z}+\sum_{n=1}^{\infty} a_n z^n,\ a_n \geq 0$ be regular in $E$. Then $f(z) \in \sum_p^*(\alpha,\beta,\sigma)$ if and only if (6) is satisfied.

Proof. In view of Theorem 2.1, it is sufficient to show that only if part.
Let us assume that $f(z)=\frac{1}{z}+\sum_{n=1}^{\infty} a_n z^n,\ a_n \geq 0$ is in $\sum_p^*(\alpha,\beta,\sigma)$.
Then $\left|\frac{\frac{z[I^\sigma f(z)]'}{I^\sigma f(z)}+1}{\frac{z[I^\sigma f(z)]'}{I^\sigma f(z)}+2\alpha-1}\right| = \left|\frac{\sum_{n=1}^{\infty}(n+1)\left[\frac{1}{n+2}\right]^\sigma a_n z^n}{2(1-\alpha)\frac{1}{z}-\sum_{n=1}^{\infty}(n+2\alpha-1)\left[\frac{1}{n+2}\right]^\sigma a_n z^n}\right| < \beta$, for all $z \in E$.
Using the fact that $Re(z) \leq |z|$, if follows that

$$Re\left\{\frac{\sum_{n=1}^{\infty}(n+1)\left[\frac{1}{n+2}\right]^\sigma a_n z^n}{2(1-\alpha)\frac{1}{z}-\sum_{n=1}^{\infty}(n+2\alpha-1)\left[\frac{1}{n+2}\right]^\sigma a_n z^n}\right\} < \beta,\ z \in E \tag{8}$$

Now choose the values of $z$ on the real axis so that $\frac{z[I^\sigma f(z)]'}{I^\sigma f(z)}$ is real.
Upon clearing the denominator in (8) and letting $z \to 1$ through positive values,
we obtain $\sum_{n=1}^{\infty}(n+1)\left[\frac{1}{n+2}\right]^\sigma a_n \leq \beta\left\{2(1-\alpha)-\sum_{n=1}^{\infty}(n+2\alpha-1)\left[\frac{1}{n+2}\right]^\sigma a_n\right\}$
$\Rightarrow \sum_{n=1}^{\infty}[(1+\beta)n+(2\alpha-1)\beta+1]\left[\frac{1}{n+2}\right]^\sigma |a_n| \leq 2\beta(1-\alpha)$.
Hence the theorem. □

**Corollary 2.3.** If $f(z)=\frac{1}{z}+\sum_{n=1}^{\infty} a_n z^n,\ a_n \geq 0$ is in $\sum_p^*(\alpha,\beta,\sigma)$ then

$$a_n \leq \frac{2\beta(1-\alpha)(n+2)^\sigma}{(1+\beta)n+(2\alpha-1)\beta+1},\ n=1,2,\cdots \tag{9}$$

with equality for each $n$, for function of the form

$$f_n(z)=\frac{1}{z}+\frac{2\beta(1-\alpha)(n+2)^\sigma}{(1+\beta)n+(2\alpha-1)\beta+1}z^n,\ n=1,2,\cdots \tag{10}$$

If $\beta=1$ in the above theorem, we get the following result of Atshan et al. [6].

**Corollary 2.4.** If $f(z) \in \sum_p^*(\alpha,\beta,\sigma)$ then

$$a_n \leq \frac{(1-\alpha)(n+2)^\sigma}{n+\alpha},\ n=1,2,\cdots$$

The result is sharp for the functions $f_n(z)$ is given by

$$f_n(z)=\frac{1}{z}+\frac{(1-\alpha)(n+2)^\sigma}{n+\alpha}z^n,\ n=1,2,\cdots$$

## 3. Distortion properties and radius of convexity estimates

In this section we prove the Distortion Theorem and radius of convexity estimates for the class $\sum_p^*(\alpha,\beta,\sigma)$.

**Theorem 3.1.** Let $f(z) \in \sum_p^*(\alpha,\beta,\sigma)$. Then for $0<|z|=r<1$,

$$\frac{1}{r}-\frac{3^\sigma\beta(1-\alpha)}{1+\alpha\beta}r \leq |f(z)| \leq \frac{1}{r}+\frac{3^\sigma\beta(1-\alpha)}{1+\alpha\beta}r \tag{11}$$

with equality for the function

$$f(z)=\frac{1}{z}+\frac{3^\sigma\beta(1-\alpha)}{1+\alpha\beta}z, \text{ at } \ z=r, ir \tag{12}$$

PROOF. Suppose $f(z) \in \sum_p^*(\alpha,\beta,\sigma)$. In view of Theorem 2.2, we have

$$\sum_{n=1}^{\infty} a_n \leq \frac{3^\sigma\beta(1-\alpha)}{1+\alpha\beta} \tag{13}$$

Thus for $0<|z|=r<1$,

$$\begin{aligned}|f(z)| &= \left|\frac{1}{z}+\sum_{n=1}^{\infty}a_n z^n\right| \\ &\leq \frac{1}{|z|}+\sum_{n=1}^{\infty}a_n|z|^n \\ &\leq \frac{1}{r}+r\sum_{n=1}^{\infty}a_n \\ &\leq \frac{1}{r}+\frac{3^\sigma\beta(1-\alpha)}{1+\alpha\beta}, \quad \text{by (13)}\end{aligned}$$

This gives the right hand side of (11). Also,

$$\begin{aligned}|f(z)| &= \left|\frac{1}{z}-\sum_{n=1}^{\infty}a_n z^n\right| \\ &\geq \frac{1}{|z|}-\sum_{n=1}^{\infty}a_n|z|^n \\ &\geq \frac{1}{r}-r\sum_{n=1}^{\infty}a_n \\ &\geq \frac{1}{r}-\frac{3^\sigma\beta(1-\alpha)}{1+\alpha\beta}\end{aligned}$$

which gives the left hand side of (11) . □

**Theorem 3.2.** Let the function $f(z)$ be in $\sum_p^*(\alpha,\beta,\sigma)$. Then for $f(z)$ is meromorphically convex of order $\delta(0\leq\delta<1)$ in $|z|<r=r(\alpha,\beta,\sigma,\delta)$, where

$$r(\alpha,\beta,\sigma,\delta)=\inf_n\left\{\frac{(1-\delta)[(1+\beta)n+(2\alpha-1)\beta+1}{2\beta(1-\alpha)n(n+2-\delta)(n+2)^\sigma}\right\}^{\frac{1}{n+1}}, \quad n=1,2,\cdots \tag{14}$$

The bound for $|z|$ is sharp for each $n$ with the extremal function being of the form (10).

PROOF. Let $f(z) \in \sum_p^*(\alpha,\beta,\sigma)$. Then by Theorem 2.2

$$\sum_{n=1}^{\infty}\frac{(1+\beta)n+(2\alpha-1)\beta+1}{2\beta(1-\alpha)(n+2)^\sigma}\ a_n \leq 1 \tag{15}$$

In view of (3) , it is sufficient to show that

$$\left|2+\frac{zf''(z)}{f'(z)}\right| \leq 1-\delta, \quad \text{for} \quad |z|<r(\alpha,\beta,\sigma,\delta)$$

or equivalently to show that

$$\left|\frac{f'(z)+(zf'(z))'}{f'(z)}\right| \leq 1-\delta, \quad \text{for } |z|<r(\alpha,\beta,\sigma,\delta) \tag{16}$$

Substituting the series expansions for $f'(z)$ and $(zf'(z))'$ in the left hand side of (16) then we get

$$\left|\frac{\sum\limits_{n=1}^{\infty} n(n+1)a_n z^{n-1}}{-\frac{1}{z^2}+\sum\limits_{n=1}^{\infty} na_n z^{n-1}}\right| \leq \frac{\sum\limits_{n=1}^{\infty} n(n+1)a_n|z|^{n+1}}{1-\sum\limits_{n=1}^{\infty} na_n|z|^{n+1}}$$

This will be bounded by $(1-\delta)$ if

$$\sum_{n=1}^{\infty}\frac{n(n+2-\delta)}{1-\delta}a_n|z|^{n+1} \leq 1 \tag{17}$$

In view of (15), it follows that (17) is true if

$$\frac{n(n+2-\delta)}{1-\delta}|z|^{n+1} \leq \frac{(1+\beta)n+(2\alpha-1)\beta+1}{2\beta(1-\alpha)(n+2)^{\sigma}}, \ n=1,2,\cdots$$

$$\Rightarrow \ |z| \leq \left\{\frac{(1-\delta)[(1+\beta)n+(2\alpha-1)\beta+1]}{2\beta(1-\alpha)n(n+2-\delta)(n+2)^{\sigma}}\right\}^{\frac{1}{n+1}}, \ n=1,2,\cdots \tag{18}$$

Setting $|z|=r(\alpha,\beta,\sigma,\delta)$ in (18), the result follows.

The result is sharp, the extremal function being of the form

$$f_n(z)=\frac{1}{z}+\frac{2\beta(1-\alpha)(n+2)^{\sigma}}{(1+\beta)n+(2\alpha-1)\beta+1}z^n, \ n=1,2,\cdots$$

□

## 4. Convex linear combinations

In this section we prove that the class $\sum\limits_{p}^{*}(\alpha,\beta,\sigma)$ is closed under convex linear combinations.

**Theorem 4.1.** Let $f_0(z)=\frac{1}{z}$ and $f_n(z)=\frac{1}{z}++\frac{2\beta(1-\alpha)(n+2)^{\sigma}}{(1+\beta)n+(2\alpha-1)\beta+1}z^n, \ n=1,2,\cdots$. Then $f(z)\in\sum\limits_{p}^{*}(\alpha,\beta,\sigma)$ if and only if it can be expressed in the form $f(z)=\sum\limits_{n=0}^{\infty}\lambda_n f_n(z)$, where $\lambda_n\geq 0$ and $\sum\limits_{n=0}^{\infty}\lambda_n=1$.

PROOF. Let $f(z) = \sum_{n=0}^{\infty} \lambda_n f_n(z)$, where $\lambda_n \geq 0$ and $\sum_{n=0}^{\infty} \lambda_n = 1$. Then

$$\begin{aligned}
f(z) &= \sum_{n=0}^{\infty} \lambda_n f_n(z) = \lambda_0 f_0(z) + \sum_{n=1}^{\infty} \lambda_n f_n(z) \\
&= \left[1 - \sum_{n=1}^{\infty} \lambda_n\right] f_0(z) + \sum_{n=1}^{\infty} \lambda_n f_n(z) \\
&= \left[1 - \sum_{n=1}^{\infty} \lambda_n\right] \frac{1}{z} + \sum_{n=1}^{\infty} \lambda_n \left[\frac{1}{z} + \frac{2\beta(1-\alpha)(n+2)^\sigma}{(1+\beta)n + (2\alpha-1)\beta + 1} z^n\right] \\
&= \frac{1}{z} + \sum_{n=1}^{\infty} \lambda_n \frac{2\beta(1-\alpha)(n+2)^\sigma}{(1+\beta)n + (2\alpha-1)\beta + 1} z^n
\end{aligned}$$

$$\begin{aligned}
\text{Since } &\sum_{n=1}^{\infty} \left\{\frac{(1+\beta)n + (2\alpha-1)\beta + 1}{2\beta(1-\alpha)(n+2)^\sigma}\right\} \lambda_n \frac{2\beta(1-\alpha)(n+2)^\sigma}{(1+\beta)n + (2\alpha-1)\beta + 1} \\
&= \sum_{n=1}^{\infty} \lambda_n = 1 - \lambda_0 \leq 1
\end{aligned}$$

Therefore $f(z) \in \sum_p^*(\alpha, \beta, \sigma)$.

Conversely suppose that $f(z) \in \sum_p^*(\alpha, \beta, \sigma)$.

Since $a_n \leq \frac{2\beta(1-\alpha)(n+2)^\sigma}{(1+\beta)n+(2\alpha-1)\beta+1}$, $n = 1, 2, \cdots$.

Setting $\lambda_n = \frac{(1+\beta)n+(2\alpha-1)\beta+1}{2\beta(1-\alpha)(n+2)^\sigma} a_n$, $n = 1, 2, \cdots$ and $\lambda_0 = 1 - \sum_{n=0}^{\infty} \lambda_n$.

It follows that $f(z) = \sum_{n=0}^{\infty} \lambda_n f_n(z)$. This completes the proof of the theorem. □

**Theorem 4.2.** The class $\sum_p^*(\alpha, \beta, \sigma)$ is closed under convex linear combination.

PROOF. Let the function $F_k(z)$ be given by

$F_k(z) = \frac{1}{z} + \sum_{n=1}^{\infty} f_{n,k} z^n$, $k = 1, 2 \cdots, m$ be in the class $\sum_p^*(\alpha, \beta, \sigma)$.

Then it is enough to show that the function

$$H(z) = \lambda F_1(z) + (1-\lambda)F_2(z), \ (0 \leq \lambda \leq 1)$$

tt is also in the class $\sum_p^*(\alpha, \beta, \sigma)$. Since for $0 \leq \lambda \leq 1$,

$$H(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \left[\lambda f_{n,1} + (1-\lambda) f_{n,2}\right] z^n$$

We observe that

$$\begin{aligned}
&\sum_{n=1}^{\infty} [(1+\beta)n + (2\alpha-1)\beta + 1] \frac{1}{(n+2)^\sigma} [\lambda f_{n,1} + (1-\lambda) f_{n,2}] \\
&= \lambda \sum_{n=1}^{\infty} [(1+\beta)n + (2\alpha-1)\beta + 1] \frac{1}{(n+2)^\sigma} f_{n,1} \\
&\qquad + (1-\lambda) \sum_{n=1}^{\infty} [(1+\beta)n + (2\alpha-1)\beta + 1] \frac{1}{(n+2)^\sigma} f_{n,2} \\
&\leq \ 2\beta\lambda(1-\alpha) + (1-\lambda)2\beta(1-\alpha) = 2\beta(1-\alpha)
\end{aligned}$$

By Theorem 2.2, we have $H(z) \in \sum_p^*(\alpha, \beta, \sigma)$. □

## 5. Integral transforms

In this section, we consider integral transforms of functions in $\sum_p^*(\alpha, \beta, \sigma)$.

**Theorem 5.1.** If $f(z)$ is in $\sum_p^*(\alpha, \beta, \sigma)$ then the integral transforms

$$F_c(z) = c\int_0^1 u^c f(uz)du, \ 0 < c < \infty \tag{19}$$

$$\text{are in } \sum_p^*(\delta) \text{ , where } \delta = \delta(\alpha, \beta, \sigma, c) = \frac{(1+\alpha\beta)(c+2) - 3^\sigma \beta c(1-\alpha)}{(1+\alpha\beta)(c+2) + 3^\sigma \beta c(1-\alpha)} \tag{20}$$

The result is best possible for the function $f(z) = \frac{1}{z} + \frac{3^\sigma \beta(1-\alpha)}{(1+\alpha\beta)} z$.

Proof. Suppose $f(z) \in \sum_p^*(\alpha, \beta, \sigma)$. We have

$$F_c(z) = c\int_0^1 u^c f(uz)du = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{ca_n}{n+c+1} z^n$$

It is sufficient to show that

$$\sum_{n=1}^{\infty} \frac{(n+\delta)}{(1-\delta)} \frac{ca_n}{(n+c+1)} \leq 1 \tag{21}$$

Since $f(z) \in \sum_p^*(\alpha, \beta, \sigma)$, we have

$$\sum_{n=1}^{\infty} \frac{(1+\beta)n + (2\alpha-1)\beta + 1}{2\beta(1-\alpha)(n+2)^\sigma} a_n \leq 1 \tag{22}$$

Thus (21) will be satisfied if $\frac{(n+\delta)}{(1-\delta)} \frac{c}{(n+c+1)} \leq \frac{(1+\beta)n+(2\alpha-1)\beta+1}{2\beta(1-\alpha)(n+2)^\sigma}$, for each $n$

$$\Rightarrow \delta \leq \frac{[(1+\beta)n + (2\alpha-1)\beta + 1][n+c+1] - 2\beta(1-\alpha)nc(n+2)^\sigma}{[(1+\beta)n + (2\alpha-1)\beta + 1][n+c+1] + 2\beta(1-\alpha)nc(n+2)^\sigma} \tag{23}$$

Since the right hand side of (23) is an increasing function of $n$, putting $n = 1$ in (23), we get

$$\delta \leq \frac{(1+\alpha\beta)(c+2) - 3^\sigma \beta(1-\alpha)c}{(1+\alpha\beta)(c+2) + 3^\sigma \beta(1-\alpha)c}$$

Hence the theorem. □

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